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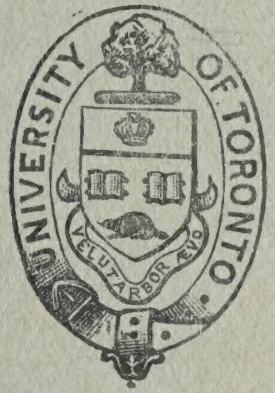

BULLETIN NO. 8

1928

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AN INVESTIGATION OF THE DRAFT, TEMPERATURE AND COMBUSTION CONDITIONS IN COMMERCIAL KILNS BURNING BUILDING BRICK

By H. P. Kimbell, M.A.

Introduction

Burning is without doubt the most important part of any ceramic manufacturing process. Without careful operation, large losses will occur in fuel, time, and first quality ware. Draft, temperature, and combustion conditions are the important features, and an attempt will be made in this report to determine these conditions in some brick kilns of Toronto plants.

Draft.

In order to burn ceramic ware, it is necessary that hot gases from the fuel burned should pass through the mass of ware set in a kiln, and be discharged at a point where they are not objectionable. This necessitates the establishment of a pressure difference between the grates and the point of exit of the gases. This difference must be sufficiently large to enable the gases to overcome the friction of the ware, floor, flues, and stack. The pressure difference is obtained in three ways:

(1) Forced draft.
(2) Induced draft.
(3) Natural draft.

Forced draft consists in creating a pressure greater than atmospheric at the grates, by means of a fan. Induced draft is produced by a fan at the base of the stack, which produces a pressure less than atmospheric by forcing the gases up the stack. Natural draft is simply the draft produced by a column of hot gases in a tall stack as opposed to the column of cold air on the outside.

The disadvantage of natural draft arises from the fact that during watersmoking, the period during which a strong draft is necessary to remove water vapor, a poor draft, or none at all, exists, by reason of the cold stack. This condition is offset by induced or forced draft. In natural draft it is often remedied to some degree by building a fire in the base of the stack. It is also aided by having more than one kiln to a stack. In this case, if there is a hot kiln on the stack, it will have heated the stack, and thus created a draft which can be immediately utilized by a kiln just started. This is not always in favour of the progress of the hot kiln, however.

In the watersmoking period, as aforementioned, it is desirable to have a pull throughout as much of the kiln as possible, in order to remove the water vapor, which might otherwise condense in the cooler part of the kiln. This is necessary, but to a lesser extent, in the oxidation period. After the oxidation period, it is advisable to have a very slight pressure in as much of the kiln as possible, to prevent the infiltration of cold air.[1]

Temperature.

Pyrometers are a valuable aid in burning. They indicate the rate of rise in temperature, which, if too great, may cause trouble in expelling the water too fast, and similar trouble in the oxidation period in some cases. Time-Temperature curves are used to advantage in this connection. Pyrometers also serve to indicate the finishing temperature, although it is recommended that pyrometric cones, or draw trials, or both, be used as an aid to them in this respect.

Occasionally, the "settle", or total amount of shrinkage of the ware in a kiln, is used as an indication of the finishing temperature. The correct amount of settle is determined by experience.

By the installation of thermocouples in the bottom of the kilns, too great a difference between top and bottom temperature is prevented. If the top temperature is rising too rapidly with respect to the bottom, soaking is necessary. A thermocouple in the flue outside of the kiln gives an indication of the length of the watersmoking period. While a large amount of water vapor is escaping through the flue, it will not register a temperature much higher than 212°F.; when most of the vapor is expelled, the temperature will begin to rise, and is not far below the bottom temperature of the kiln. A flue couple, with flue gas analysis, and a fuel record, will also indicate the sensible heat lost in the flue gases.

Combustion.

Combustion is simply a rapid oxidation, which causes an evolution of heat. Oxygen of the air is required to oxidize the combustibles of the fuel, the chief products of oxidation being carbon dioxide, water vapor, and the nitrogen brought in with the air. It is these hot products of combustion which do the work required.

The two great combustion losses are due either to an excess or deficiency of combustion air. If more air is used than is necessary for the combustion, the excess is needlessly heated and expelled at the stack.

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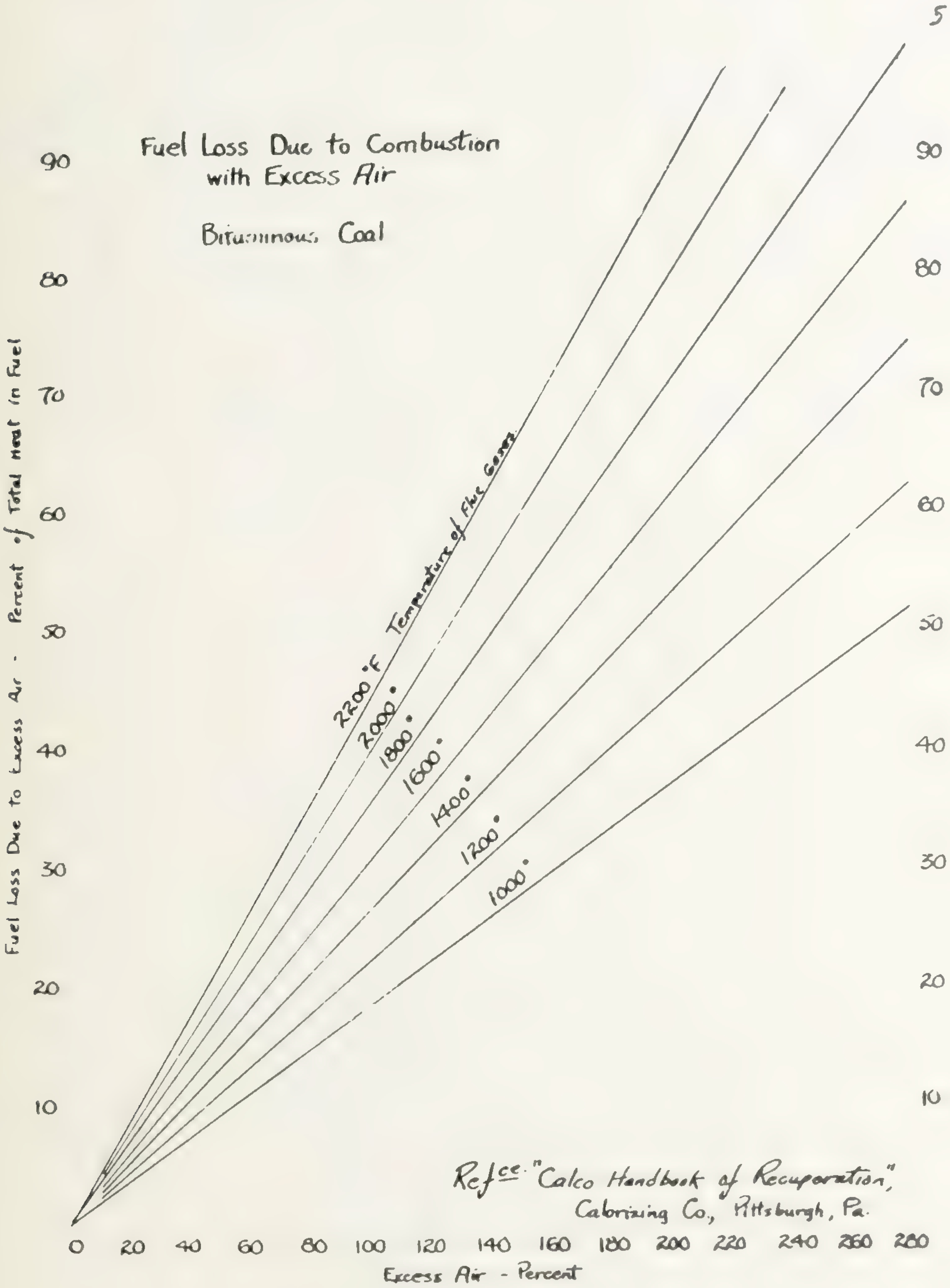
[1] Jour. Am. Cer. Soc. Vol. 7, p. 255.

This represents a considerable loss, depending on the amount of excess air, and the temperature to which it is heated. (See curve.)

The other loss, due to a deficiency of air, is made apparent by the presence of carbon monoxide in the flue gases. If this gas is present, it is evidence that the carbon has not been completely oxidized to carbon dioxide, but only to the intermediate product, carbon monoxide. This results in a considerable loss in heat value, as is shown from the following considerations:



One pound of carbon combines with 1.33 pounds of oxygen, and liberates 4,382 B.T.U., giving carbon monoxide.

One pound of carbon combines with 2.66 pounds of oxygen, and liberates 14,544 B.T.U., giving carbon dioxide.

If insufficient oxygen is present to produce complete oxidation to carbon dioxide, a loss results of 10,162 B.T.U. per pound of carbon burned.

It is possible to have both carbon monoxide and oxygen in a flue gas, which seems to indicate both an excess and a deficiency of air. This condition results from poor mixing of the air with the fuel at the point of combustion. It is impossible to get perfect mixture, so that even with a theoretical amount of air, combustion is never perfect.

In many furnace operations, it is not practical to attempt to attain theoretical conditions, since it would be injurious to the efficiency of the process. This is true in ceramic work. An oxidizing atmosphere is nearly always required in a ceramic kiln; therefore an excess of combustion air is necessary. This is true especially during the first part of the burn, during the watersmoking and oxidation periods. The excess air should then be reduced.

In the tests of this report, bituminous coals were used, of which an average analysis is:

Carbon	79.86%
Hydrogen	5.02
Nitrogen	1.86
Sulphur	1.18
Oxygen	4.27
Ash	7.81

The quantity of air required for the theoretically perfect combustion of one pound of fuel may be calculated as follows:

The combustibles are carbon, hydrogen, and sulphur. The amount of air required to burn these constituents is:

	Wt. per lb. of coal—lbs.		Wt. of air req'd for perfect combustion of 1 lb. combustible		Wt. of Air lbs.
Carbon.......	.7986	×	11.594	=	9.26
Hydrogen....	.0502	×	34.780	=	1.74
Sulphur......	.0118	×	4.350	=	0.051
			Total weight of air	=	11.051

From this there must be subtracted the air represented by the oxygen in the fuel, namely,

$$\frac{100}{23} \times 0427 = 0.186 \text{ lbs.}$$

$$\therefore \text{ Air required} = 10.865 \text{ lbs.}$$

At 62°F. and atmospheric pressure, the volume of 10.865 lbs. is 142.8 cubic feet.

The analysis of the products of combustion (counting SO_2 as CO_2, as is usual) is:

CO_2	2.948 lbs.
H_2O	.452 "
N_2	8.375 "
Total......	11.755 "

The percentages by weight are:

CO_2	25.03
H_2O	3.83
N_2	71.10

Converting these to percentages by volume (below 100°C.):

CO_2	18.3
N_2	81.5

This would be the composition, then, of a flue gas, after theoretically perfect combustion.[2]

EXPERIMENTAL WORK.

Tests were made during an entire burn on four different kilns, two at the plant of the Don Valley Brick Works, Limited, and two at the John Price plant of the Toronto Brick Company, Limited.

Readings of temperature, and draft pressure in various parts of the kilns were taken every two hours for the period from 6 a.m. until 12 midnight, daily throughout the burn. Flue gas analyses were also made every two hours.

A description and list of data regarding the four kilns are shown on the following page.

Kilns *A* and *B* were burned with forced draft, while *C* and *D* had natural draft which was sometimes augmented by a stack fan. The battery of kilns of which *A* and *B* form a part are equipped with the Minter Waste Heat System, by means of which the heat from a cooling kiln is utilized in watersmoking another kiln.

The actual readings taken are recorded hereunder for each kiln. The top temperatures are those recorded on plant pyrometers. In kilns

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[2] "Calco Handbook of Recuperation"—Calorizing Co., Pittsburgh, Pa.

Type of Kiln	KILN *A* No. 8 Tor. Brick Co.	KILN *B* No. 4 Tor. Brick Co.	KILN *C* No. 2 Don Valley	KILN *D* No. 7 Don Valley
	Round Down-Draft	Round Down-Draft	Rect. Down-Draft	Rect. Down-Draft
Dimensions (inside)	Diameter 30 ft.	Diameter 30 ft.	14′ × 60′	16′ × 60′
No. of furnaces	10	12	24	24
Grates	Horizontal	Horizontal	Horizontal	Horizontal
No. of stacks	1	1	1	1
Setting	70,000 Soft mud 14,000 Wire-cut	70,000 Soft mud 14,000 Wire-cut	36,500 Wire-cut 74,500 Pressed	137,500 Wire-cut
Coal	Kentucky, Slack	Kentucky, Slack	Penn., Lump	Penn., Lump
Flue System	See Fig. 1	Fig. 1	Fig. 2	Fig. 2
Duration (Hours)				
Waste Heat	0	25	0	0
Wood	30	12	42	42
Coal	121	103	144	144
Total	151	140	186	186
Final Temp. °F.	2000	2000	1865	1890
Fuel used:				
Wood	12 cords	5 cords	10 cords	8 cords
Coal	49,900 lbs.	54,000 lbs. Waste Heat	94,600 lbs.	103,400 lbs.

A and *B*, the thermocouples were situated at a point in the crown three feet below the centre, where it is intended that they register the temperature at the top of the setting. In kilns *C* and *D*, the couples were in the centre of the crown, one at each end. The recording instrument used with kilns *A* and *B* was compensated for the cold junction reading, which was maintained constant in a well dug for that purpose. The instrument with kilns *C* and *D* had no such correction; readings of the cold junction temperature were therefore taken, and a correction made. This correction was also made for bottom and flue gas temperatures.

The bottom temperatures were recorded with a thermocouple at a point ten feet inside the kiln through a hole in the bottom of the wicket. When the kiln became too hot for this unprotected couple, an optical pyrometer was used. The latter records the temperature of the ware, whereas the former records the temperature of the kiln gases, a fact which should be noted.

The stack temperatures, in the case of kilns *A*, *B* and *D*, are the temperatures of the flue gases immediately on leaving the kiln, *i.e.*, they were taken in the flue close to the kiln. In the case of kiln *C*, the stack temperature was recorded at a point 75 ft. from the point where the gases leave the kiln.

The top gauge readings were in all cases taken by inserting an open-end iron pipe through the centre of the crown. A rubber tube extended from the iron pipe to the ground, where the gauge was attached. At first, the pipe was allowed to remain in the kiln, but as the higher temperatures were reached it was necessary to remove it after each reading. The bottom and stack gauge readings were taken in the same manner, at the same points at which the temperatures were recorded.

The specially designed draft gauge[3] used to measure the pressures, consists essentially of a glass U-tube so mounted that it may be inclined to any desired angle in order that small differences in pressure may make a large difference in the reading of the manometer. Gasoline of sp. gr. 0.715 (water = 1) is used in the gauge to further increase its sensitivity. Scales used with the various angles of inclination are so calibrated that the readings are obtained directly in inches of water. At the smallest angle of inclination the instrument is sensitive to one one-thousandth of an inch of water.

The flue gas samples were taken in each case from the point in the flue at which the stack temperature and pressure were read. An Orsat apparatus[4] was used for analysis.

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[3] Jour. Amer. Cer. Soc. 7, 257.

[4] Lord and Demorest—"Metallurgical Analysis", p. 319-323.

Kiln *A*

Hours on	Temperatures, °F			Gauge Readings Inches of Water			Flue Gas Analysis				
Fire	Top	Bottom	Stack	Top	Bottom	Stack	CO_2	O_2	CO	Air Ratio	Remarks
											Wood fire
6	300	157	107	+.070	−.035	−.140	1.0	19.6	0	14.7	Damper 24″
8	380	107	82	+.070	−.040	−.190	1.7	19.0	0	10.6	Damper 30″
10	460	117	127	+.075	−.035	−.170	2.0	18.8	0	9.7	
12	520	189	139	+.080	−.045	−.180	1.8	18.8	0	9.45	
14	540	193	121	+.080	−.040	−.180					
16	630	289	119	+.10	−.055	−.200	3.0	17.6	0	6.15	
18	670	317	177	+.10	−.040	−.180	2.5	18.1	0	7.3	
20	660	457	212	+.110	−.050	−.190	6.0	14.6	0	3.3	Kiln just fired
22	720	544	222	+.115	−.040	−.185	5.5	15.3	0	3.7	
24	760	615	288	+.115	−.040	−.190					
30	800	673	283	+.13	−.030	−.180	2.0	18.6	0	8.7	Coal firing
32	820	700	340	+.05	−.08	−.22	2.2	17.9	0	6.55	Doors open 1/5
34	1025	706	296	+.06	−.10	−.24	1.2	18.8	0	8.9	Doors open 1″
35	1075		216	+.04	−.13	−.26					
36	1110	734	196	+.03	−.14	−.25	1.8	17.8	0	6.1	
38	1200	739	224	+.02	−.15	−.25	2.6	17.2	0	5.3	Damper dropped 6″
40	1240	808	240	+.09	−.125	−.245	4.4	15.8	0	4.0	
41	1280	808	266	+.05	−.130	−.225					Three shovels coal every 15 mins each furnace
42	1330	743	263	+.05	−.130	−.230	3.9	16.8	0	5.0	
44	1360	794	274	+.06	−.145	−.250	6.8	13.4	0	2.7	
46	1420	860	260		−.185	−.295	6.2	14.0	0	3.0	Doors closed
48	1440	872	208		−.185	−.285					

54	1600	953	503	−.15	−.240	6.8	14.5	0	3.3	Damper dropped 3″
56	1625	996	546	−.16	−.260	10.9	8.7	0	1.7	Damper dropped 3″
58	1750	962	222	−.11	−.190	8.6	12.3	0	2.4	Damper raised 2″
60	1775	1009	404	−.10	−.210	14.8	4.0	0	1.23	After fire
61						11.6	10.0	0	2.0	Clear stack
62	1800	1056	467	−.145	−.230	9.8	12.2	0	2.4	Grates cleaned
64	1810	1061	639	−.130	−.245	14.7	5.3	0	1.33	Just fired
66	1840	1106	719	−.155	−.195	8.5	13.7	0	3.0	Just before firing
68	1850	1139	732	−.125	−.195	10.3	11.3	0	2.2	
70	1840	1284	767	−.105	−.140	9.1	12.8	0	2.6	Grates cleaned
72	1860	1257	747	−.120	−.200	10.3	10.8	0	2.07	
78	1850	1353	783	−.120	−.170	9.2	10.2	0	1.9	
80	1880	1378	788		−.200	6.6	14.2	0	3.1	Damper dropped 1½
82	1920	1500	1040		−.160	8.7	10.9	0	2.05	Opt. Pyro. Bot. temp.
84		1525	1053		−.180	7.2	13.2	0	2.67	2½ shovels coal per furnace every 15 minutes.
89	1955	1570	1148		−.170	5.7	14.9	0	3.44	Just before firing
91	1950	1580	1171		−.190	5.5	15.9	0	3.5	Settle ½″
93	1950	1580	1217		−.160	8.2	11.8	0	2.26	2 shovels per fire
95	1950	1585	894		−.155	14.2	2.8	0	1.15	Just after fire
97	1960	1592	932		−.160					
102	1990	1715	1068		−.140	7.2	12.6	0	2.46	Settle 1″
104	2010	1739	1048		−.160	10.7	7.3	0	1.51	
106	2030	1745	1124		−.140	8.4	11.6	0	2.2	Damper dropped 1″
108	1990	1763	1225		−.150	6.9	13.4	0	2.75	
110	1990	1756	1272		−.130	9.2	9.0	0	1.88	
112	1990	1763	1287		−.120	9.0	10.5	0	2.0	Settle 1¾″
114	2000	1763	1318		−.125	12.4	7.2	0	1.51	During firing
116	1990	1775	1276		−.130	10.2	8.8	0	1.7	During cleaning
118	1990	1763	1335		−.145	9.8	9.8	0	1.86	Settle 2½″
120	2000	1775	1333		−.145					Settle 2¾″

KILN *A* (*Continued*)

Hours on	Temperature, °F			Gauge Readings Inches of Water			Flue Gas Analysis				
Fire	Top	Bottom	Stack	Top	Bottom	Stack	CO_2	O_2	CO	Air Ratio	Remarks
126	1990	1799	1342	+.170		−.180	7.4	5.4	0	1.3	Settle 3½″
128	1985	1799	1439	+.140	−.050	−.120	9.8	8.6	0	1.66	
130	2000	1811	1423	+.150	−.060	−.170	8.6	10.4	0	1.94	
132	2000	1817	1423	+.080	−.070	−.140	10.7	6.5	0	1.42	
134	2000	1829	1433	+.120	−.075	−.130	17.7	1.2	0	1.06	During firing
135							8.1	11.5	0	2.18	After smoke off
136	2000	1840	1307	+.130	−.055	−.115	7.9	11.5	0	2.17	Settle 4 5/8″
138	1985	1825	1440	+.120	−.070	−.135	6.8	13.3	0	2.7	Top = 1920° O.P.
140	1975	1846	1476	+.145	−.050	−.120	6.6	13.7	0	2.86	Settle 5 7/8″
142	1965	1835	1486	+.135	−.065	−.125	9.0	11.0	0	2.08	During firing
144	1975	1823	1473	+.130	−.075	−.140	7.0	13.4	0	2.74	
146	1975	1823	1466	+.120	−.080	−.140	10.2	7.6	0	1.54	After firing
147			1496								Settle 6 5/8″
151	Finished. Settle 7½″										

Plus sign (+) indicates pressure.

Minus sign (−) indicates pull.

The readings of top and bottom draft pressure, which are omitted in the above, were unreliable. Instead of an open end pipe, one with 1/16″ dia. holes along the circumference had been used. It was found that these holes became stopped up. Thereafter, an open end pipe was used.

"Air Ratio" refers to the air present in terms of the air necessary for combustion, *e.g.*, 14.7 means that there was present 14.7 times the amount of air necessary. It is calculated from the following formula:

$$\text{Air Ratio} = \frac{N_2}{N_2 - (3.78 > 0_2)}$$

where N_2 = nitrogen present in flue gas.

O_2 = oxygen present in flue gas.

3.78 = ratio of nitrogen to oxygen in air.

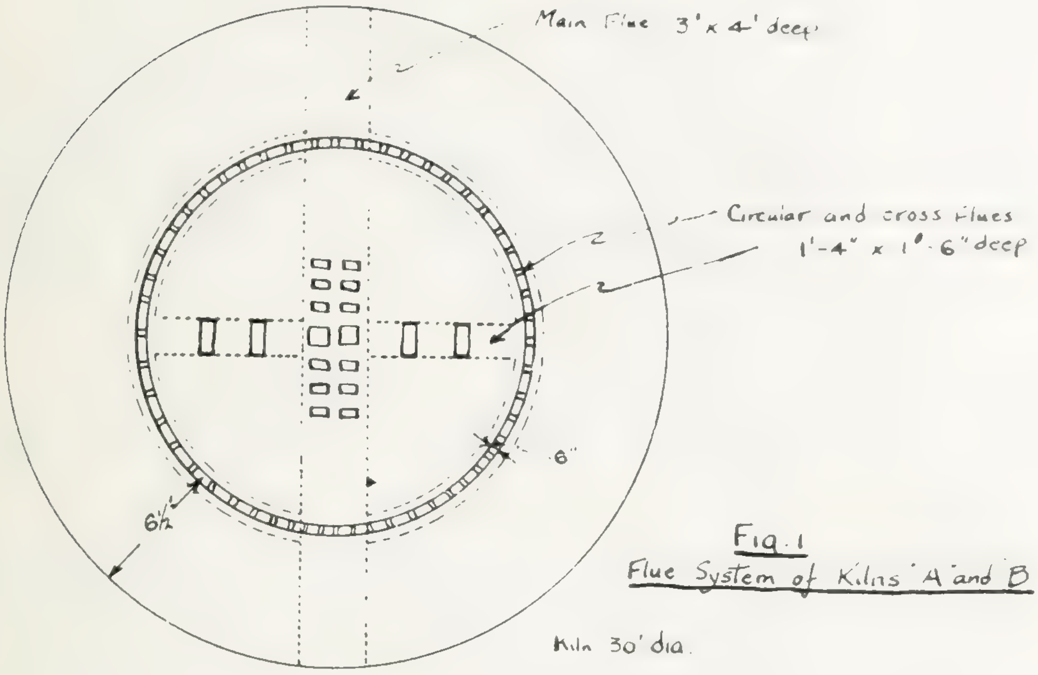

Fig. 1
Flue System of Kilns A and B

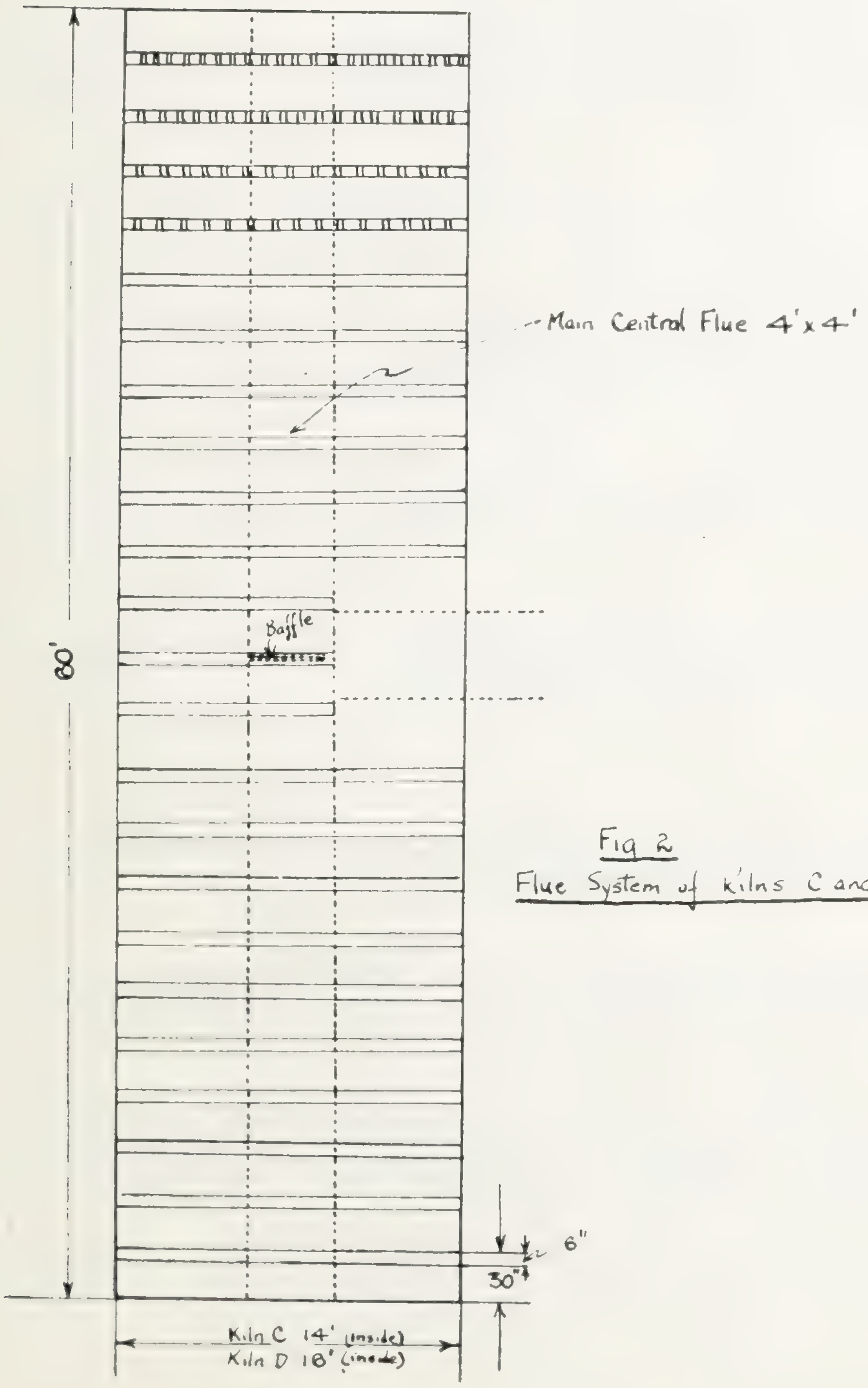

Fig 2
Flue System of Kilns C and D

KILN *B*

Hours on Fire	Temperatures, °F.			Gauge Readings Inches of Water			Flue Gas Analysis				% Loss of Wt.	Remarks
	Top	Bottom	Stack	Top	Bottom	Stack	CO_2	O_2	CO	Air Ratio		
13	110	74	179	−.47	−.53	−.71		Waste heat			0.34	
15	125	79	184	−.45	−.52	−.70		Waste heat			0.34	
25	185	114	184	−.52	−.62	−.85		Waste heat			1.7	
26	195	116	236	+.07	−.02	−.52	1.8	19.2	0	12.1	1.7	Wood fire
28	230	151	256	+.07	−.03	−.51	1.3	19.6	0	15.8	2.0	
30	270	156	261	+.10	−.04	−.55	1.8	19.2	0	12.1	2.3	
32	330	196	276	+.09	−.04	−.53	1.8	19.0	0	10.7	2.6	
34	390	226	296	+.09	−.04	−.50					2.8	
36	525	294	356	+.08	−.045	−.52	4.3	16.4	0	4.6	3.1	
38	560	359	387	+.095	−.05	−.52	3.2	17.1	0	5.3	3.1	Coal fire
40	635	437	467	+.02	−.14	−.54	4.4	16.0	0	4.2	3.5	
42	800	549	529	−.10	−.27	−.62	4.8	15.9	0	4.1		
49	970	761	521	+.06	−.11	−.38	2.9	17.7	0	6.35	4.2	
51	1090	773	573	+.06	−.11	−.42	3.0	17.8	0	6.65	4.2	
53	1165	783	573	+.05	−.13	−.40	3.1	17.9	0	6.93	4.4	
54	1230	803	603	+.05	−.15	−.40	2.4	18.7	0	9.6	4.4	
56	1335	833	603	+.04	−.15	−.43	3.6	16.9	0	5.1		
58	1390	861	551	+.006	−.19	−.48	3.5	17.7	0	6.6		
60	1460	886	561	+.01	−.19	−.50	3.4	18.0	0	7.5		
62	1500	912	577	+.07	−.125	−.36	4.0	17.6	0	6.6		
64	1550	931	586	+.12	−.125	−.36	4.8	16.7	0	5.1		
66	1600	976	591	+.07	−.12	−.36						
73	1695	1100	692	+.10	−.09	−.32	4.6	16.8	0	5.2		Estimated
75	1730		684	+.11	−.09	−.31	5.7	15.3	0	3.7		During fire

76	1750		727	+.04	−.11	−.35	4.4	17.2	0	5.85	
78	1790		739	+.06	−.08	−.33	4.8	16.6	0	4.9	
80	1870		761	+.06	−.13	−.37	4.0	17.6	0	6.6	
82	1875		751	+.09	−.12	−.36	5.2	15.2	0	3.6	Just after fire
83	1880		751	+.09	−.13	−.38	3.3	18.3	0	8.5	Just before
84	1895	1368	741	+.08	−.13	−.40	3.0	18.5	0	9.2	Before fire
86	1910	1368	781	+.065	−.135	−.38	4.8	15.5	0	3.8	After fire
88	1910	1409	811	+.08	−.125	−.38	2.6	18.5	0	8.9	
90	1925	1409	848	+.08	−.12	−.38	3.4	17.7	0	6.6	Before fire
97	1920	1585	968	+.10	−.10	−.35	3.8	16.2	0	4.25	Settle ¾″
99	1935	1592	985	+.09	−.09	−.30	4.9	15.3	0	3.6	
100	1935	1600	992	+.09	−.08	−.30	1.9	19.1	0	11.6	
102	1930	1630	1019	+.10	−.09	−.32	4.4	15.8	0	4.0	After fire
104	1935	1687	972	+.10	−.08	−.35	3.0	17.8	0	6.65	
106	1940	1687	1017	+.08	−.09	−.40	3.7	16.9	0	5.1	Settle 1¼″
108	1950	1727	1014	+.08	−.105	−.39	3.3	17.5	0	6.05	
110	1960	1751	1040	+.115	−.055	−.34	4.4	16.0	0	4.2	
112	1975	1751	1073	+.09	−.08	−.39	4.0	16.0	0	4.1	Settle 1 7/8″
114	1985	1763	1068	+.09	−.09	−.41	3.5	17.2	0	5.5	
121	1980	1799	1138	+.11	−.08	−.41	2.8	18.0	0	7.1	Settle 3 5/8″
123	1985	1799	1137	+.11	−.07	−.36	5.0	15.8	0	4.06	During fire
124	1980	1799	1124	+.12	−.06	−.43	5.4	15.2	0	3.6	During fire
126	1990	1799	1166	+.15	−.07	−.46	3.6	16.6	0	4.7	Settle 5¼″
131	1990	1835	1234	+.15	−.055	−.38	3.0	17.6	0	6.15	
133	1975	1846	1180	+.14	−.065	−.39	3.0	17.7	0	6.4	
135	1975	1846	1268	+.13	−.07	−.39	7.2	12.6	0	2.46	After fire
137	1980	1846	1275	+.11	−.09	−.48	5.3	14.1	0	2.95	Settle 6 7/8″
139	2000	1846	1307	+.115	−.095	−.47	7.6	12.1	0	2.3	After fire
140	Finished	Settle 8″									

The flue gas analyses varied with the time the sample was taken relative to the time of firing. See notations "before fire" and "after fire". During the waste heat period, the doors were tightly mudded, and the stack damper low, giving a high draft through the kiln.

KILN *C*

Hours on Fire	Temperature, °F.					Gauge Readings Inches of Water				Flue Gas Analysis				Remarks
	Top N.	Top S.	Bottom N.	Bottom S.	Stack	Top	Bottom N.	Bottom S	Stack	CO_2	O_2	CO	Air Ratio	
2	160	170				+.035	+.011		−.165	2.2	18.4	0	8.1	Wood fire
4	210	210				+.040	+.010		−.165	1.1	19.8	0	18.8	
6	240	240				+.045	+.011		−.150	3.2	17.3	0	5.7	
8	280	260				+.045	+.008		−.155					
14	357	307	86	66		+.05	−.004	+.010	−.16	2.3	18.4	0	8.17	
16	388	343	83	59		+.05	+.008	+.006	−.14	1.9	18.3	0	7.6	
18	366	336	106	63		+.05	+.012		−.15	2.6	18.0	0	7.0	
20	398	398	108	93	120	+.05	+.04	+.012	−.16	2.6	17.7	0	6.2	
22	415	415		95	120	+.07		+.018	−.15	2.8	17.6	0	6.0	
24			70	90	120	+.06	+.016	+.016	−.155	3.4	17.7	0	6.6	
26	450	470			120	+.075	+.012		−.150	3.8	16.8	0	5.0	
28	424	444		76	120	+.075	+.010	+.015	−.150	3.3	17.3	0	5.67	
30	444	454		74	120	+.07		+.013	−.140	3.7	17.1	0	5.4	
32	475	475	85	86	120	+.07	+.018	+.017	−.135					
38	580	560	70		140	+.07	+.015		−.14	3.0	17.6	0	6.15	
40	575	560	100	90	140	+.07	+.015	+.008	−.20	3.6	16.8	0	4.95	
42	610	610	100	90	140	+.08	+.015	+.015	−.21	4.3	16.1	0	4.25	Coal Firing
44	715	675	115	95	150	+.09	+.020	+.010	−.21	4.0	16.0	0	4.1	on one side
47	883	903	108		155	+.09	+.020		−.21	6.5	13.3	0	2.7	
48	883	913	103	101	160	+.105	+.018	+.016	−.21					All coal
51	913	943	117	103	165	+.095	+.009	+.010	−.205	7.6	11.6	0	2.2	Fires
53	981	961	99	97	175	+.10	+.009	+.006	−.190	7.4	12.4	0	2.4	
55	1059	1019		99	190	+.105		+.005	−.21	7.4	12.8	0	2.54	

62	1105	1045	225	115	220	+.10	+.015	+.003	−.19	6.1	14.3	0	3.1	Steam gone Kiln dry
64	1140	1070	190	160	240	+.11	+.01	−.006	−.21	7.3	13.1	0	2.6	
66	1115	1065	235	200	260	+.04	−.02	−.03	−.22	9.4	9.0	0	1.7	Doors closed
68	1211	1151	191	241	280	+.08	−.017	−.025	−.22	9.2	11.2	0	2.1	
70	1221	1141	201	321	300	+.095	−.017	−.018	−.215					
72	1221	1181	201	311	340	+.10	−.012	−.012	−.17	13.5	6.3	0	1.42	After fire 1 side of kiln fired 1½ hrs.
74	1330	1260	230	430	380	+.095	−.019	−.021	−.40	10.1	11.0	0	2.1	
76	1340	1310	370	510	420	+.11	−.014	−.018	−.40	9.7	12.3	0	2.5	
78	1426	1386	406	686	460	+.105	−.011	−.019	−.33	8.7	13.8	0	3.05	
80	1366	1366	506	586	500	+.105	−.021	−.022	−.425					
86	1461	1431	611	726	631	+.08	−.05	−.04	−.37	13.3	7.0	0	1.5	Just fired
88	1542	1552	642		631	+.12	−.015	−.015	−.39	9.0	13.0	0	3.1	
90	1544	1534	734	694	664	+.15	−.015	−.015	−.38	12.4	8.9	0	1.75	
92	1624	1624												
97	1568	1568												
99	1618	1608	668	788	748	+.13	−.01	−.012	−.33	10.3	12.4	0	2.54	
102	1688	1668	788	788	768	+.12	−.018	−.014	−.33	13.6	8.2	0	1.65	After fire
104	1648	1648	818	768	783	+.12	−.018	−.018	−.32					
110	1753	1763	863	773	813	+.10	−.03	−.03	−.36	8.8	13.5	0	2.9	
112	1710	1720	850	780	830	+.10	−.025	−.03	−.32	11.4	10.0	0	1.9	
114	1720	1740			830	+.14	−.025	−.015	−.31	12.2	8.4	0	1.67	
116	1740	1770			840	+.11	−.015	−.025	−.25	8.9	12.9	0	2.65	
119	1750	1750			758	+.135	−.02	−.02	−.29	12.7	7.8	0	1.6	During fire
121	1736	1756			841	+.11	−.023	−.023	−.285	8.0	12.8	0	2.57	
123	1735	1735			845	+.13	−.022	−.024	−.28	6.1	15.2	0	3.7	
125	1699	1709			864	+.115	−.022	−.027	−.26	7.0	13.8	0	2.9	
127	1702	1722			882	+.12	−.022	−.027	−.265	8.4	13.0	0	2.66	
134	1722	1852	1502	1485	932	+.11	−.01	−.01	−.25	9.3	11.2	0	2.1	
136	1791	1791	1485	1485	931	+.12	−.01	−.01	−.25	8.7	9.9	0	1.85	

KILN *C* (*Continued*)

Hours on	Temperature, °F.					Gauge Readings Inches of Water				Flue Gas Analysis				
Fire	Top N.	Top S.	Bottom N.	Bottom S.	Stack	Top	Bottom N.	Bottom S.	Stack.	CO_2	O_2	CO	Air Ratio	Remarks
139	1822	1837	1600	1536	952	+.13	−.008	−.006	−.21	9.3	9.0	0	1.7	
141	1834	1834	1673	1585	974	+.12	−.008	−.013	−.19	6.8	14.1	0	3.07	
145	1835	1835	1694	1577	1015	+.13	−.010	−.014	−.17	6.5	14.4	0	3.2	
158	1806	1751	1708	1673	1156	+.25	−.002	−.015	−.16	8.8	11.9	0	2.3	
160	1795	1775	1715	1687	1165	+.14	−.015	−.010	−.24	9.8	8.4	0	1.6	
162	1816	1801	1727	1687	1176	+.12	±.002	−.015	−.32	7.0	11.9	0	2.24	
164	1811	1851	1739	1701	1216	+.12	±.005	−.010	−.32	10.0	8.2	0	1.6	
166	1828	1828	1721	1687	1203	+.12	−.016	−.02	−.32	5.8	14.6	0	3.26	
169	1801	1801	1715	1687	1201	+.14	−.012	−.014	−.32	10.6	9.8	0	1.87	
171	1783	1763	1721	1715	1218	+.16	−.008	−.016	−.32	6.2	12.6	0	2.4	
173	1761	1761	1721	1681	1231	+.125	−.02	−.02	−.325	5.8	14.2	0	3.0	
176	1734	1759	1727	1687	1249	+.14	−.018	−.021	−.32	6.7	12.9	0	2.5	
182	1807	1837	1757	1745	1217	+.14	−.010	−.015	−.32	7.5	13.0	0	2.6	
184	1816	1822	1757	1757	1271	+.12	−.020	−.025	−.59	9.7	7.0	0	1.47	Last fire
186	1768	1803	1757	1757	1358	+.12	−.030	−.010	−.46	13.0	3.4	0	1.18	After fire
188	1808	1818	1757	1757	1268	+.10	−.030	−.02	−.47	6.0	14.8	0	3.41	
190	1758	1728	1727	1701	1268	+.115	−.03	−.035	−.47	6.0	15.1	0	3.6	Doors pugged

The difference between final top and bottom temperatures as recorded above is very small. Although the temperatures do come close, the amount shown is open to question, due to the different method of measuring the two temperatures. The top was measured with a noble metal thermocouple and the bottom with the optical instrument.

The draft conditions throughout this burn, particularly at the start, were rather poor, as shown by the manometer readings. A pressure was indicated on the bottom of the kiln for the first sixty hours. Steam issued from the holes in the wicket in which the thermocouple was placed.

KILN *D*

Hours on Fire	Temperature, °F.					Gauge Readings Inches of Water				Flue Gas Analysis				Remarks
	Top N.	Top S.	Bottom N.	Bottom S.	Stack	Top	Bottom N.	Bottom S.	Stack	CO_2	O_2	CO	Air Ratio	
17	430	410	100	100		+.07	+.003	+.002						Wood fire
22	465	465	95	85		+.06	−.006	−.010						
26	561	551	101	101		+.07	−.003	−.003	−.075	3.7	16.9	0	5.1	
41	590	580	145	150		+.08	−.002	−.004	−.18	2.9	17.6	0	6.1	Coal fire
47	863	863	163	223		+.09	−.006	−.016	−.11	6.2	13.1	0	2.6	
66	1057	1052	552	662	602	+.06	−.07	−.05	−.20	5.6	14.3	0	3.07	Dpr. open
69	1214	1204	554	714	604	+.04	−.06	−.06	−.21	2.0	18.9	0	10.4	Before fire
75	1272	1232	662	792	672	+.05	−.07	−.06	−.245	6.3	14.5	0	3.25	
89	1653	1618	668	838	808	+.08	−.04	−.035	−.22	2.8	18.1	0	7.46	
95	1614	1614	814	854	854	+.075	−.06	−.06	−.265	8.6	12.8	0	2.6	
101	1668	1648	818	958	918		−.032	−.027	−.30	4.2	16.2	0	4.35	
114	1740	1755	950	1080	900	+.09	−.04	−.04	−.30	7.4	12.9	0	2.6	
120	1734	1774		1447	1054	+.10	−.05	−.04	−.17	6.0	14.8	0	3.4	
138	1774	1789	1300	1485	1314	+.09	−.04	−.035	−.31	11.4	6.9	0	1.47	Estimated
144	1746	1761	1510	1630	1301	+.10	−.045	−.045	−.33	4.8	15.6	0	3.86	Opt. Pyro.
151	1839	1794	1536	1683	1349	+.10	−.04	−.028	−.29	5.5	15.1	0	3.56	
162	1826	1831	1687	1727	1361	+.10	−.02	−.015	−.14	7.3	13.4	0	2.76	
169	1741	1771	1673	1694	1441	+.095	−.03	−.019	−.19	14.5	3.9	0	1.22	During fire
171	1794	1804	1673	1739	1419	+.11	−.025	−.018	−.19	10.6	7.7	0	1.55	After fire
173	1782	1792	1687	1739	1419	+.095	−.025	−.02	−.19	6.0	14.9	0	3.5	
175	1787	1772	1701	1751	1459	+.09	−.035	−.027	−.18	5.6	15.1	0	3.57	Before fire
183	1838	1843	1727	1763	1503	+.07	−.05	−.04	−.27	6.2	13.1	0	2.6	
185	1784	1789	1727	1763	1520	+.05	−.06	−.05	−.36	4.4	15.3	0	3.56	Before fire
186	Last fire — (Flash)						+.005							

Note:—Kilns C and D were on fire at the same time, which accounts for the relatively small number of readings taken for Kiln D.

The results in the column headed "% Loss of Wt." were obtained as follows:

A brick from the dryer was weighed and suspended by an iron wire through a hole in the crown of the kiln. The brick was within 2 ft. of the kiln floor. The weight of the brick, plus the wire, was then found with a balance. Weights were then taken at intervals during the first part of the burn to determine the loss in weight and thence the end of the watersmoking period.

A variable and considerable error was introduced into the weight readings on account of wind. Some protection should be used in future experiments of this sort. Also, an alloy wire should be used instead of iron, as there is oxidation of the latter.

From the results given, it would seem that water smoking was complete after 50 hours.

An analysis of the coals sampled at each plant gave the following results:

	Kilns *A* and *B*	Kilns *C* and *D*
Volatile Combustible Matter	37.0%	34.9%
Fixed Carbon	52.37%	54.85%
Ash	10.63%	10.25%
Sulphur	1.04%	2.21%
Heat Value	13,758 B.T.U./lb.	13,820 B.T.U./lb.

The moisture content of the bricks when set, and the weights of the green and burned bricks were determined, with the following results:

		Moisture Content	Green Weight	Burned Weight
Kilns *A* and *B*	Soft Mud	3.11%	5.5 lbs.	5.13 lbs.
	Wire-cut	2.47%	6.24 "	5.7 "
Kilns *C* and *D*	Pressed	9.25%	6.58 "	5.54 "
	Wire-cut	1.24%	6.39 "	5.88 "

The efficiency, based on the heat used to heat up the ware, compared to the heat input, was calculated for each kiln, and also the amount of two important kiln losses, namely, the heat lost in dry flue gases up the stack, and the heat lost in water vapour up the stack.

A sample calculation for Kiln *A* follows:

1. *Efficiency*[5]—

Setting Record: 70,000 soft mud brick
14,000 wire cut brick

Fuel Used:

Wood — 12 cords = 46,500 lbs.
(Assume wood[6] is 50% beech and 50% maple; 1 cord maple weighs 4500 lbs.; 1 cord beech weighs 3250 lbs.)
Coal: 49,900 lbs.

Calorific Value:

Wood: 5600 B.T.U./lb.
Coal: 13,758 B.T.U./lb.

Weight of Green Bricks:

Soft Mud: 5.5 lbs.
Wire-cut: 6.24 lbs.

Total weight in Kiln:

70,000 × 5.5 = 385,000 lbs.
14,000 × 6.24 = 87,360 "

472,360 " = 236.18 tons

Weight of moisture in Kiln:

385,000 × 3.11% = 11,970 lbs.
87,360 × 2.47% = 2,156 "

14,126 "

$$\therefore \text{ Moisture per ton of ware} = \frac{14,126}{236.18} = 59.81 \text{ lbs.}$$

$\therefore$ Clay in 1 ton of setting = 2000 − 59.81 = 1940.2 lbs.

Initial temperature of ware = 57°F.

(*a*) Heat required to raise the clay in 1 ton of setting to final temperature:

1940.2 × (1925-57) × 0.255 = 925,000 B.T.U.

(*b*) Heat required to raise the water in 1 ton of setting to temperature of of steam formation:

59.8 × (212-57) = 9,265 B.T U.

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[5] Jour. Amer. Cer. Soc. *6*, 886; also *7*, 738.

[6] Kent's "Mechanical Engineers' Handbook"—1923, p. 878.

(*c*) Heat required to change water to steam:
$59.8 \times 967 = 57{,}800$ B.T.U.

∴ Total heat required:
(*a*) 925,000
(*b*) 9,265
(*c*) 57,800
992,065 B.T.U.

Note: In the above calculation the specific heat value of $0.192 + .000033T$ was used, being that determined by Mellor.[7]

Weight of burned brick:
Soft mud = 5.13 lbs.
Wire-cut = 5.7 "
∴ Total burned weight = $5.13 \times 70{,}000 = 359{,}000$
$5.7 \times 14{,}000 = 80{,}000$
439,000 lbs.
= 219.5 tons

Heat Input:

$$\text{Coal used per ton of ware} = \frac{49{,}900}{219.5} = 227 \text{ lbs.}$$

$$\text{Wood used per ton of ware} = \frac{46{,}500}{219.5} = 211.5 \text{ lbs.}$$

∴ Heat from coal for 1 ton of ware = $227 \times 13{,}758$
= 3,120,000 B.T.U.
Heat from wood for 1 ton of ware = 211.5×5600
= 1,185,000 B.T.U.
∴ Total Heat input per ton = 4,305,000 B.T.U.

$$\therefore \text{Efficiency of Kiln} = \frac{\text{Heat required}}{\text{Heat input}} = \frac{4{,}305{,}000}{992{,}065} = 23.0\%$$

2. *Heat Lost in Dry Flue Gases*[8]—

The source of largest heat loss in periodic kilns is the sensible heat escaping up the stack in the dry flue gases. This loss depends directly upon the mass and temperature of the gases leaving the kiln at the stack. The mass of the gases was computed from the amount of coal burned and the flue gas analyses. The formula used is as follows:

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[7] Trans. Cer. Soc. (Eng.) *12*, 279 (1912-13).
[8] Jour. Am. Cer. Soc. *7*, 738.

$H = M.S. (t_2 - t_1)$
H = Sensible heat in dry flue gases.
M = Mass of gas.
S = Mean specific heat between t_1 and t_2.
t_1 = Temperature of air entering furnaces.
t_2 = Temperature of gas at stack.

M = weight of carbon burned × weight of dry gas per lb. of carbon. Weight of carbon burned = weight of coal × carbon content + weight of wood × carbon content. Weight of dry gas per lb. of carbon $= \dfrac{44\ CO_2 + 32\ O_2 + 28\ (N_2 + CO)}{12\ (CO_2 + CO)}$

where CO_2 = percentage of CO_2 in flue gases
O_2 = percentage of O_2 in flue gases
N_2 = percentage of N_2 in flue gases
CO = percentage of CO in flue gases
44, 32, 28, 12 = molecular weights of CO_2, O_2, N_2 and C, respectively.

The values used for S,[9] the mean specific heat, were:

Temperature, t°F.	Mean specific heat between 32° F. and t° F.
200	0.240
400	0.242
600	0.245
800	0.247
1000	0.250
1500	0.256
2000	0.262

For purposes of this calculation, and also that following, namely, heat lost in water vapor up the stack, the burn under consideration must be divided into periods to account for changes in the flue gas analysis. It is sufficient in this case to divide the burn into two periods, the first covering the first 42 hours, until the furnace doors were closed, the second being the remainder of the burn.

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[9] Jour. Amer. Cer. Soc. *7*, 744.

First Period.—

Average percentage of carbon in wood = 49.7%

Percentage of carbon in coal (estimated from proximate analysis and list of ultimate analyses of Kentucky coals)[10] = 68%

Weight of carbon burned:

In wood $= 46{,}500 \times .497 = 23{,}120$ lbs.

In coal (first 12 hours) $= \frac{12}{121} \times 49{,}900 \times .68 = 3{,}365$ lbs.

(This assumes an equal rate of consumption of coal throughout, which is not exactly correct.)

$\therefore$ Total carbon burned $= 26{,}485$ lbs.

Weight of dry gas per lb. of carbon:

$$\frac{44\ CO_2 + 32\ O_2 + 28\ (N_2 + CO)}{12\ (CO_2 + CO)}$$

Average CO_2 content in period = 2.8%

Average O_2 content in period = 17.6%

CO_2 content = 0

$\therefore$ Average N_2 content in period = 79.6%

$$\therefore \frac{44 \times 2.8 + 32 \times 17.6 + 28 \times 79.6}{12 \times 2.8} = 86.8 \text{ lbs.}$$

$$\begin{aligned} \therefore\ M &= 26{,}485 \times 86.8 = 2{,}295{,}000 \\ H &= M.S.\ (t_2 - t_1) \\ &= 2{,}295{,}000 \times .240 \times (213 \times 58) \\ &= 2{,}295{,}000 \times .240 \times 155 \\ &= 85{,}400{,}000 \text{ B.T.U.} \end{aligned}$$

Second Period.—

$$\begin{aligned} \text{Weight of carbon burned} &= \frac{109}{121} \times 49{,}900 \times .68 \\ &= 30{,}600 \text{ lbs.} \end{aligned}$$

Average CO_2 content = 9.2%

Average O_2 content = 10.0%

Average N_2 content = 80.8%

CO content = 0

$\therefore$ Weight of dry gas per lb. of carbon =

$$\frac{44 \times 9.2 + 10.0 \times 32 + 28 \times 80.8}{12 \times 9.2} = 27 \text{ lbs.}$$

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[10] "Ceramic Industry", Vol. 7, No. 5, Nov. 1926.

$\therefore$ M = 30,600 × 27 = 826,000 lbs.

H = 826,000 × .247 × (859 − 54)

= 826,000 × .247 × 805

= 164,400,000 B.T.U.

Total of two periods = 249,800,000 B.T.U.

Loss per ton of ware = 1,137,000

Percent of heat input = $\frac{1{,}137{,}000}{4{,}305{,}000}$ = 26.4%

3. *Heat Lost in Water Vapor in Flue Gases.*[11]

There is always a loss of heat in the water vapor escaping up the stack with the flue gases. This is dependent on the hydrogen content of the fuel, and is calculated from the formula:

$H = W[h^t - (t_1 - 32)]$

W = weight of water vapor

t = mean temperature of flue gas.

t_1 = mean air temperature.

h_t = total heat of steam at temperature "t".

The weight of water vapor is calculated from the amount of hydrogen in the coal. It is assumed that all the hydrogen is present in the coal as water vapor, or is burned to water vapor. The total heat of steam is calculated from Pier's formula, as follows:

$h = 1058.68 + 0.44701t + 0.000{,}01541t^2$
$-0.000{,}000{,}0002432t^3 + 1.9 \times 10^{-12} \times t^4$ where
t is the temperature in degrees Fahrenheit.

First Period.—

Average percentage of hydrogen in wood = 6.06

Estimated percentage of hydrogen in coal = 5.52

Water vapor from wood = 46,500 × .0606 × 9

= 25,400 lbs.

Water vapor from coal = $\frac{12}{121}$ × 49,900 × .0552 × 9

= 2,460 lbs.

Total = 27,860 lbs.

t = 213°F.; t_1 = 58°F.; h_t = 1154.78 B.T.U./lb.

$\therefore$ H = $W[h_t - (t_1 - 32)]$

= 27,860 [1128.78]

= 31,400,000 B.T.U.

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[11] Jour. Amer. Cer. Soc. *7*, p. 738.

Second Period.—

$$W = \frac{109}{121} \times 49{,}900 \times .0552 \times 9 = 22{,}390 \text{ lbs.}$$

$$t = 859°\text{F.}; \quad t_1 = 54°\text{F.}; \quad h_t = 1454.9 \text{ B.T.U./lb.}$$

$$\therefore H = 22{,}390 \times 1432.9$$

$$= 32{,}100{,}000 \text{ B.T.U.}$$

Total of two periods = 63,500,000 B T.U.

$$\therefore \text{ Loss per ton of ware } = \frac{63{,}500{,}000}{219.5} = 289{,}000 \text{ B.T U.}$$

$$\text{Percentage of heat input } = \frac{289{,}000}{4{,}305{,}000} = 6.72\%$$

The period into which burns *B*, *C*, and *D* were divided are as follows:

	Kiln *B*	Kiln *C*	Kiln *D*
Period 1...........	Wood Fire	Wood Fire—42 hrs.	Wood Fire
Period 2...........	Remainder of Burn	First 24 hours Coal	Remainder of Burn
Period 3...........		Remainder of Burn	

SUMMARY OF RESULTS

		Kiln *A*	Kiln *B*	Kiln *C*	Kiln *D*
Efficiency		23%	25.2%	19.9%	22.7%
Loss in Dry Flue Gas		26.4%	61.8%	23.6%	46.7%
Loss in Water Vapor		6.72%	6.0%	5.4%	5.6%
Fuel per ton of ware	Coal:	227 lbs.	245 lbs.	302 lbs.	256 lbs.
	Wood:	.054 cords	.023 cords Waste Heat	.032 cords	.02 cords
Heat input, per ton		4,305,000 B.T.U.	3,981,000 B.T.U.	4,865,000 B.T.U.	3,969,000 B.T.U.
Heat used to raise temperature of brick, per ton		1,558,400 B.T.U.	1,571,600 B.T.U.	1,414,900 B.T.U.	1,495,600 B.T.U.
Heat used to evaporate moisture, per ton		67,865 B.T.U.	68,850 B.T.U.	152,610 B.T.U.	28,320 B.T.U.
Moisture content, per ton		59.81 lbs.	59.81 lbs.	133.4 lbs.	24.8 lbs.
Heat lost in flue gases, per ton of ware		1,137,000 B.T.U.	2,390,000 B.T.U.	1,150,000 B.T.U.	2,060,000 B.T.U.
Carbon content of coal		68%	68%	75%	75%
Average CO_2 content of flue gas	Period 1	2.8%	1.7%	2.8%	3.3%
	Period 2	9.2%	4.0%	6.1%	6.6%
	Period 3			9.3%	
Weight of dry gas, per lb. of carbon	Period 1	86.8 lbs.	142.3 lbs.	86.8 lbs.	73.75 lbs.
	Period 2	27.0 "	61.1 "	40.35 "	37.3 "
	Period 3			26.81 "	
Average temperature of flue gases	Period 1	213°F.	280°F.	120°F.	210°F.
	Period 2	859°F.	846°F.	177°F.	1028°F.
	Period 3			853°F.	
Temperature of air entering furnaces	Period 1	58°F.	36°F.	50°F.	55°F.
	Period 2	54°F.	41°F.	57°F.	47°F.
	Period 3			46°F.	
Heat lost in water vapor in flue gases		289,000 B.T.U.	232,000 B.T.U.	263,000 B.T.U.	229,500 B.T.U.
Hydrogen content of coal		5.52%	5.52%	5.03%	5.03%

DISCUSSION AND CONCLUSIONS.

The numerical results of efficiency and the two heat losses calculated, seem to be somewhat inconsistent. For instance, Kiln *B*'s efficiency is greater than that of Kiln *A*, but the heat lost in flue gases is more than twice as great in Kiln *B*. How can this be explained? It would also seem, from the results, that the fuel used per ton of ware seems only slightly less for Kiln *B* than for Kiln *A*, but this is probably explained by the fact that an inaccurate estimate of the waste heat used was made. There was no way of measuring this amount. The greater efficiency of Kiln *B* is partly explained by its less burning time (see curve sheet No. 3). Kiln *B* was over 1800°F. for 61 hours, while Kiln *A* was over 1800° for 88 hours. On the other hand, the amount of heat lost up the stack is undoubtedly higher in Kiln *B* than in Kiln *A*, although there might be a possible error in the numerical value, due to failure to get a good average of the flue gas analysis. It would be desirable to take a flue gas sample for analysis much oftener, but in this case sufficient help was not available. The greater heat loss in Kiln *B* was due to a large excess of combustion air, which in turn was due to a too high draft (see curve sheets numbers 2 and 5). Although the high draft during the waste heat watersmoking period produced an excellent condition during this period, the kiln efficiency would no doubt have been increased if the draft had been reduced during the later part of the burn. During the watersmoking period, the force draft proved its value, the water vapour being immediately swept out of the kiln.

It will be seen from the above that it is difficult to compare the efficiencies of kilns, due to the many variable factors affecting them. Following are some of the variables:

(1) Time of Burn—
    (*a*) Coal consumption,
    (*b*) Turnover.
(2) Loss in Flue Gases—
    (*a*) Draft intensity,
    (*b*) Flue Gas Analysis,
    (*c*) Temperature of Gases.
(3) Moisture Content of Ware—
(4) Amount of Ware Set.
(5) Clay.
(6) Fuel.

Each of these variables affects the efficiency of a burn, and must be considered. In future work it would be desirable to maintain some constant if possible, to determine the effect of others.

A variation in the clay also played a part in affecting the efficiency of Kilns *A* and *B*. At this plant, the kilns are finished according to the amount of settle of the ware. If, therefore, there is more sand in the clay at one time than another, the ware will not settle as quickly; it might even expand at first. It was noticed that in Kiln *B* the settle was normal, but in *A* it occupied more time than usual.

Kiln *C* showed a very poor condition during watersmoking. There was a pressure at the bottom of the kiln for the first 63 hours of the burn (see curve sheet No. 8). At this time in the burn there should be as strong a draft as possible, to remove the moisture from the kiln, and prevent condensation. Due to this poor draft, the watersmoking period lasted from 70-75 hours (see curve sheet No. 6). The flue gas temperature remained down to 200°F. for this length of time, indicating the continued presence of moisture in the flues. The top temperature in the kiln had reached 1200°F. before the bottom reached 250°F. If there had been a higher draft during the watersmoking period, the time of the burn could probably have been reduced by a day and a half.

A somewhat better condition existed in Kiln *D* (see curve sheets Nos. 9 and 10), there being a pressure for only 18 hours at the bottom of the kiln, although there was only a very slight pull for the next 30 hours. The draft condition at the plant of kilns *C* and *D* varies according to the other kilns on fire, and also according to a smoke nuisance problem.

The same condition regarding efficiency and heat lost in flue gases, exists for kilns *C* and *D* as for kilns *A* and *B*. With a higher loss in Kiln *D*, there is a greater efficiency. The same remarks apply. It is very difficult to compare the two burns, due to the difference in type of ware and capacity of kiln. Kiln *C* has a smaller capacity, and was set with pressed brick containing 9.25% of water, while Kiln *D* was set with wire-cut brick containing 1.24% of water. There is a still greater possibility of an error arising in the calculations of Kiln *D* as a result of poor averages of flue gas analyses. Insufficient analyses were made, but more were not possible, due to a lack of time and help. It seems evident, however, that the flue gas loss should be higher than in Kiln *C*, although the amount is in doubt.

The method of firing kilns *A* and *B* gave a much more even rise in temperature than the method of firing kilns *C* and *D*. In the former case, every firebox was fired with a small amount of fuel every fifteen minutes, while in the latter case, one side of the kiln was fired every 90 minutes, with a relatively large amount of fuel. (Cf. curves Nos. 3 and 6).

In spite of this, a more even final temperature from top to bottom was recorded for kilns *C* and *D* than for kilns *A* and *B*. The difference between top and bottom temperatures of kilns *C* and *D* was from 35° to 60°F., whereas for kilns *A* and *B*, the difference recorded was about

150°F. As mentioned before, it is rather doubtful if the temperatures of kilns *C* and *D* were as nearly coincident as recorded. The following porosity tests show, however, that a more even temperature actually was attained in kilns *C* and *D* than in kilns *A* and *B*.

	D	S	W	%P
Kilns *A* and *B*, Top, (Soft Mud)	17.6	10.55	19.36	20.0
Middle, " "	15.85	9.52	18.07	26.0
Bottom, (Wire-Cut)	18.0	11.25	21.68	35.3
Kiln *C*, Top, (Pressed)	19.51	12.2	23.65	36.2
Middle, "	22.19	14.03	27.49	39.4
Bottom "	21.71	13.85	26.73	39.0
Top, (Wire-Cut)	35.58	22.03	39.59	22.8
Bottom, "	29.0	18.02	32.2	22.6
Kiln *D*, Top	18.86	11.18	20.18	14.7
Middle	29.07	17.69	31.46	17.3
Bottom	30.41	18.97	34.09	24.4

The porosity of bricks made by different processes cannot, of course, be compared, but it will be seen that the difference in porosity from the top of the kiln to the middle was greater for kilns *A* and *B* than for kilns *C* and *D*, indicating a greater difference of temperature in the former.

The specific heat value used in the calculations was that determined by Mellor. This value is for burned firebrick and is probably low when used for clay. A later value determined is .43[12] for fireclay, and takes into account the exothermic and endothermic reactions which take place in a clay when heated. This value gives the following results for efficiency:

Kiln *A*	*B*	*C*	*D*
37.8%	42.5%	32.2%	38.4%

These results seem too high, as the totals of efficiency and the two losses calculated are too close to 100%; in the case of kiln *B*, over 100%. This value of the specific heat is probably not applicable to impure clays. Another reference[13] gives a value of $0.22 + 0.00012t$, where t is in degrees Centigrade.

The other kiln losses, in addition to the two calculated in this report, may be enumerated as follows:

(1) Heat in combustible in ash.
(2) Heat in combustible in flue gases.
(3) Heat used to raise the temperature of the walls and crown of the kiln.
(4) Heat lost by radiation and convection from the walls and crown of the kiln.
(5) Heat given up by gases between bottom of kiln and stack.

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[12] Jour. Amer. Cer. Soc. *9*, 206.

[13] "Industrial Stoichiometry"—Lewis and Radasch, p. 109.

An average total of these losses, according to previous work,[14] is at least 25% of the heat input. If this amount be added to the values of efficiency, plus the two losses, as given in the summary, the total for kiln *B* is over 100%, and for *D* exactly 100%. This is another indication of a probable inaccuracy in the values for flue gas loss, due to a poor average of flue gas analysis.

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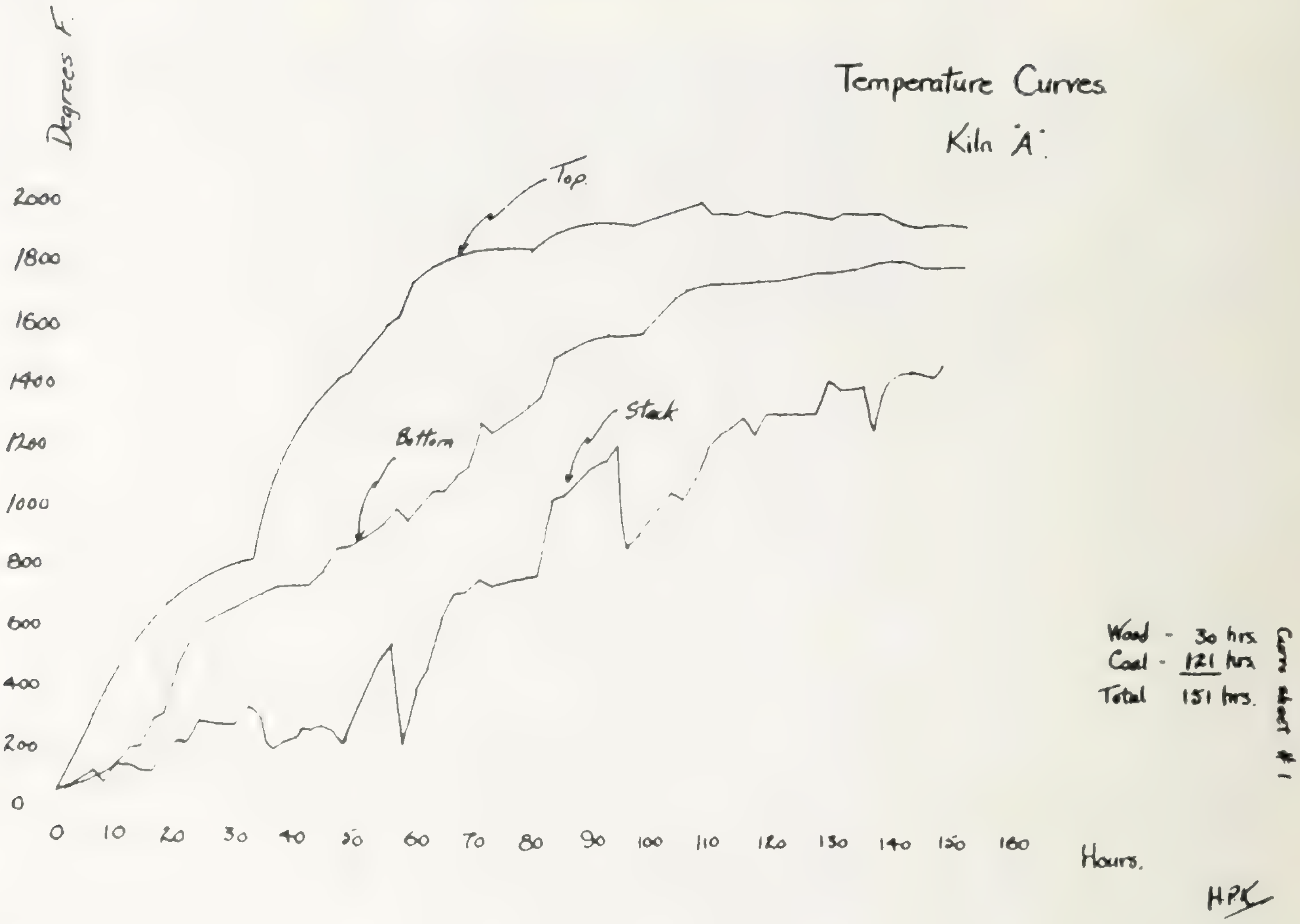
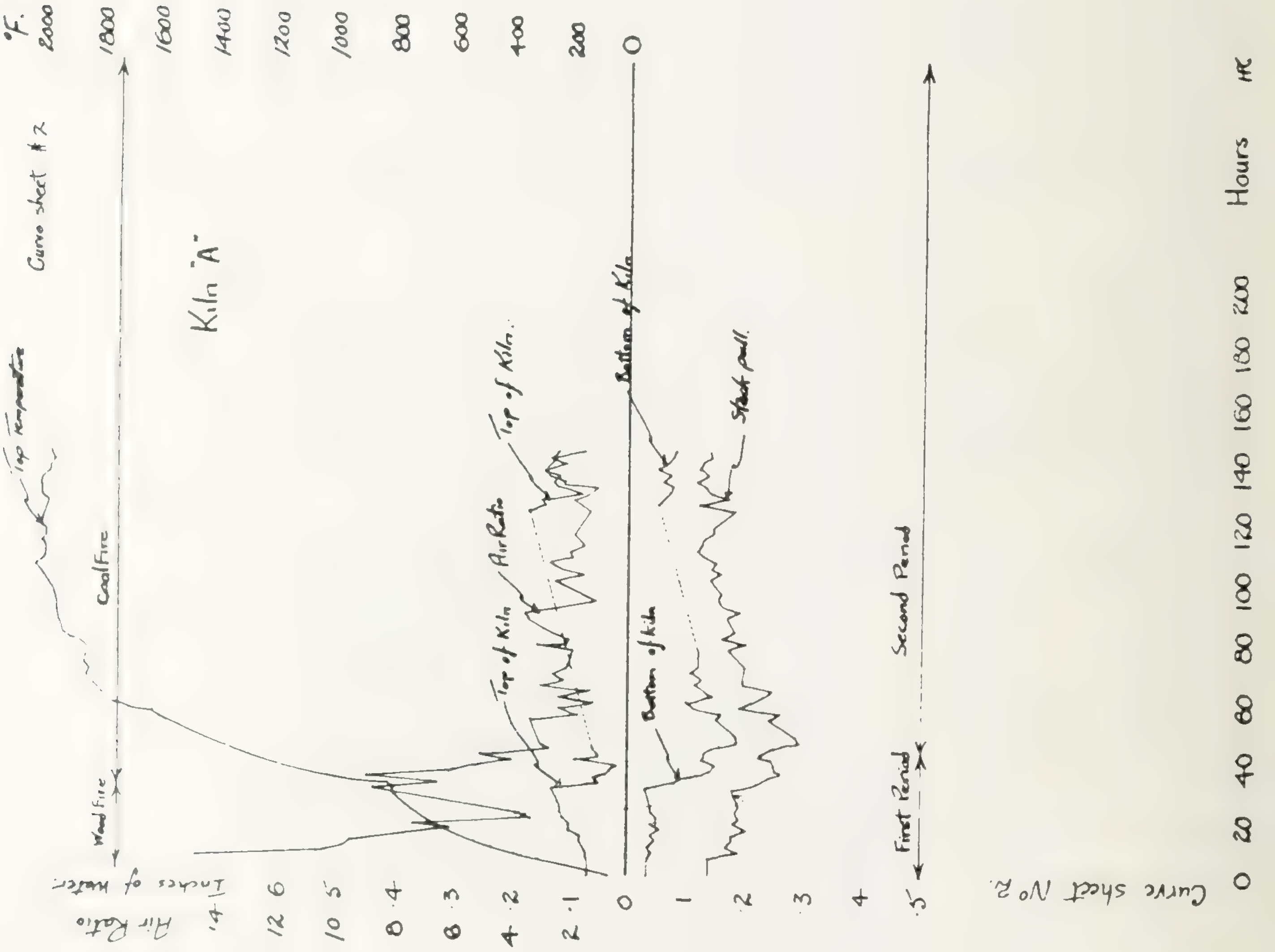
[14] Jour. Amer. Cer. Soc. 7, 738.



Curve Sheet No. 1



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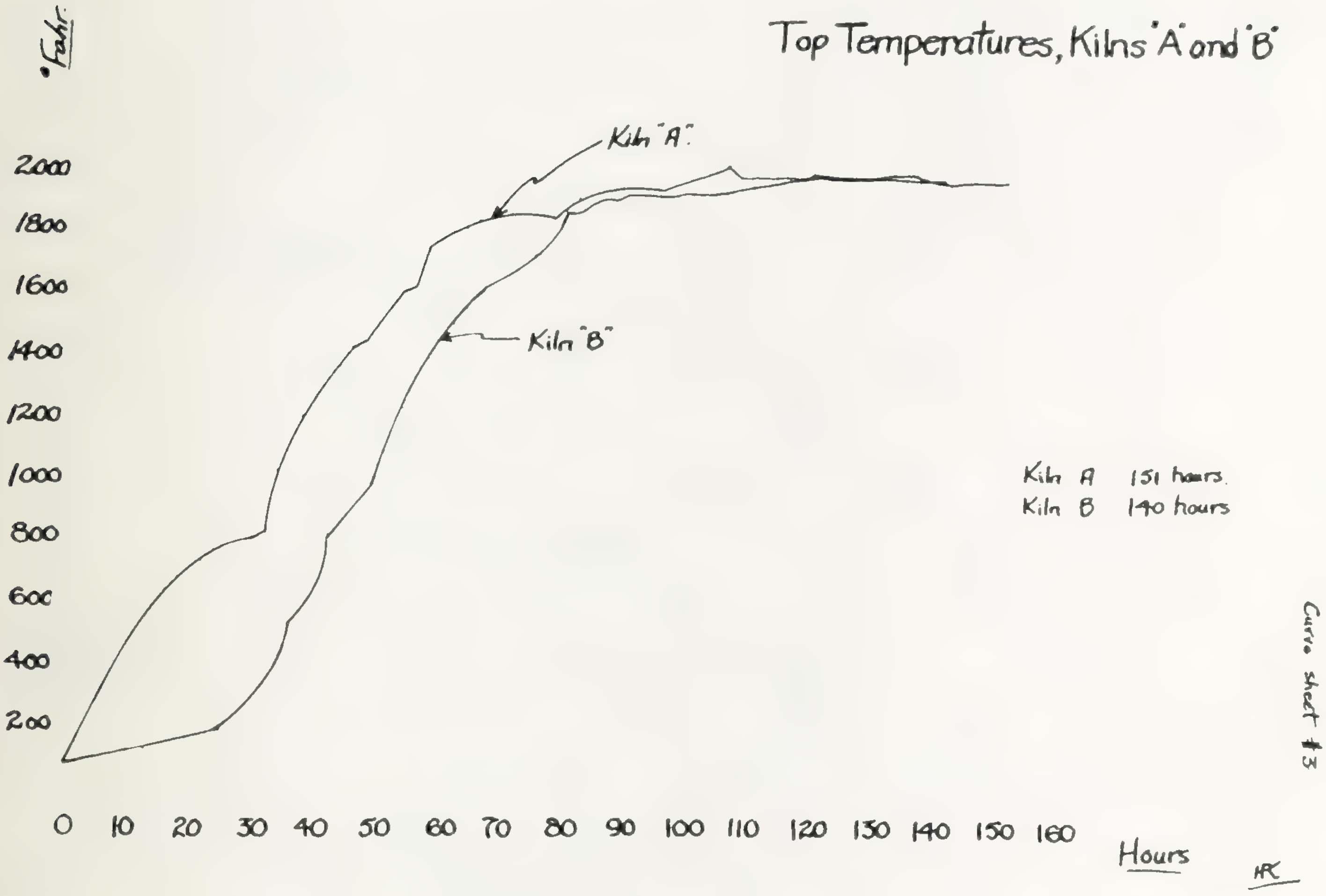

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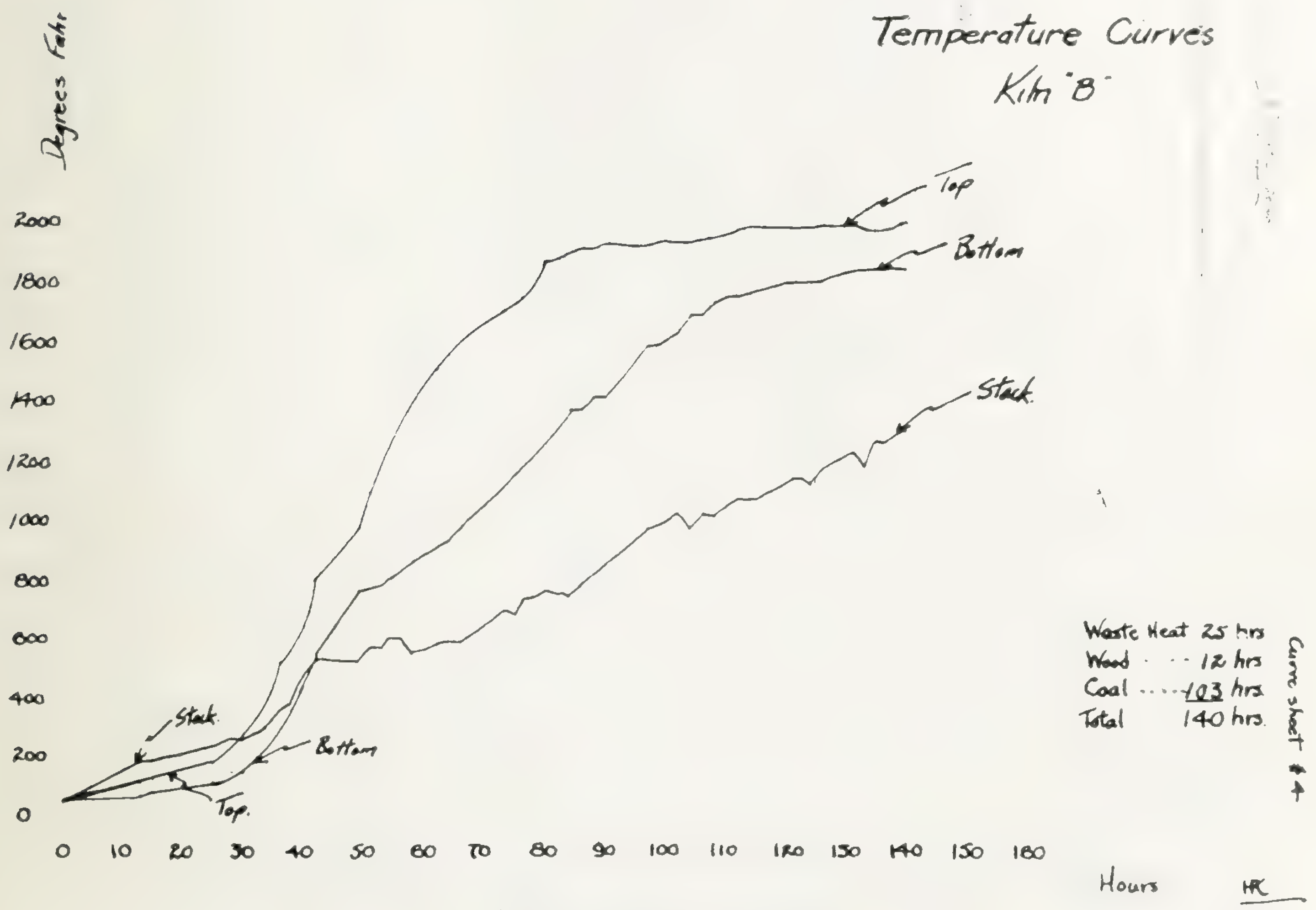

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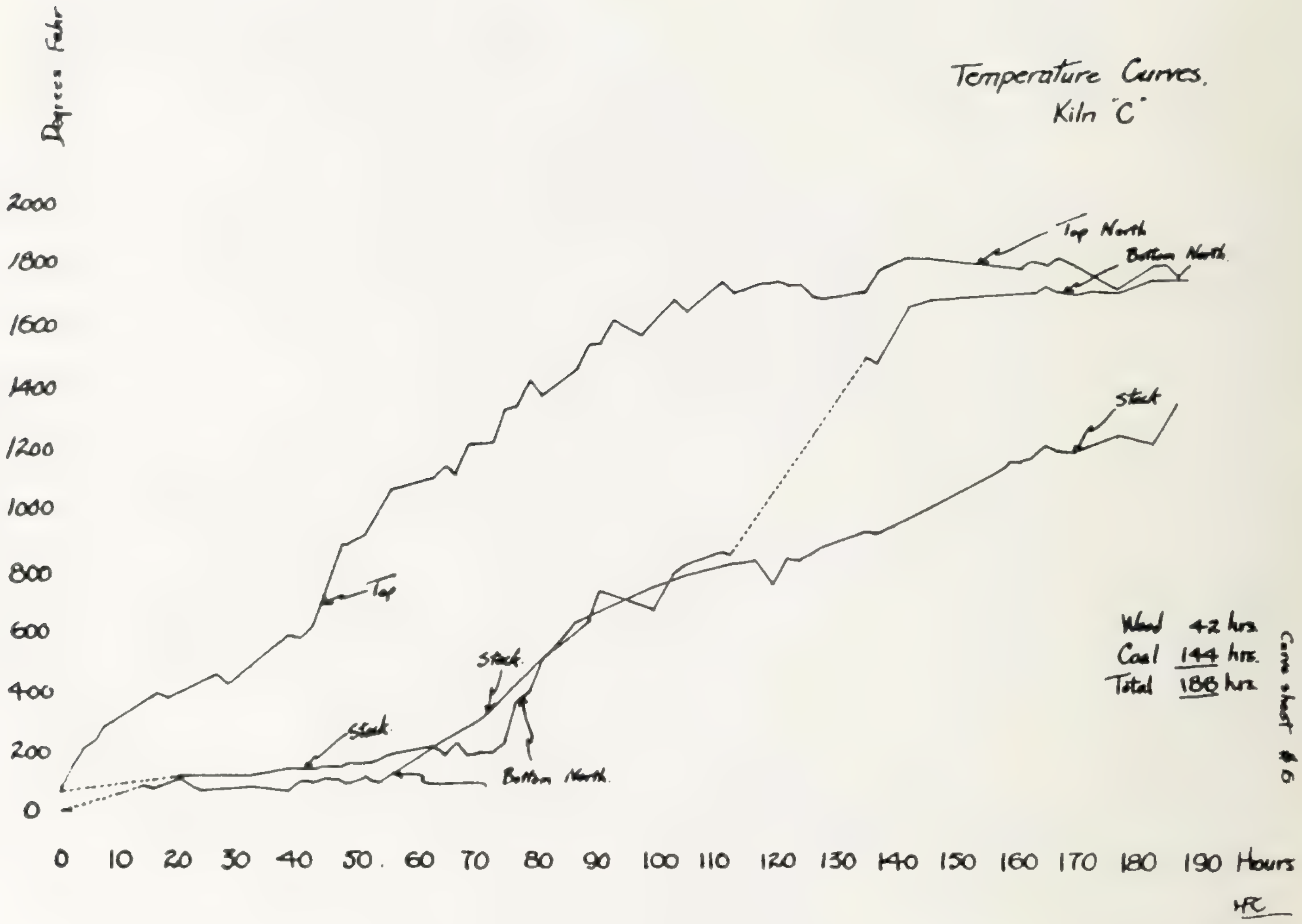
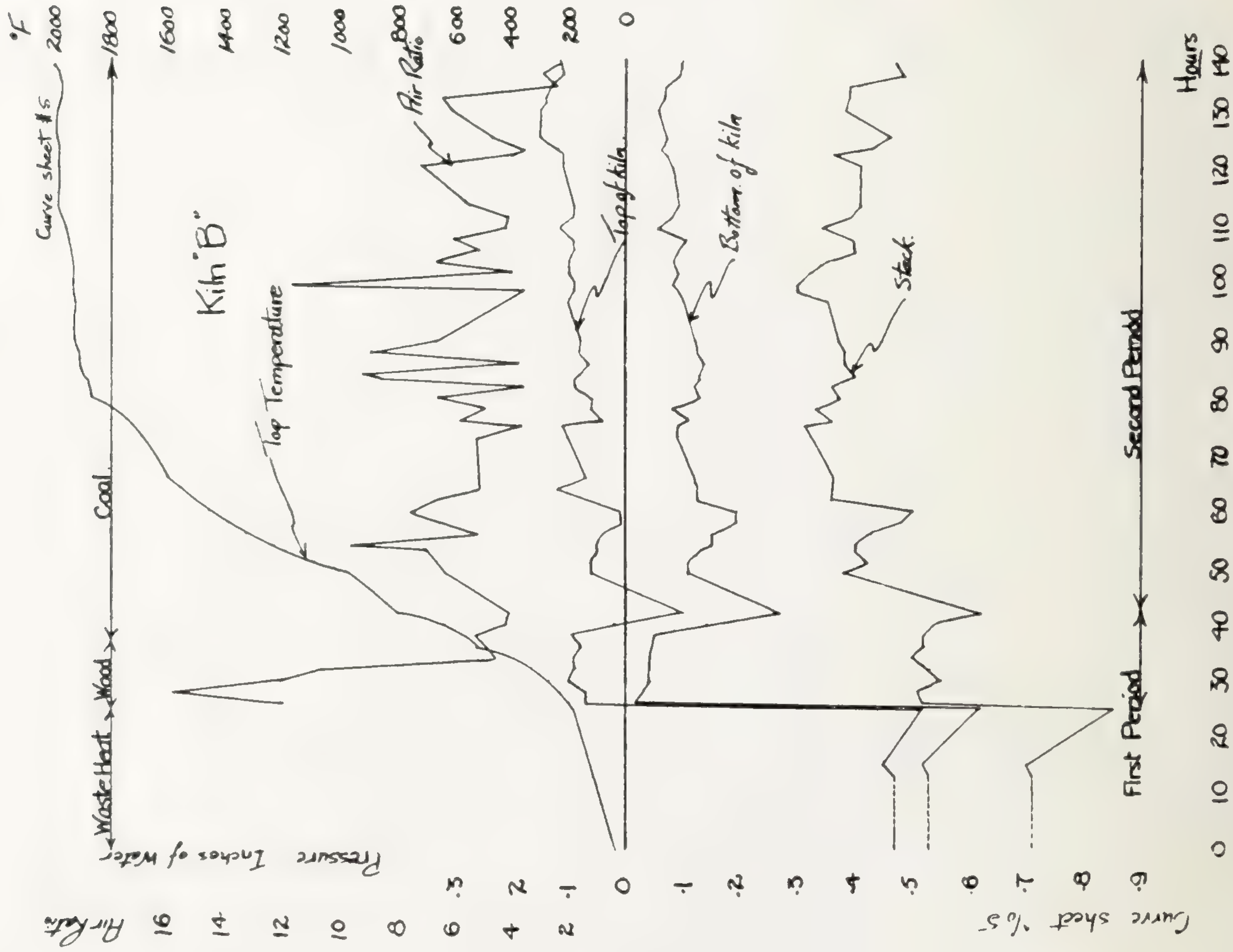
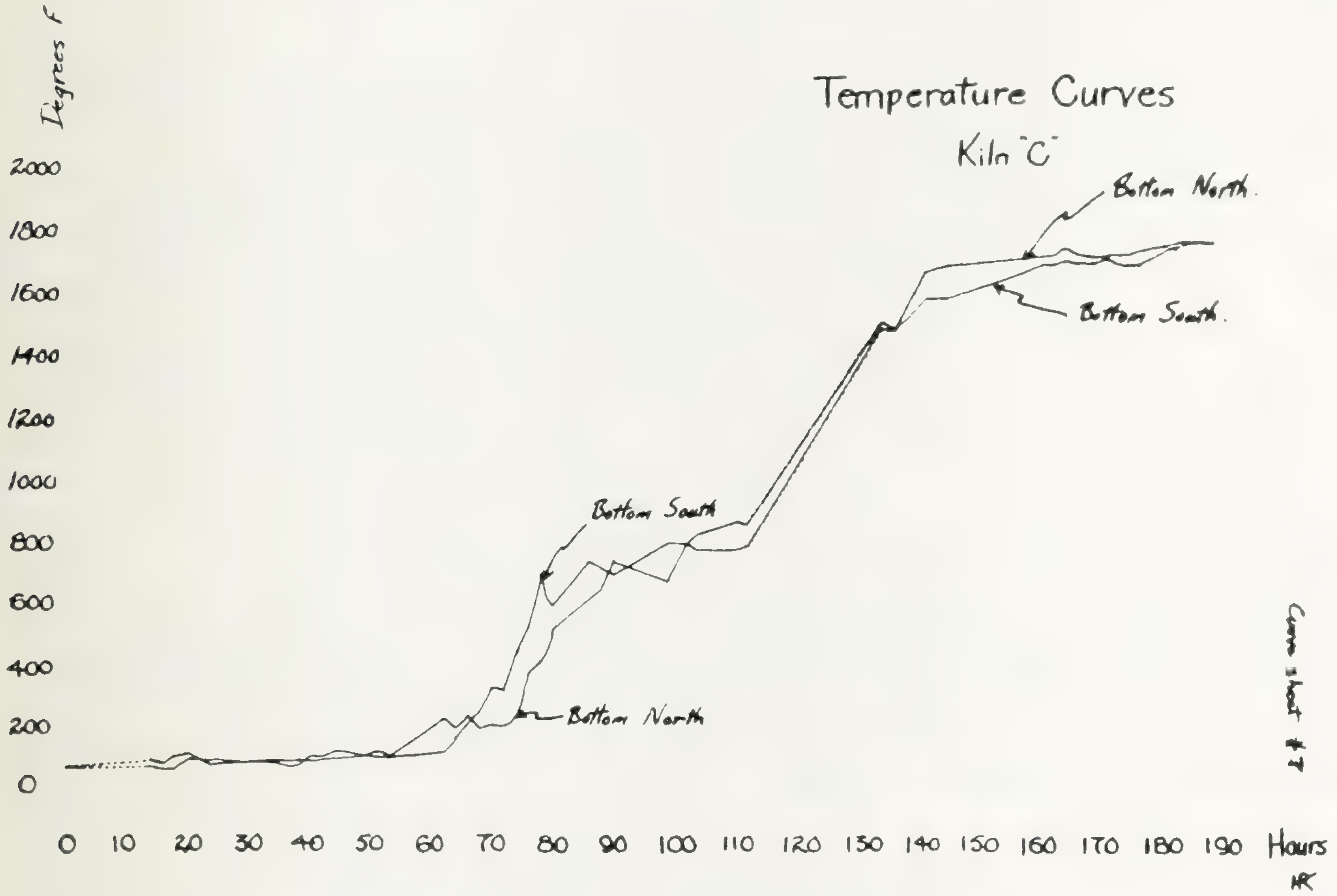


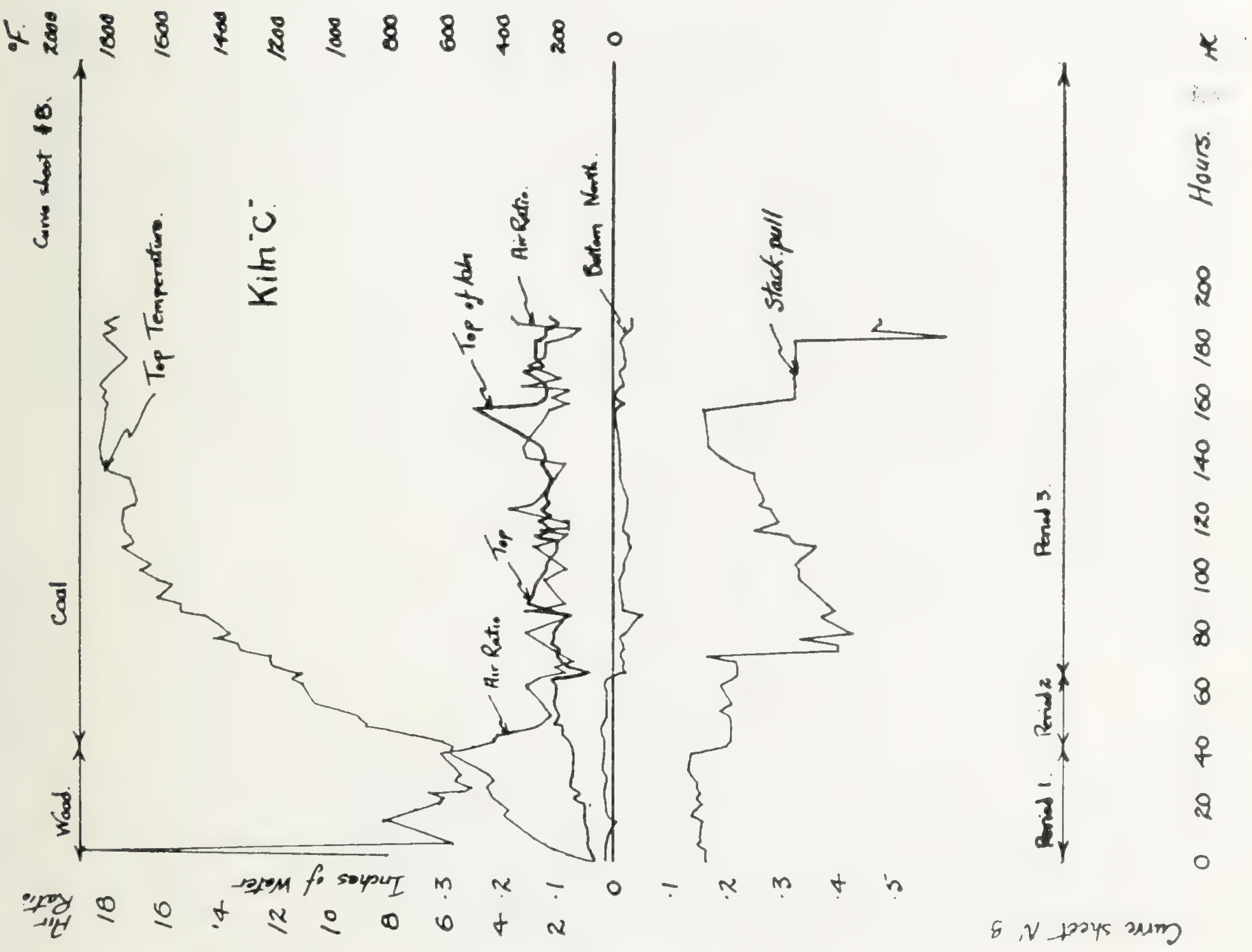
Curve Sheet No. 5



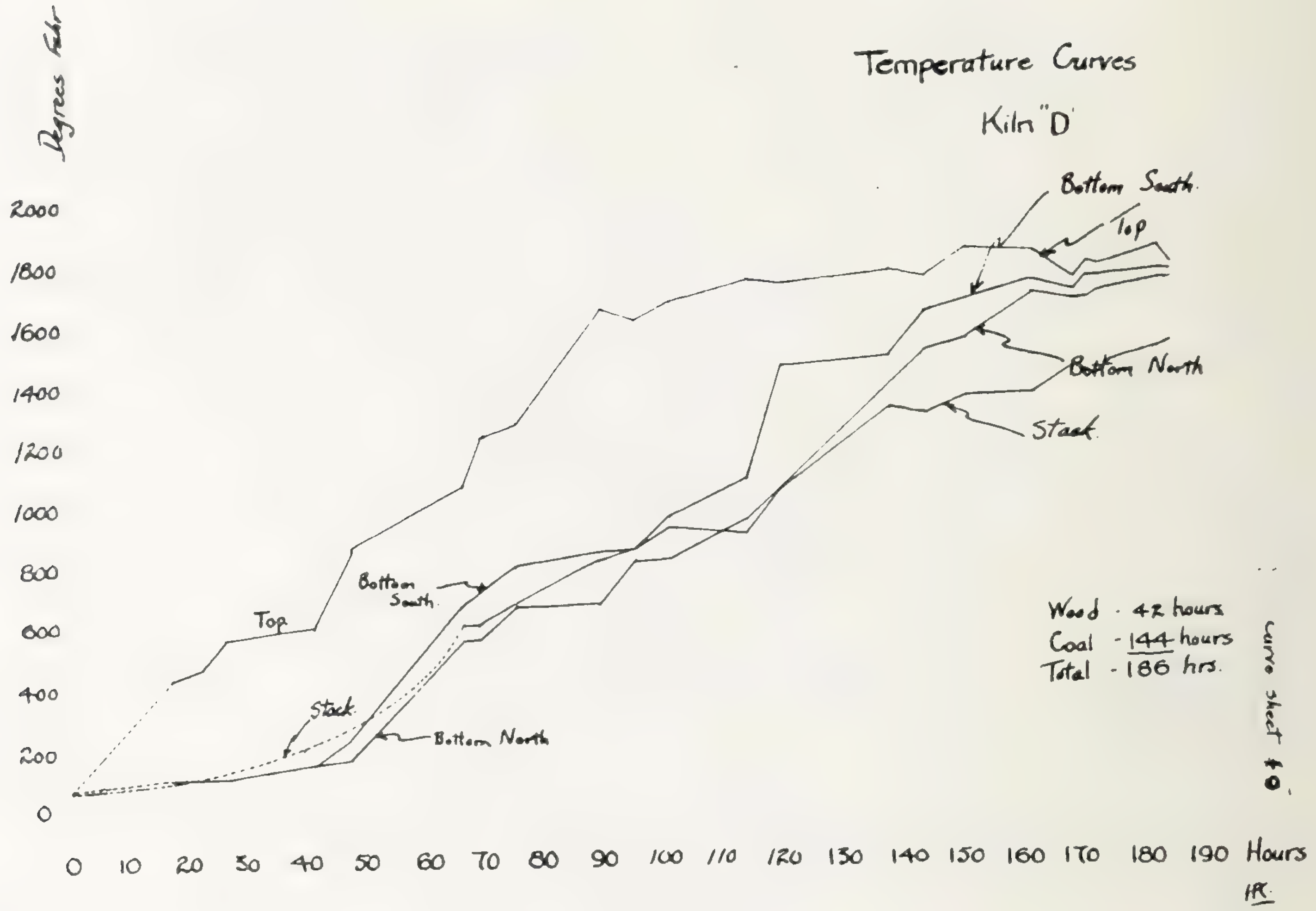
Curve Sheet No. 6



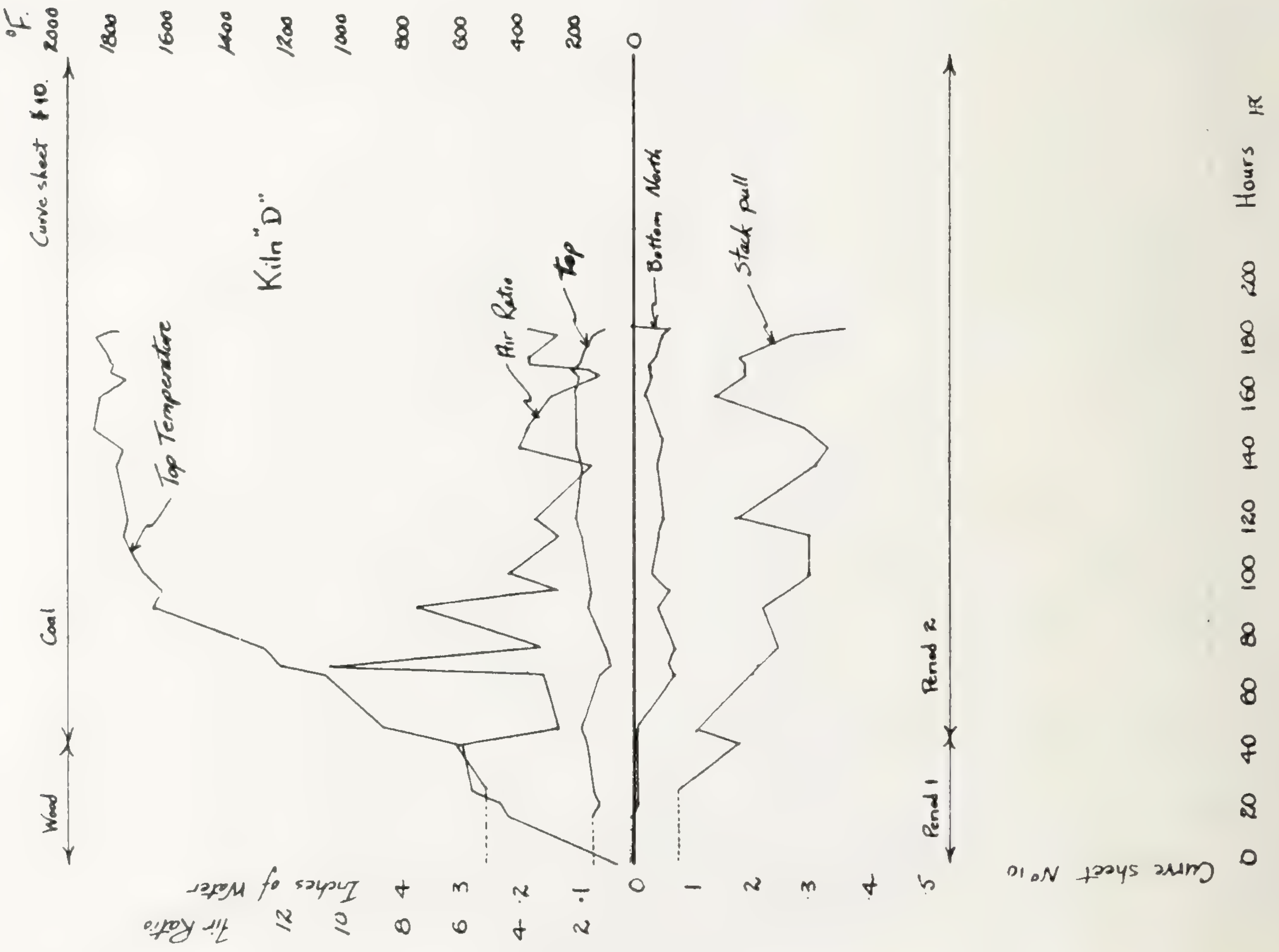
Curve Sheet No. 7



Curve Sheet No. 8



Curve Sheet No. 9



Curve Sheet No. 10

REPORT ON PILOT TESTS CONDUCTED FOR THE STRUCTURAL STEEL WELDING COMMITTEE OF THE AMERICAN BUREAU OF WELDING AT THE UNIVERSITY OF TORONTO, 1927-28

Peter Gillespie, C. A. Hughes, K. B. Jackson, J. H. Fox

Foreword

The Structural Steel Welding Committee of the American Bureau of Welding desires to record its great appreciation of the generous cooperation of Professor Gillespie and the authorities of the University of Toronto in carrying out these "pilot" tests, the thorough manner in which they were carried out and the excellent report of the results which has been prepared.

Professor Gillespie's tests have admirably served the purpose for which they were made and which is described in the introduction to this report. The Committee feels that the main programme has now had a sufficient preliminary trial to insure that conclusions drawn from the final test results of the entire series will be thoroughly dependable for use in the designing of welded steel structures.

The Committee desires particularly to emphasize the fact that these "pilot" tests were made only as a preliminary trial of the main programme. General conclusions should not be drawn from this very limited amount of data in regard to unit strengths, relative performance of different forms of joint, relative showing of the two welding processes, or other similar features. Such deductions should be deferred until the much greater amount of data from a larger number of welders in different fabricating plants, as contemplated by the main programme of the Committee, is available.

J. H. Edwards, *Chairman,*
Structural Steel Welding Committee,
American Bureau of Welding

Introduction

The American Bureau of Welding, which is the Research Department of the American Welding Society, has prepared an extensive programme of systematic investigation of the various types of welded joint that might be used in the construction of steel structures. The primary purpose of this investigation is to obtain reliable information and data upon which safe unit working stresses for the designing of welded structures can be based when using welding as done at the present time in the fabricating plants of the country.

In order to obtain enough data to yield reliable averages under normal conditions of commercial welding, the preparation of the large number of specimen joints of the various forms which are to be studied, (57 forms of joint in 191 sizes), is to be done, not in one fabricating plant, but divided among a large number of such plants throughout the United States and Canada. This plan involved the preparation of rather elaborate instructions in order that the final results of work done in so many different places would be carried out in the same manner and would be reasonably comparable. The instructions include such matters as qualification of welders, welding specifications for test specimens, inspection and supervision of the welding of the test specimens and the procedure for testing the specimens.

Because of the magnitude of the undertaking and the somewhat complex nature of the instructions it was deemed advisable to give the proposed instructions a trial. This would also, incidentally, afford an opportunity to give a little practical experience as to the welding and behaviour of certain unusual forms of joint which had been proposed. The authors offered to make these trial or "pilot" tests, and this report is a record of the work done and the results deduced therefrom.

In the work reported on, the whole-hearted co-operation of certain Toronto firms in supplying labour, plant and material at or below cost is gratefully acknowledged. The Canadian General Electric Company, manufacturers of heavy electric equipment, McGregor & McIntyre, structural steel contractors, The Dominion Oxygen Company, manufacturers of compressed gases, welding equipment and supplies, and Drummond McCall & Co., iron and steel merchants, are the firms referred to. The necessary funds were supplied by the School of Engineering Research, University of Toronto, and by the National Research Council, Ottawa.

Programme

From the original programme of the Structural Steel Welding Committee of the American Bureau of Welding, having regard to the time and facilities available for the work, a selection of 57 different types of test specimen was made. As each type was fabricated in triplicate and as the work was done by arc and by gas in parallel, this involved the making and testing of 342 specimens.

Material

Steel was to be of the grade employed for building purposes and in conformity with A.S.T.M. Specification A-9-21. At the time the tests were begun the steel which had been contracted for was not available and it was necessary to obtain a substitute in the open market. Bar steel was in consequence obtained from McGregor & McIntyre, and was subjected to the usual laboratory tests. While these indicated that the metal generally was of the quality desired, certain fabricated specimens subsequently in testing displayed a brittleness and lack of ductility which showed that it was not entirely of the quality expected. In not more than four instances did this brittleness reveal itself.

Welding wire was supplied, partly by the fabricating firms and partly by Messrs. Drummond McCall & Co. In the gas process a special high test rod was used. In the arc process a rod conforming to the American Welding Society Specification E1A was employed.

Fabrication

The arc fabrication was done at the plants of McGregor & McIntyre and the Canadian General Electric Company. The gas work was done at the plant of the Dominion Oxygen Company. The following is the allotment of the work of fabrication:

In the tabulations which follow, the work of these firms is indicated by the letters M, G and D respectively.

WELDING SCHEDULE
MCGREGOR & MCINTYRE (ELECTRIC ARC)
DESIGNATION LETTER, M

Series	Dimensional Detail	No. of Specimens
1000	22 to 66	15
1600	33 to 63	12
1900	22 to 66	15
2100	22 to 66	15
2300	22 to 66	15
2500	43 & 63	6
2700	43 & 63	6
	Total..........	84

CANADIAN GENERAL ELECTRIC (ELECTRIC ARC)
DESIGNATION LETTER, G

Series	Dimensional Detail	No. of Specimens
2900	22 & 23	6
3000	44 & 66	6
3100	44 & 66	6
5300	22 & 33	6

Series	Dimensional Detail	No. of Specimens
5400	44 & 66	6
5500	22 & 33	6
5600	44 & 66	6
6000	23, 43, 63	9
6100	23, 43, 63	9
6300	23, 43, 63	9
6700	25 & 35	6
6800	21 & 42	6
6900	21 & 42	6
	Total............	87

DOMINION OXYGEN COMPANY (GAS)
DESIGNATION LETTER, D

Series	Dimensional Detail	No. of Specimens
1000	22 to 66	15
1600	33 to 63	12
1900	22 to 66	15
2100	22 to 66	15
2300	22 to 66	15
2500	43 & 63	6
2700	43 & 63	6
2900	22 & 23	6
3000	44 & 66	6
3100	44 & 66	6
5300	22 & 33	6
5400	44 & 66	6
5500	22 & 33	6
5600	44 & 66	6
6000	23, 43, 63	9
6100	23, 43, 63	9
6300	23, 43, 63	9
6700	25 & 35	6
6800	21 & 42	6
6900	21 & 42	6
	Total..........	171

Welders

All welders whose work was finally accepted were qualified rigidly under the Committee's specification. Much difficulty was experienced in this regard. In the case of welders of promise who failed at first to come up to the Committee's standards, it was necessary to repeat the qualification tests several times. In other instances, where the case seemed hopeless from the first, the candidate was refused a repeat trial and another man was selected. This process of trial and elimination consumed much time and cost considerable money. Five welders were finally obtained whose work came up to the standard of the specification. These welders were designated A, B, C, D and E, the three former being gas operators and the two latter arc operators.

Inspection

The shop fabrication and subsequent inspection were supervised by men accustomed to record-making and testing, but not initially familiar with the processes of welding. The Committee's rules of procedure were faithfully followed and the records both of fabrication and test have been preserved.

A special system of numerical designation, it will be observed, has been adopted. The first two digits in a number may be considered as the series number proper. The third digit denotes plate thickness in eighths of an inch, and the fourth digit, the weld dimension (usually the side of a right angled fillet) also in eighths of an inch. A three-eighths fillet, for example, is one, the two rectangular sides of which are nominally three-eighths of an inch long.

Graphical Results

The results of the investigation and of a subsequent analysis of the test data are reported partly in graphical form in Figs. 1, 2, 3 and 4, and partly in tabular form in Tables 1 to 16 inclusive. It was recognized that theoretically, other things being equal, the strengths of fillet welds should be proportional to the linear dimensions of the fillet triangles, and that if the strengths per lineal inch were plotted against fillet dimensions (or against final digits in the series numbers) the graph would approach a straight line passing through the origin. This is illustrated particularly in Fig. 1, and to some extent also in Fig. 4. It should be pointed out that the true origin is generally not shown on the charts. For Series 2,500, 2,700, 6,000, 6,100, 1,600 and 6,300 it is obvious that the theoretic strength line is horizontal.

In Fig. 2 it will be seen that reference or standard lines have been drawn to which the actual strengths obtained may be referred. For Series 2,500 and 2,700, these lines are labelled "Specimens 1,933 (gas)" and "Specimens 1,933 (arc)", these lines showing the best strengths in both processes obtained for joints with which the 2,500 and 2,700 Series might in fairness be compared. For Series 6,000 and 6,100, the corresponding lines are designated "Specimens 2,333 (gas)" and "Specimens 2,333 (arc)". The eye has thus little difficulty in estimating the ratio of the strengths actually realized with the best obtained in both processes in other Series roughly comparable with them. In Series 5,300, 5,400, 5,500 and 5,600, appearing on Fig. 4, analogous standard lines are also plotted.

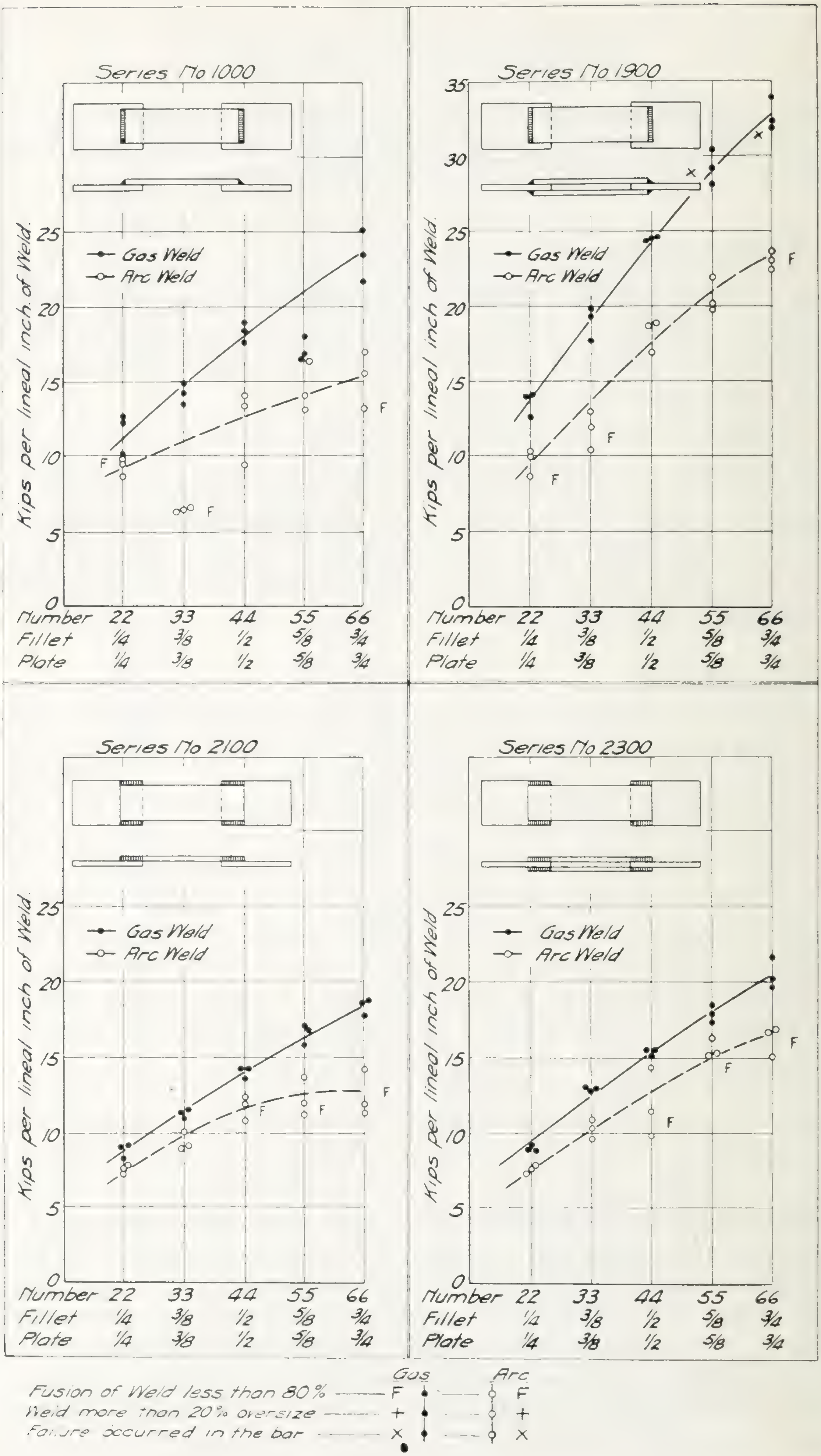


Fig. 1. Results of Tests Shown Graphically.

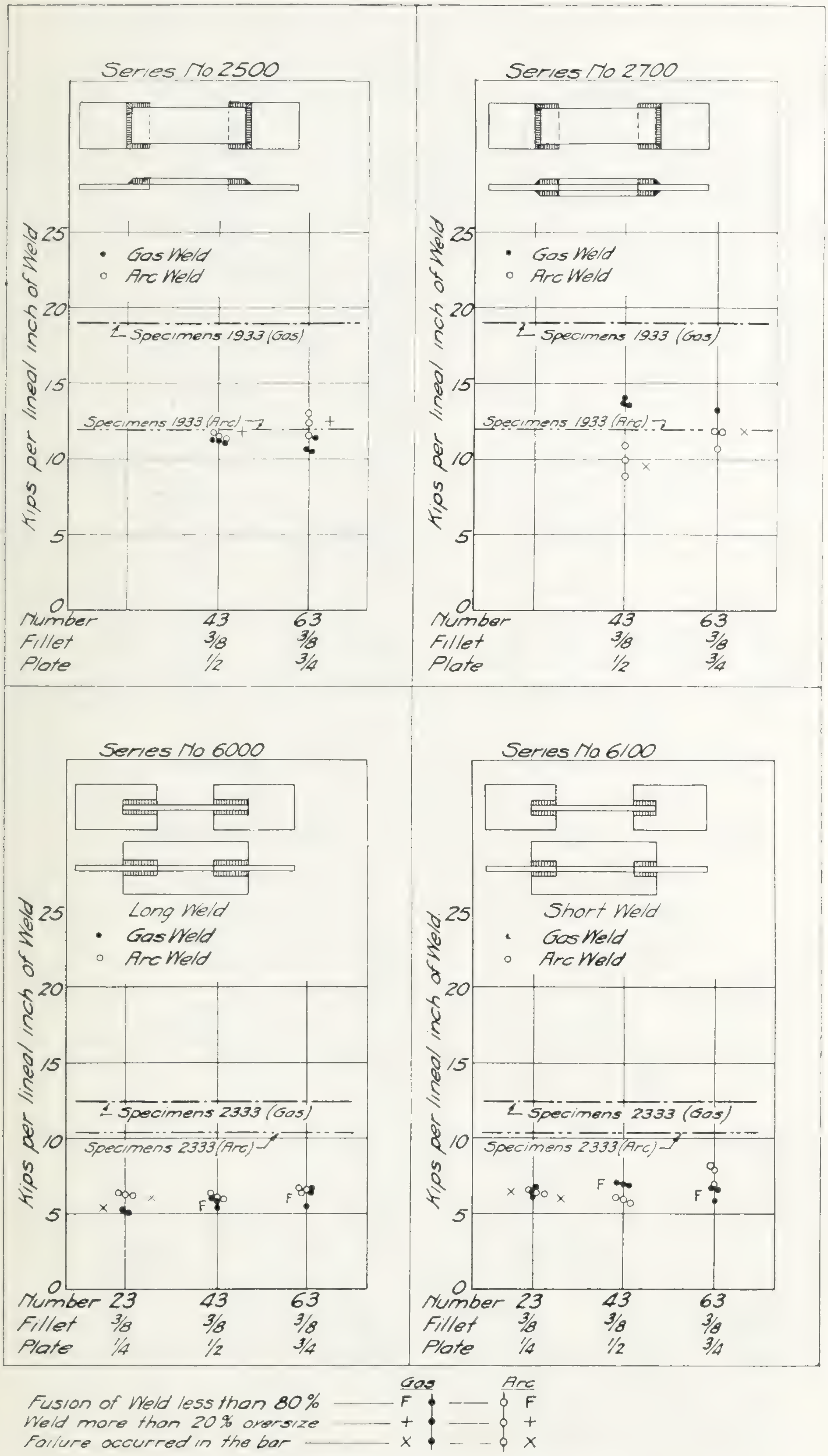


Fig. 2. Results of Tests Shown Graphically.

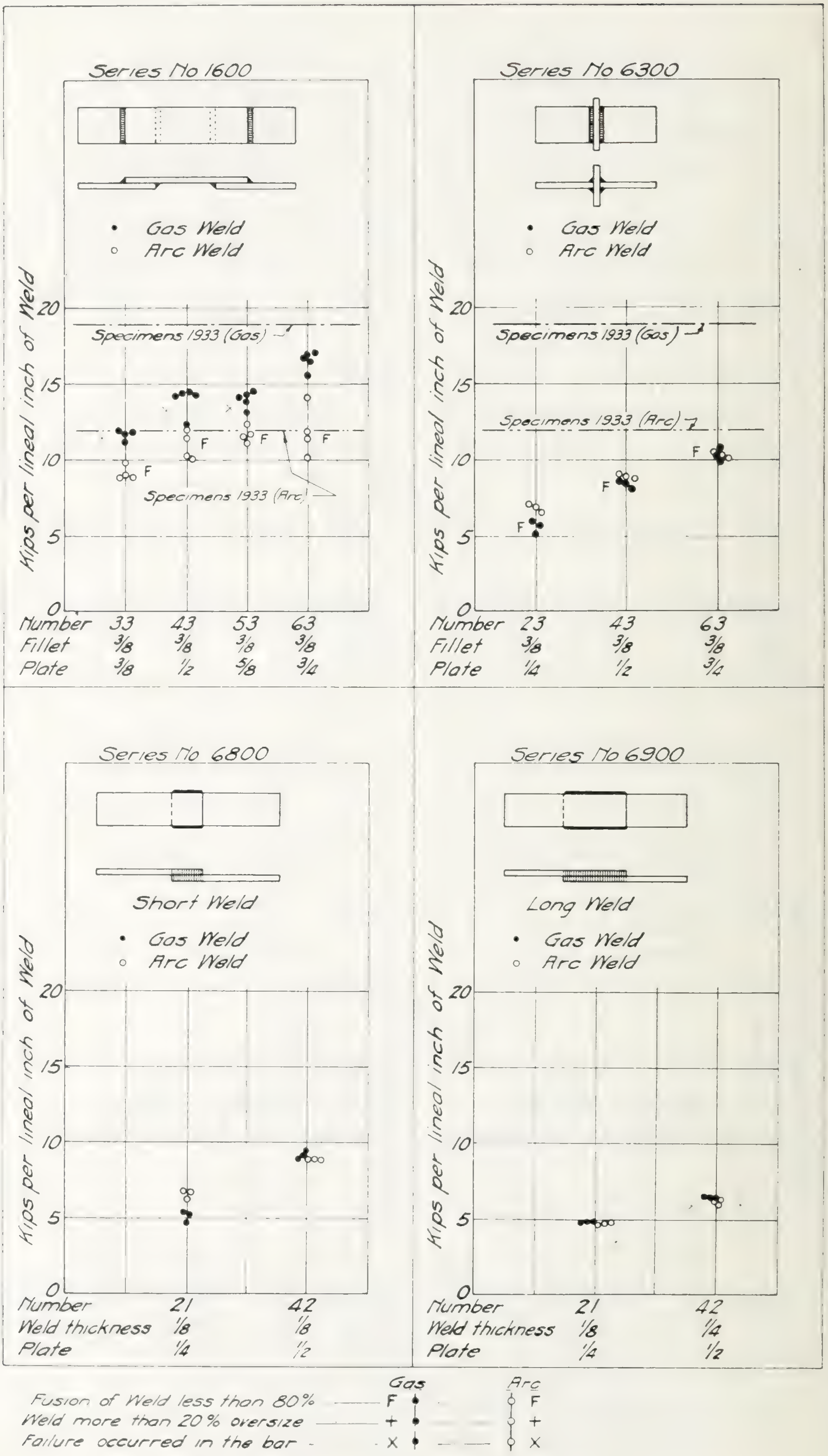


Fig. 3. Results of Tests Shown Graphically.

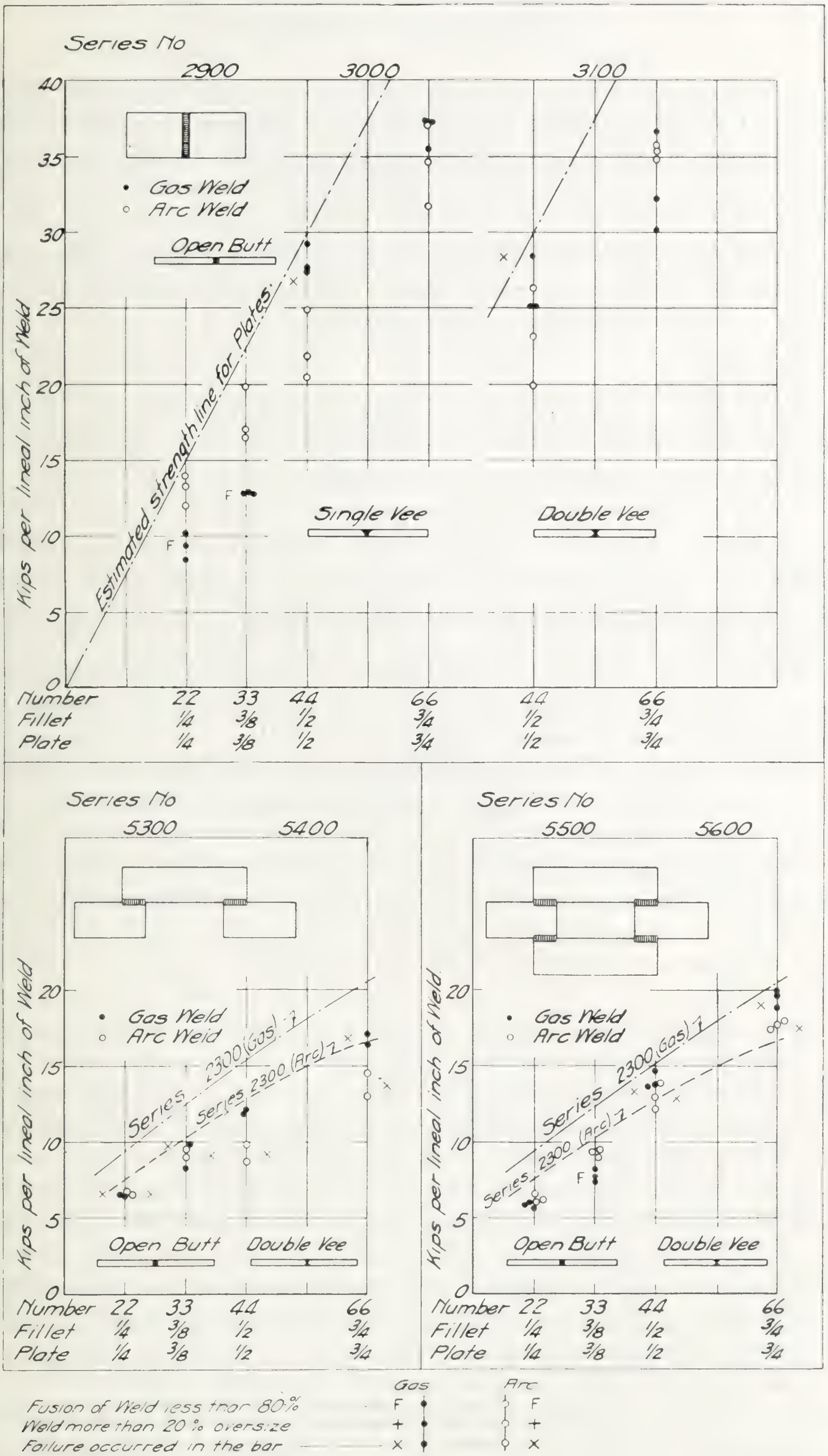


Fig. 4. Results of Tests Shown Graphically.

In the case of normal butt welds (Series 2,900, 3,000 and 3,100) a reference line or standard was found in the estimated strength of the uncut plate, which was assumed good for 60,000 lb. per sq. in. As before, the efficiency of the weld may be quickly though approximately estimated at a glance.

A few diacritical marks have been employed, the key to which will be found at the bottom of each chart. These in some instances serve to explain anomalous positions of plotted points.

Tabulated Results

In Tables 1 to 11 inclusive, entitled "Fabrication and Test Data", will be found much condensed information regarding the specimens, some of it duplicated from the charts above referred to.

It was a very easy matter to compute the length and weight of wire of any size necessary to build up a 1-inch length of fillet of any assumed dimensions. This has been done and the results appear in Tables 12 and 13. In making the computations from which the figures in these tables resulted, the dimensional standards of the Structural Steel Welding Committee were carefully followed, reasonable approximations respecting the form of the deposited material having been made to facilitate the calculation of its volume and weight.

In explanation of "Section Relative to Nominal", it may be pointed out that in Table No. 1 (Series 1,000), for example, the value of this factor for specimen 1022D1 is 1.5. After a measurement of the weld with fillet gauges the inspector estimated that the fillet was about 1.5 times its nominal section. In "normal" welds the direction of the axis of the fillet is, of course, transverse to the direction of pull. "Parallel" welds have an axis of fillet parallel to the direction of pull. A "kip" is 1,000 pounds.

Fusion or the bonding of deposited metal with base metal was appraised by the eye at the place of failure. Defective fusion in small fillets made by the arc process generally occurred at the square angle of the fillet. In the larger arc fillets a greater proportion of this defect occurred on the face of the *c* or end bar and especially at the junction of successive layers of deposited metal. In certain of the gas welds (notably the 6300 Series), to defective fusion at the rectangular apex of the fillet was added a weakness in the fillet due to marked concavity therein. A modified throat failure across the resulting small section was thus to be expected. This failure always occurred adjacent to that bar which offered the best conditions for the dissipation of heat.

Observations on the yield point were only indifferently satisfactory. Two methods of obtaining the yield point were employed—the shedding of scale from the welds and the drop of the beam. The former was difficult to observe and the latter usually indicated the yield point of the bars rather than the yield point of the weld. Both are reported in part at least.

The strength in kips "per lineal inch" of weld and "per sq. inch" of weld require little explanation. For the latter the strength per linear inch is divided by the length of the "throat" dimension and also by the ratio "Section Relative to Nominal", appearing in column 3. This use of this ratio is warranted provided the fillet be free from concavity or convexity, a condition not always realized.

Additional data respecting the character of the failure of specimens are contained in Tables 14, 15 and 16. Here an attempt to show fusion percentages in a way that permits of ready appraisal by the eye has been made. The close grouping of the fusion indicators in Series 1,022 gas, for example, is somewhat in contrast with the scattering observed in Series 1,022 arc. The remaining columns in these tables are in the main self-explanatory. It was, of course, very difficult to provide in a schedule for all possible types of failure, in consequence of which only certain frequently occurring features are provided for at the column heads. Failure through the "throat" means failure along a normal to the fillet hypotenuse through the rectangular apex. In general, normal welds broke with a crystalline fracture on the end of the *b* (or central) bar or with a silky fracture through the throat of the weld. Parallel welds in general broke with a silky fracture through the throat or on the face of the *c* (or end) bar. All specimens were designed to fail in the weld, but this did not always happen. Such failures have been carefully noted and should be considered in any attempt to generalize from the test data.

Comparison of Welders

An attempt to show some of the welders' characteristics, both graphically and numerically, was made, with results that appear in part in the tabulations. The reported percentages for fusion reflect the judgment of the tester in respect to one important evidence of skill. A value for fusion consistently high is always a favourable indication. Low values are most undesirable, as is also much variation in the fusion percentage. The mean value of this percentage was found for each welder.

The extent to which the strengths of welds of the same series of fabricated specimens varied gave a means whereby a dispersion factor for each welder could be computed. Suppose, for example, the strengths of the three specimens of a given type are 7, 11 and 12 kips per lineal inch respectively. The mean strength is 10, and the various departures from this mean are 30, 10 and 20% respectively. Ignoring the sign of the residuals the mean percentage is 20, and this may be taken as an indication of the welder's dispersion. This value was found for each group of three specimens of each type made by each welder. The mean for each welder's entire work was then obtained and this, together with the mean fusion percentage, is tabulated herewith. For the former, a small value is desirable, since it denotes uniformity and consistency; for the latter a high percentage is obviously preferred.

DISPERSION AND FUSION PERCENTAGES

Welder	A	B	C	D	E
Dispersion Percentage	3.0	3.1	6.3	5.6	4.1
Fusion Percentage	98	88	49	75	97

COMMENTS ON INDIVIDUAL SERIES

Series 1000

Of the 15 gas specimens in this Series, 5 exhibited silky fractures and 10 square breaks on the ends of the *b* bars. The lower position of 1022 D1 on the graph is explained by defective fusion and is so indicated. The 1055 D group, it will be observed, are all low in strength, but for this no explanation suggested itself on an examination of the broken specimens.

The arc welds in this Series are substantially lower in strength than the gas welds. Of the 15 tests, 13 broke square on the end of the *b* (or central) bar. The bulk of those tests which lie conspicuously below the average line can be explained by the presence of poor fusion. In five of the six tests of 1055 M and 1066 M, well defined lines of separation could be recognized in the broken fillet between successive layers of deposited metal, these fillets having been laid down in two successive operations. The union between the first layer and the second was quite incomplete.

It is thought that the large number of square breaks on the ends of *b* bars for both gas and arc in this Series is due to bending at the fillet, resulting from the unsymmetrical form of the specimens.

Series 1900

The most noticeable feature in this Series is that, in contrast to the 1000 Series, a large number of the failures occur through the throat of the fillets, and the strengths per inch of fillet are about one-third greater than in the 1000 Series. Both phenomena are doubtless due to the symmetrical form of specimen. Here again the low points on the arc process record (1922M, 1933M and 1966M) appear to be due to poor fusion. In a number of gas welds the fillet tore out a portion of the base metal in the *c* bar. This invariably accompanied high strength.

Three of the gas welds (two in 1955D and one in 1966D) failed in the *c* bars, the fracture being coarsely granular and showing little evidence of ductility. At first it was thought that perhaps the heat applied during welding had produced a change in the molecular structure of the metal. An examination of the grain size under the microscope by Professor Newcombe, however, showed no difference in structure between the metal of the *c* bar adjacent to the weld and the metal in the same bar distantly remote therefrom. The inference is that the steel from which these bars were cut was distinctly different from that contemplated in specification A-9-21, A.S.T.M.

Series 2100

It is suggested that the lesser strengths in this Series, in comparison with those of Series 1000, with which it is to some extent comparable, are due to the fact that in parallel welds the shear stress in fillets varies in intensity from point to point throughout its length, whereas in normal welds, the stress distribution is more nearly uniform. The large number of failures on the face of the *c* bars (which presented a horizontal surface to the welding process) indicated defective fusion on the *c* bar surfaces. In 2144M, 2155M and 2166M, there was a marked separation between successively placed layers of deposited material, these fillets having been laid down in three operations. This, of course, is a source of weakness.

Series 2300

The fusion in the arc specimens was often unsatisfactory, and this helps to explain the lower values for strength obtained. The gas welds failed in the throat of the fillet, or a portion of the *c* bar was detached by the fillet and came away with it. As before, these phenomena accompanied high strength. In the 2366 M specimens, characteristic divisions between successively deposited layers of fillet material were observed.

Series 2500 and 2700

In Series 2500, the strengths of gas and arc welds are more nearly equal than in any previous series, this doubtless being due to the fact that many of the arc welds were much larger than the specified size. In Series 2700, many of the test specimens broke in the bars so that deductions are difficult to make. In both cases the normal weld ruptured first, after which the parallel weld was observed to tear. In the arc process specimens, the frequently observed laminations in two and three layer fillets were again seen.

Series 5300 and 5400

These specimens were unsatisfactory in that generally they failed in the bars by tearing, due to eccentric loading.

Series 6000 and 6100

The unsymmetrical loading of the *b* bars in both these Series invited bar failure, and this actually occurred in 6023 and 6123, both gas and arc. It is believed that gas welders *B* and *C* possessed less skill than gas welder *A* since the fusion is less satisfactory. Several of the gas failures are traceable to this circumstance.

Series 6300

In practically all cases the gas welds were defective in fusion, particularly at the rectangular apex of the fillet. This was more noticeable on one side, presumably the first to be welded, than on the other. The extent to which all strengths fall below the 1933 gas and arc standards can be estimated at a glance. In the arc welds of this Series, all breaks occurred through the throat, the deposited metal being generally of poor quality.

DEDUCTIONS

1. Probably the most important influence on the strength of the work is the skill of the welder. Defective fusion accounts for more weaknesses in welds than any other condition. This defect occurs most frequently at the rectangular apex of the triangular fillet. (See Fig. 7-1044 Ml.)

2. Arc welding in two or more layers usually resulted in a line of defective work between successive layers that was a source of weakness. This may have been due to failure to thoroughly clean the first layer before depositing the second. (See Fig. 7-1055 Ml and 1066 Ml.)

3. Normal welds, other things being equal, are stronger than parallel welds.

4. There is a tendency for the normal weld to rupture first when both normal and parallel welds occur at a joint. (See Fig. 5-2543 D1.)

5. Symmetrically loaded welds are stronger than those with eccentric loading. Any joint wherein application of load of necessity means flexure in the joint is objectionable. (See Fig. 1.)

6. The practice of weighing specimens before and after welding in order to ascertain the quantity of deposited metal is nearly valueless unless the weigh scales are of more than the usual accuracy.

7. A combination of defective fusion at the rectangular apex of a fillet with a concave hypotenuse makes for very pronounced weakness. This combination was common with one of the gas welders.

8. The great difference in fusion obtained in two consecutive gas welds in close proximity presumably indicates inadequate preheating for the first weld. (See Fig. 14-6323 D1).

9. Where the end of one bar is welded to the face of another, poor fusion is more likely to occur on the face, due either to more favourable condition for the dissipation of heat or to the unsuitable direction of the electrode. (See Fig. 14-6323 D1.)

10. Those welders whose mean fusion percentage is high usually maintain a consistently high average for strength also. (See comparison of Welders.)

11. There was little evidence that marked changes in grain structure resulted from the heating of the base metal in welding.

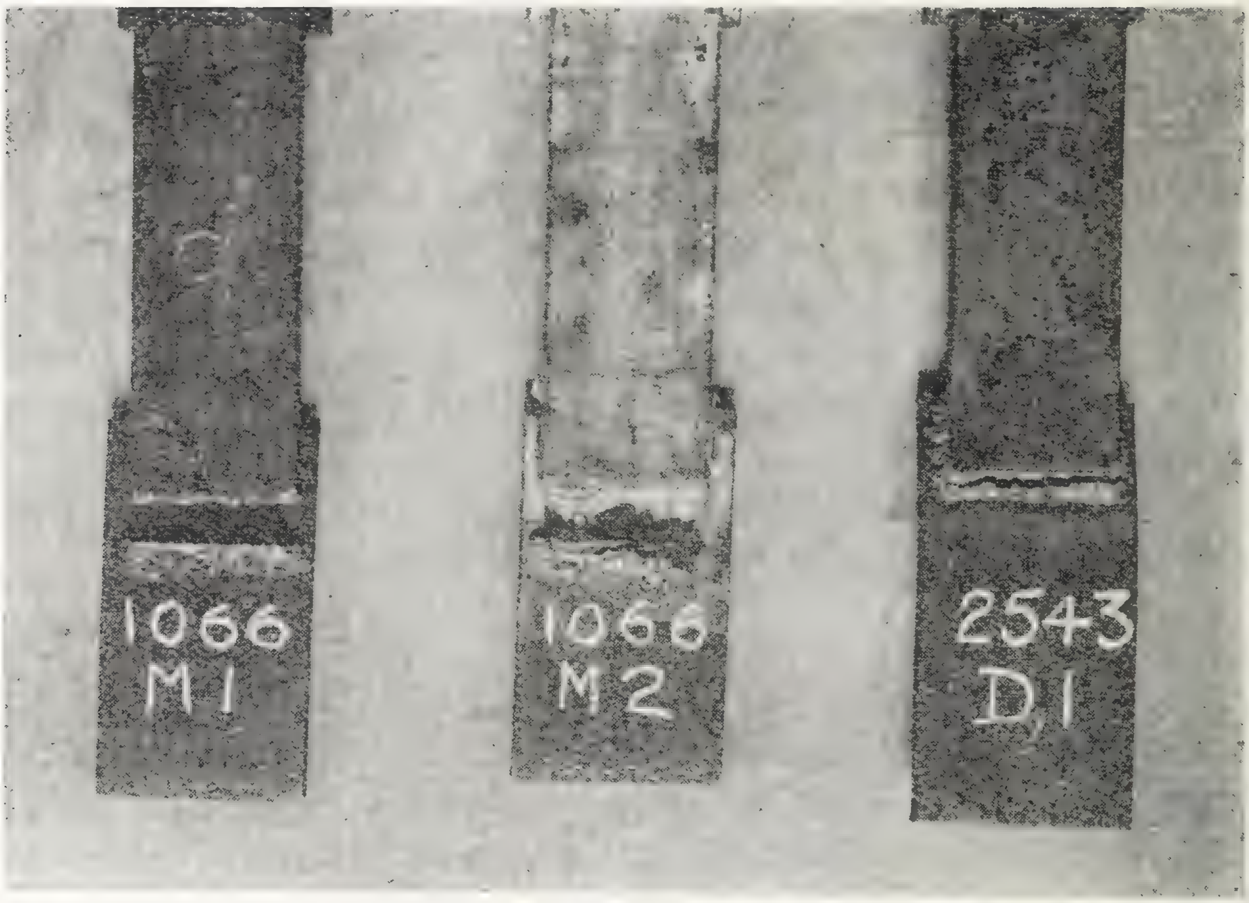


Fig. 5. 1066 M1—Crystalline Fracture in Weld at end of *b* Bar.
1066 M2—Poor Fusion at Face of *c* Bar.
2543 D1—Normal Weld Failure Preceding Parallel Weld Failure in Combined Welds.

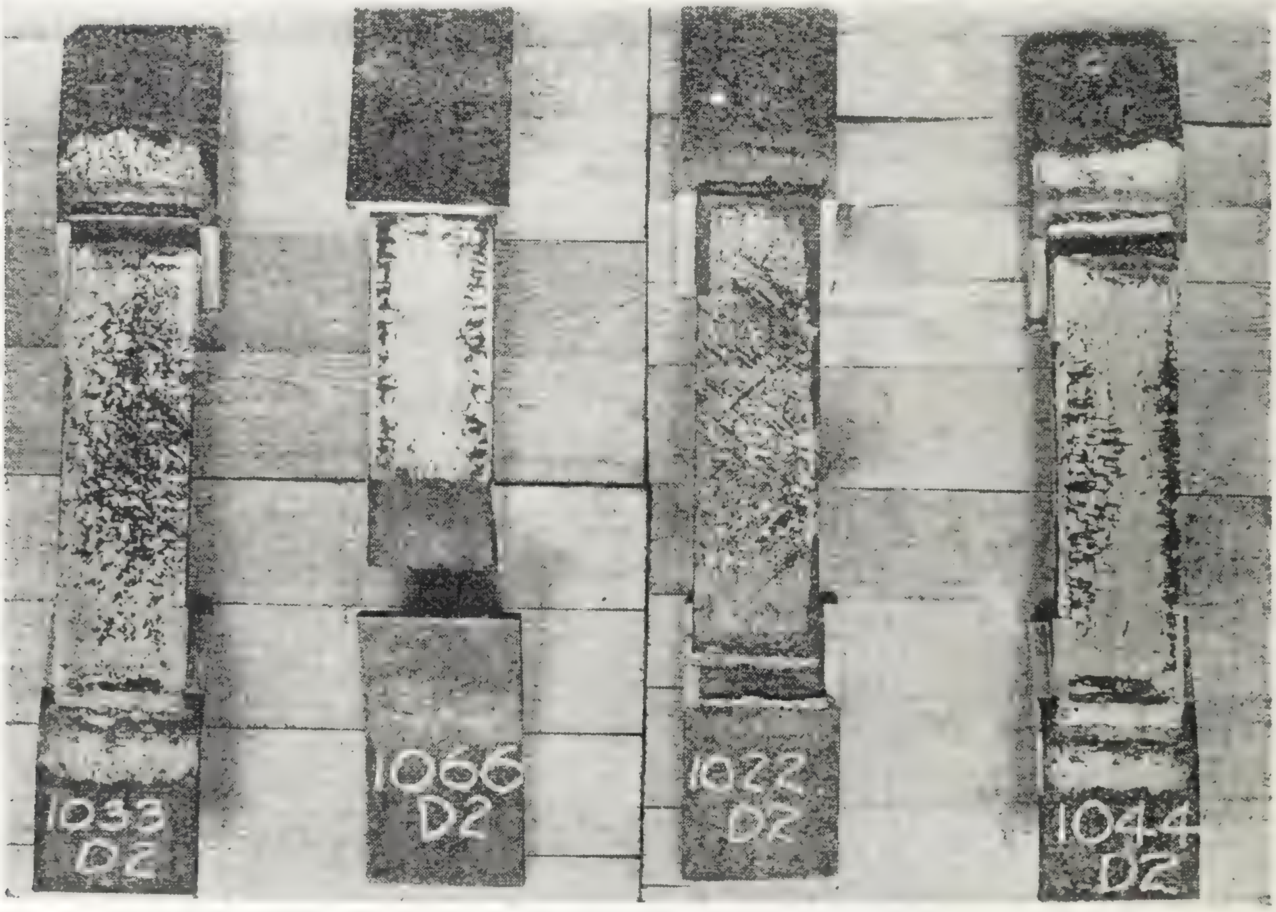


Fig. 6. Illustrating the Manner in which Whitewash is Shed.

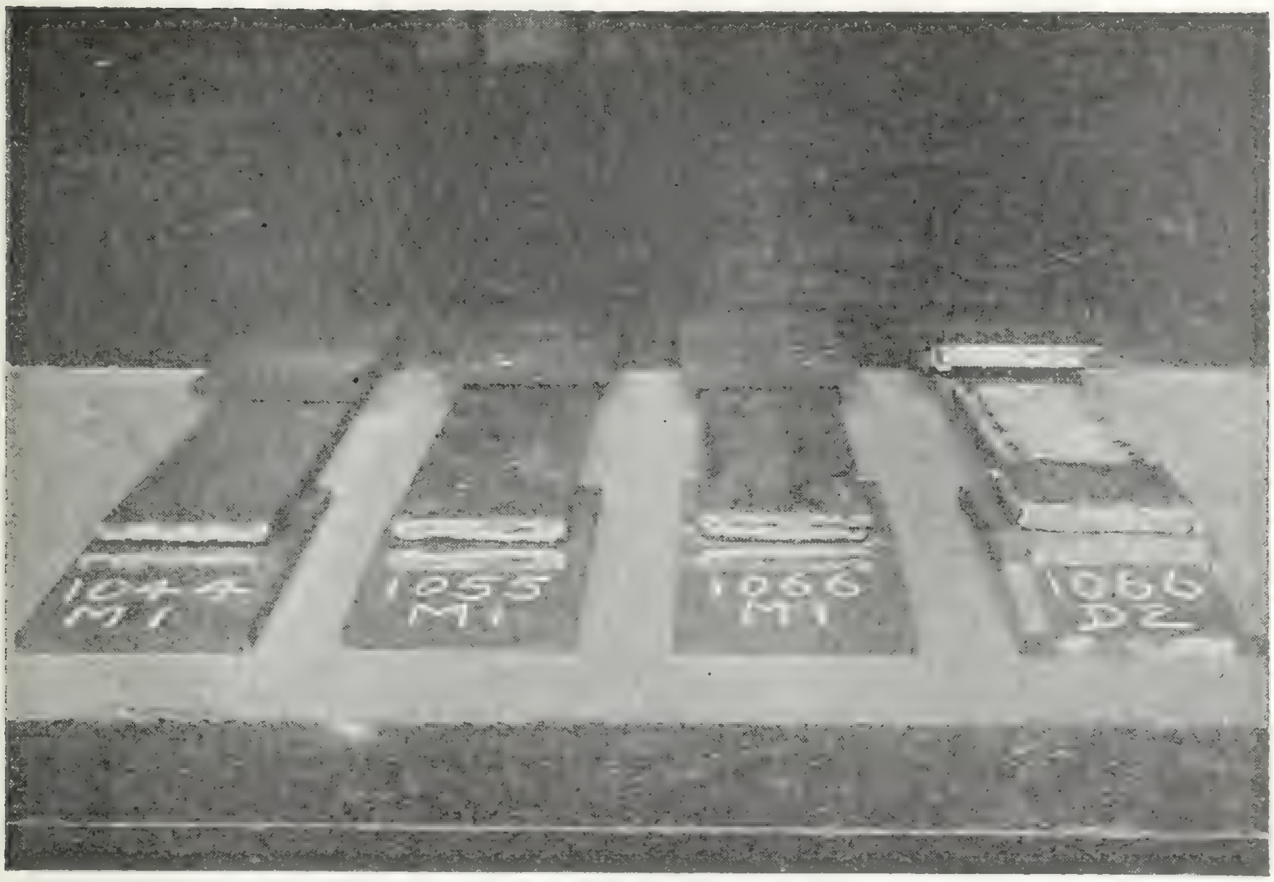


Fig. 7. 1044 M1—Poor Fusion at Square Angle of Fillet.
1055 M1 and 1066 M1—Defective Bonding Between Consecutive Layers of Deposited Metal.
1066 D2—Crystalline Fracture.
Note.—The *b* bars have been folded back from their tested positions.

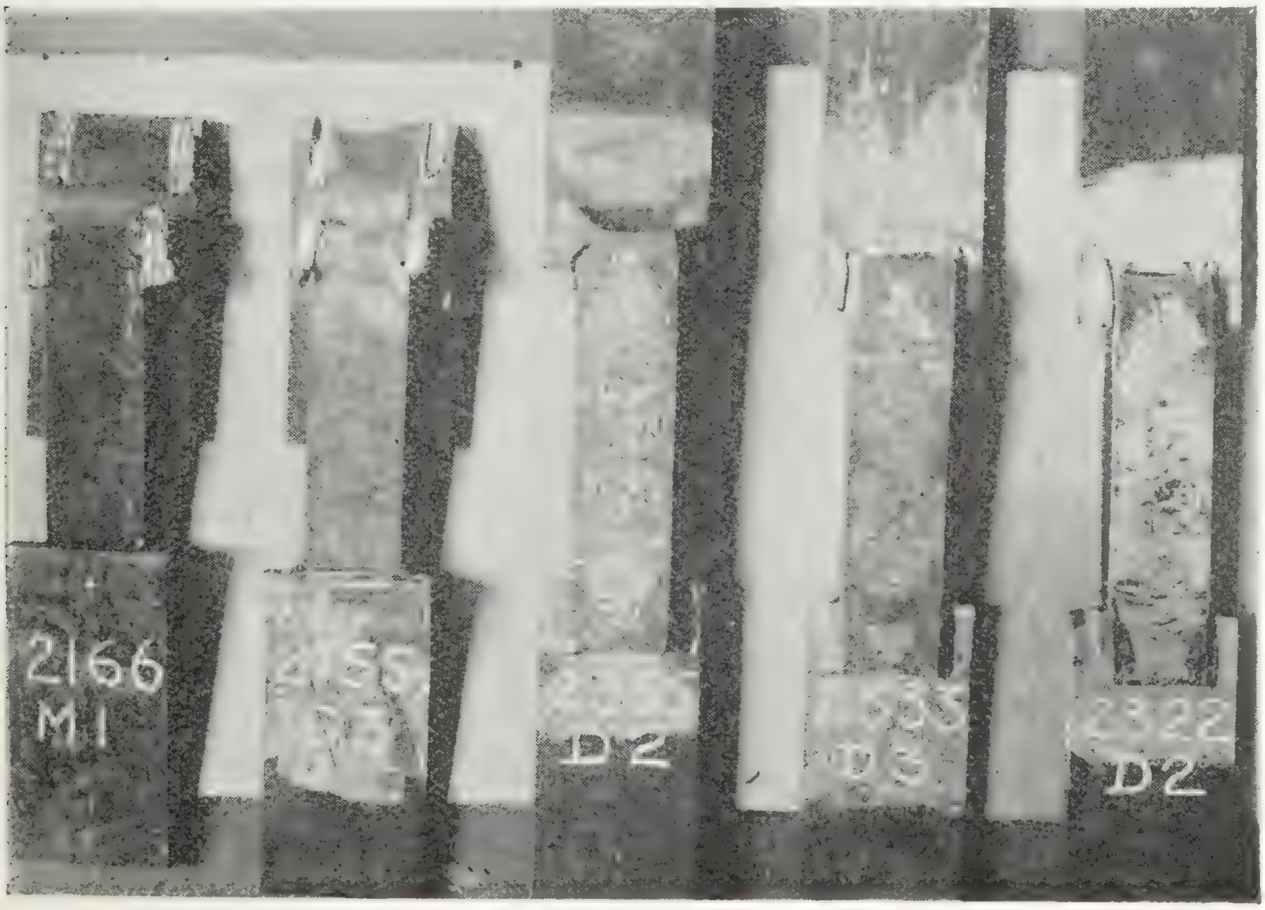


Fig. 8. 2166 M1—Poor Fusion and Consequent Failure on Face of *c* Bar.
2155 D3—Good Fusion Causing Gouging Out of *c* Bar.
2355 D2—2333 D3—2322 D2—Various Types of Failure in Parallel Welds.

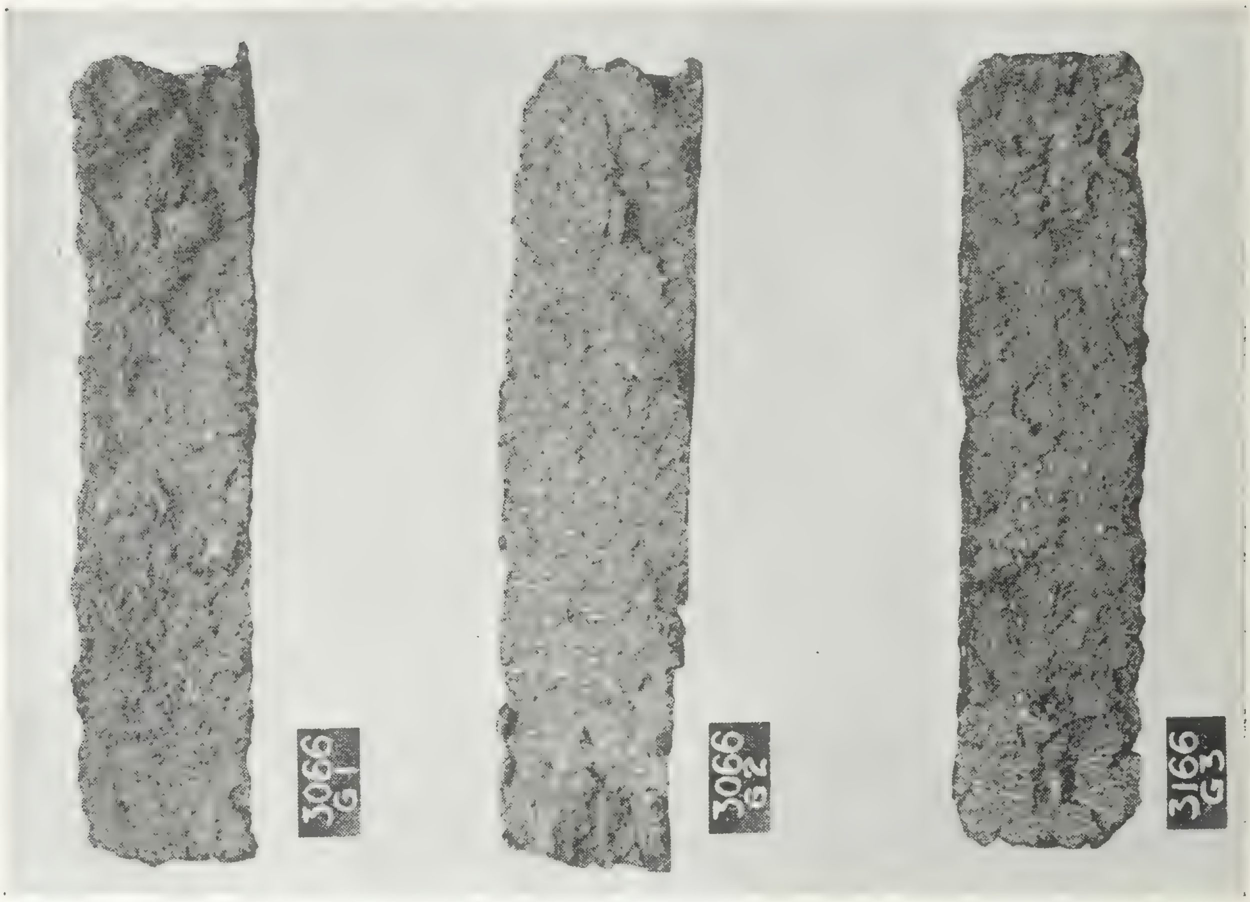


Fig. 9. Texture at Fracture in Butt Welds. 3066 G1 and 3166 G3 are porous. 3066 G2 is finely Crystalline.

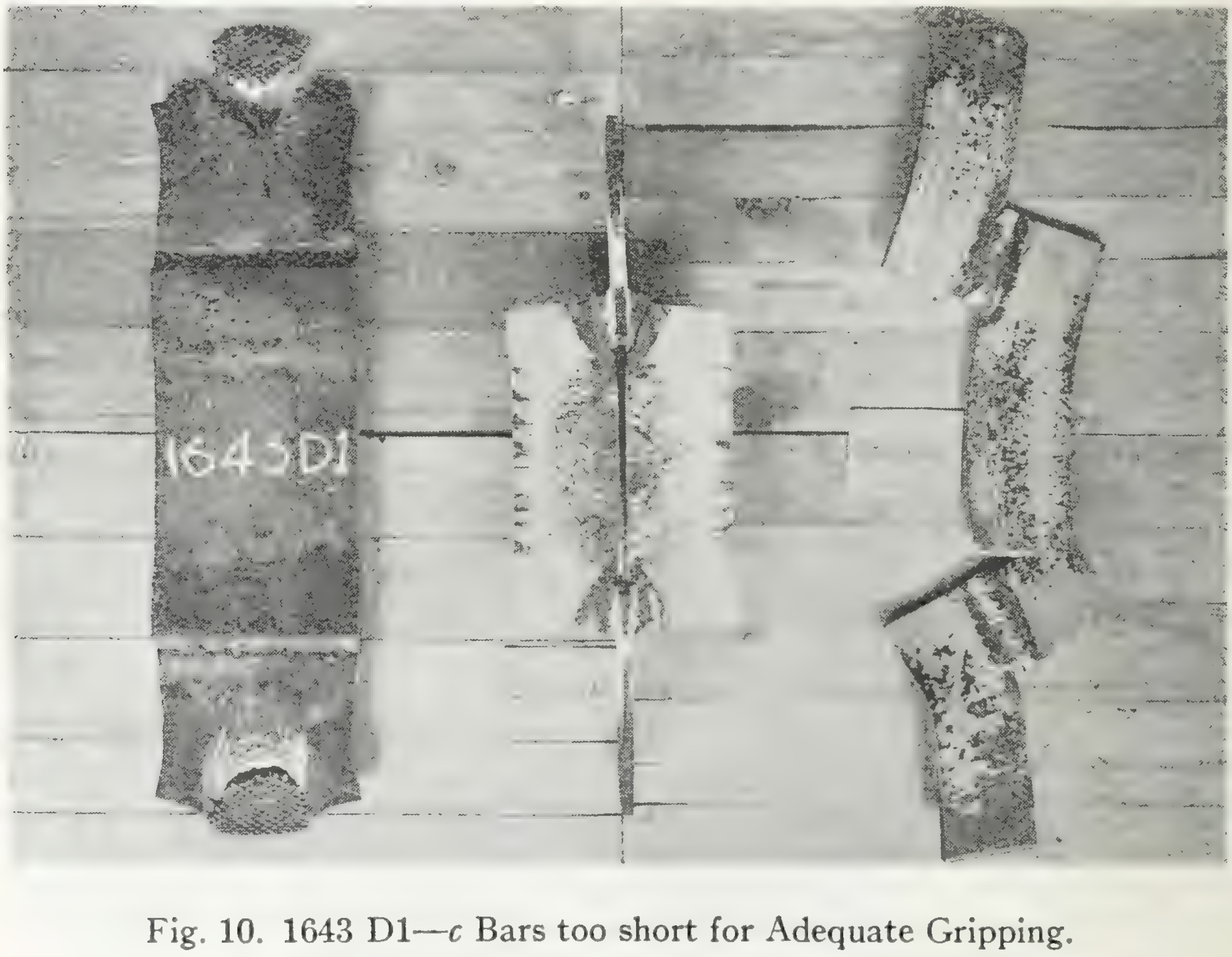


Fig. 10. 1643 D1—*c* Bars too short for Adequate Gripping.
6023 D2—Stress Distribution Illustrated by Whitewash.
5333 D2—Typical Failure by Tearing of Bars.

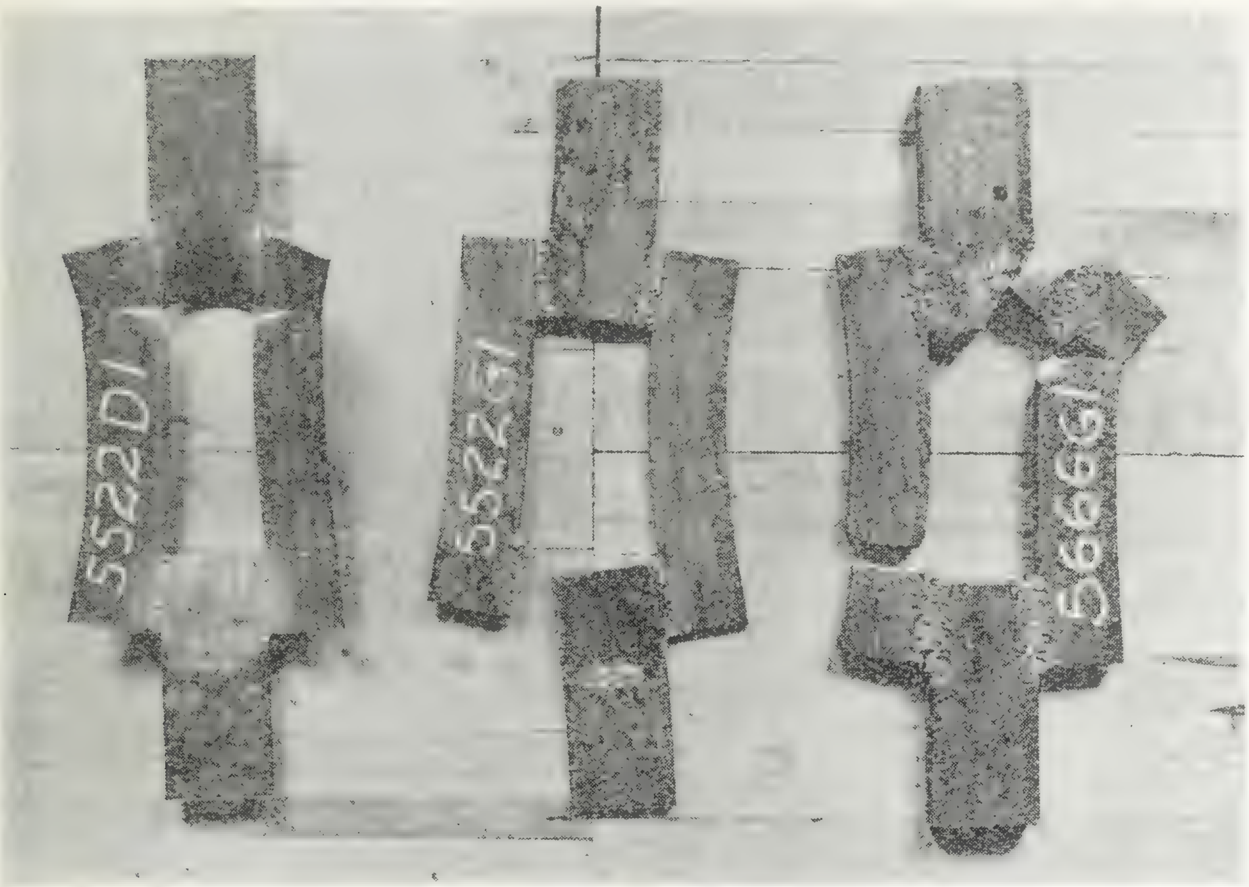


Fig. 11. Various Types of Failure.

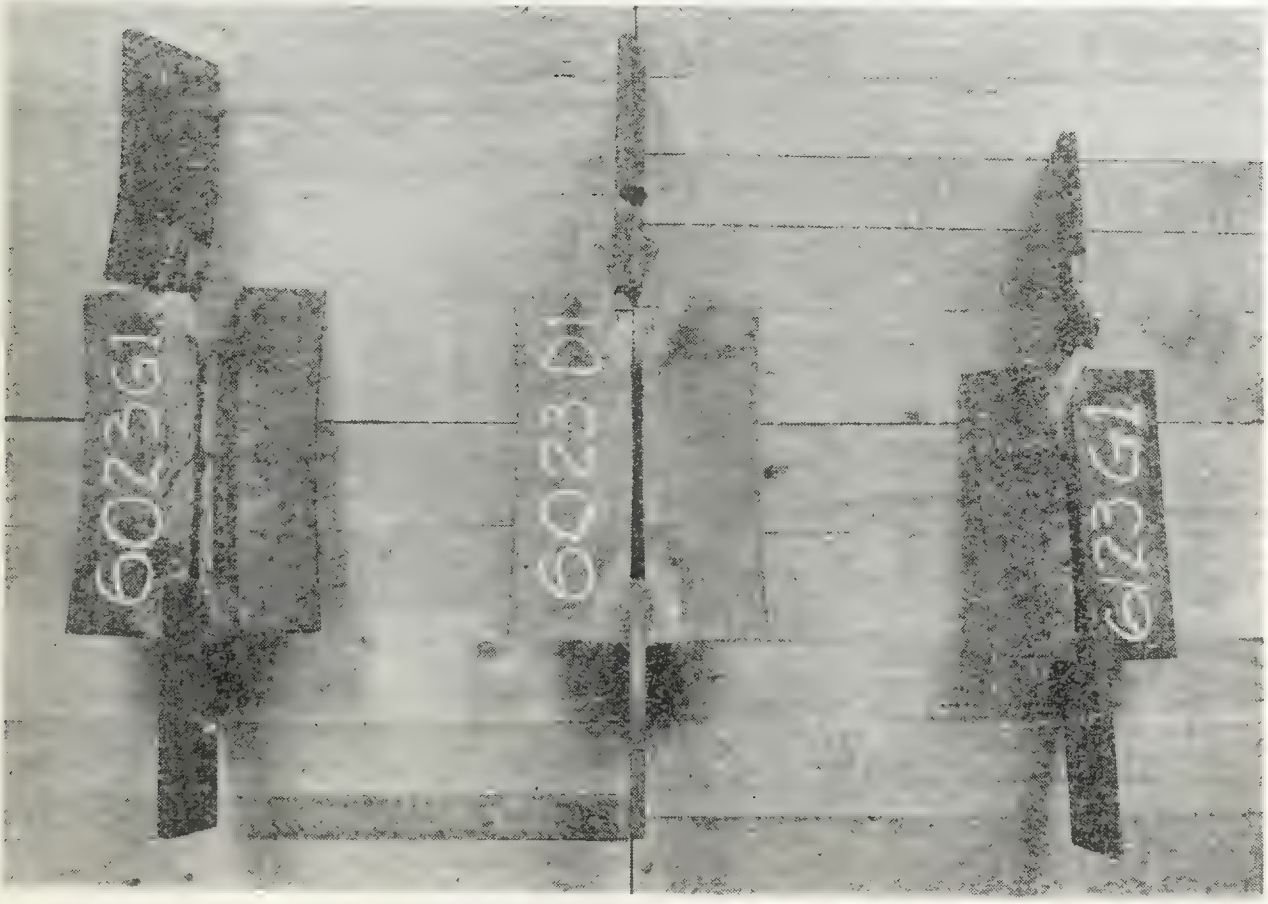


Fig. 12. Typical Failures, with Short Welds.

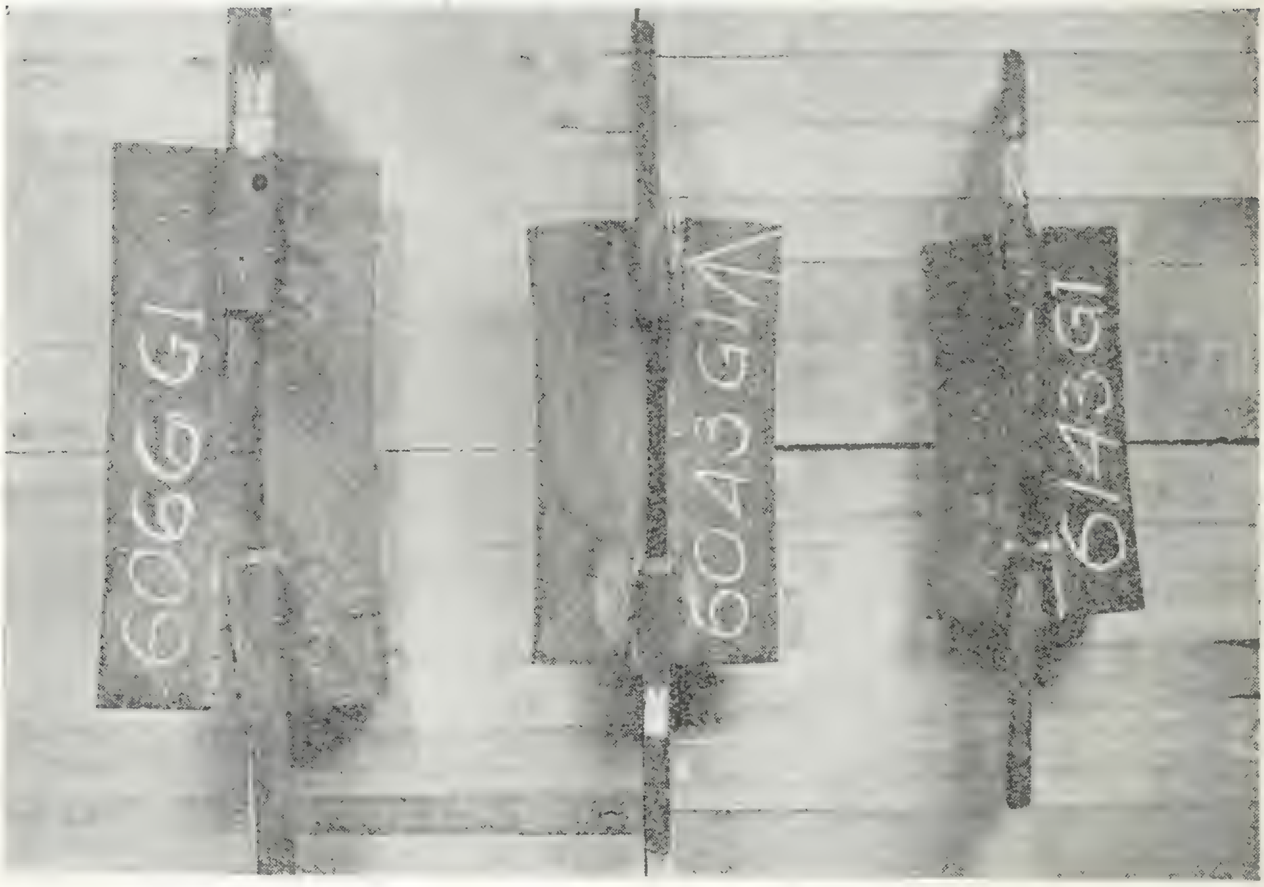


Fig. 13. Typical Failures with Long Welds.

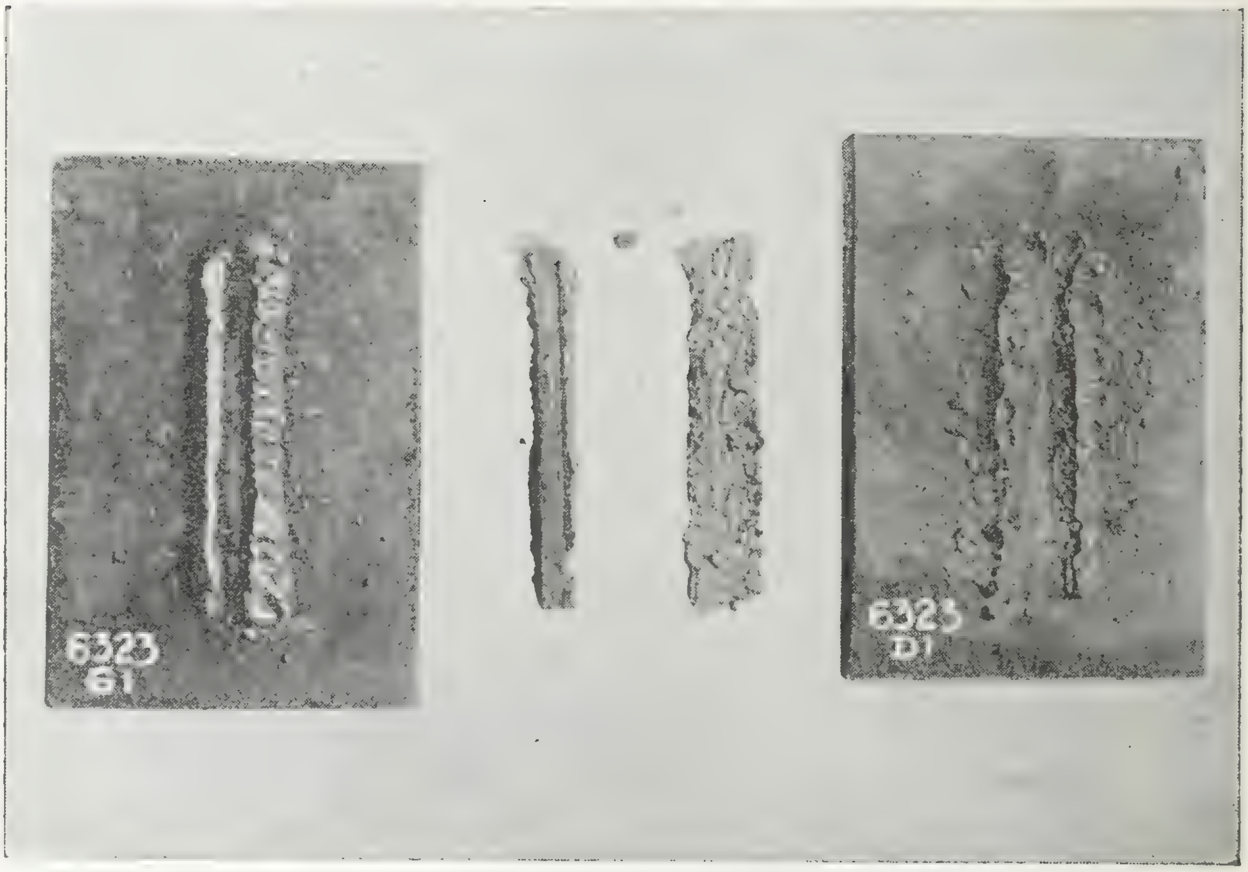


Fig. 14. 6323 G1—Throat Failure.
6323 D1—Unequal Fusion in First and Second Welds.

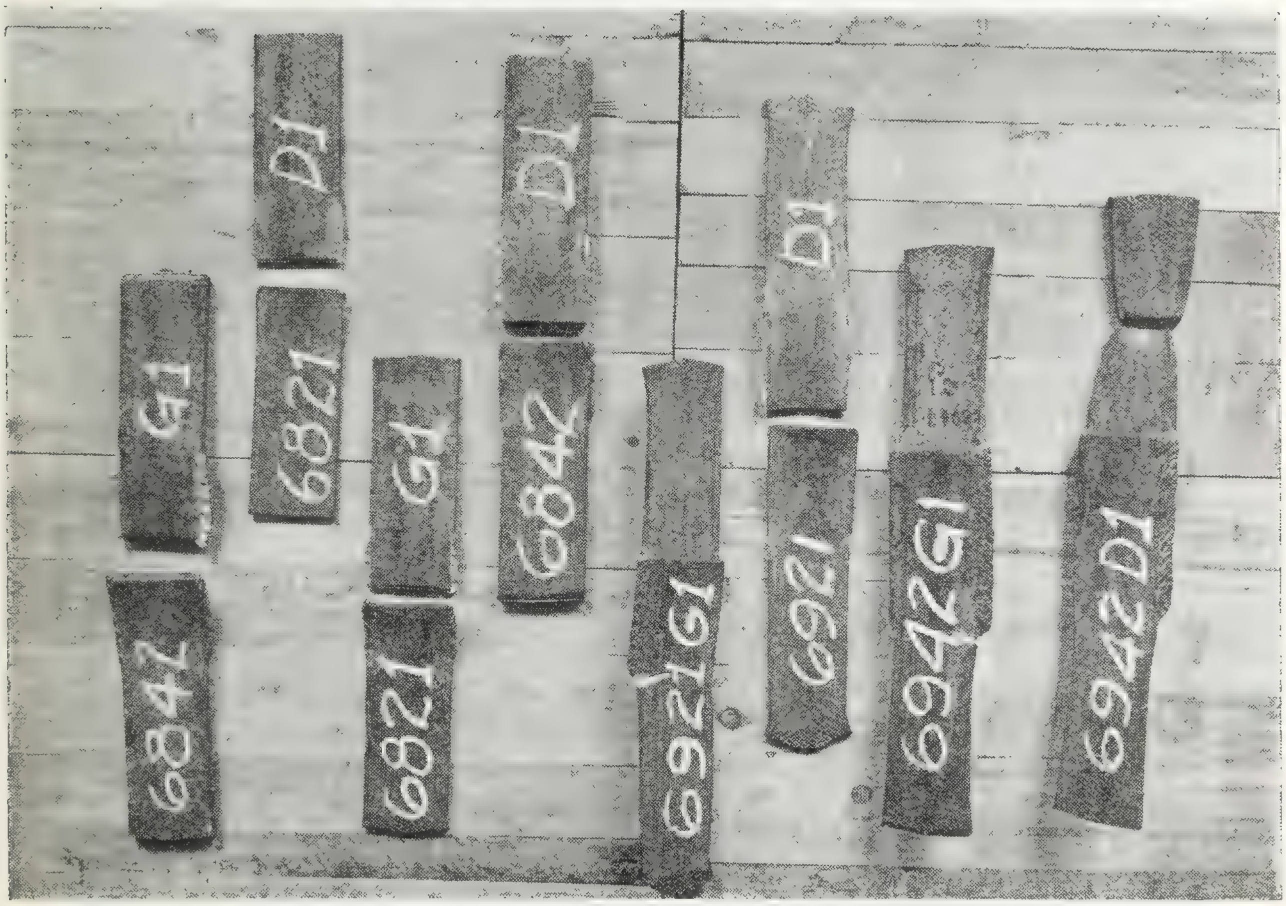


Fig. 15. Failures in Band Weld Specimens.

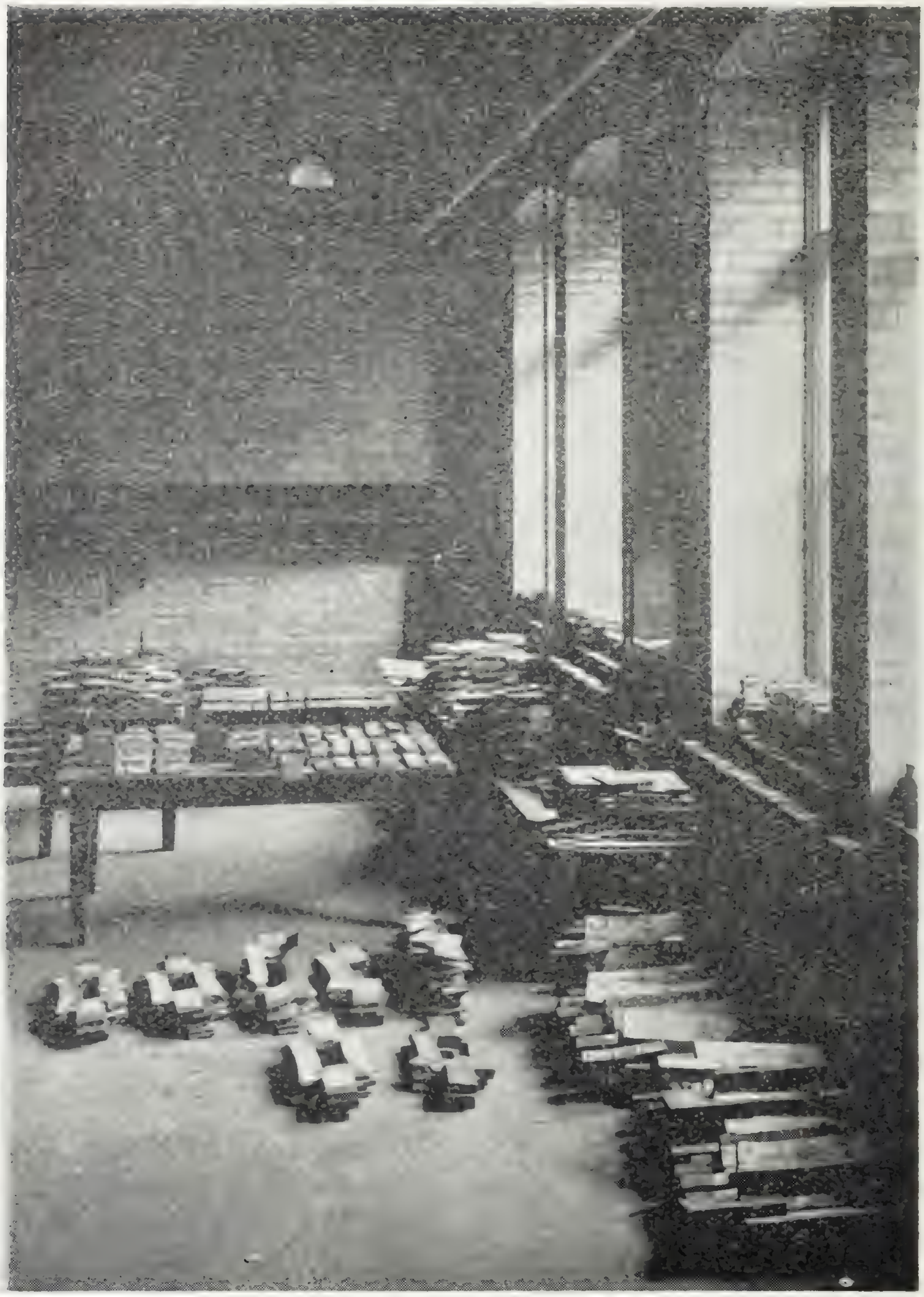


Fig. 16. Specimens after Testing.

FABRICATION AND TEST DATA

SERIES 1000.

Table No.1

Mark	WELD DETAILS						Yield Point		Strength in Kips		Remarks
	Welder	Section Relative to Nominal	Length Normal in.	Length Parallel in.	Rate of Welding Ft/hr.	Fusion %	Welds (scale) Kips/in.	Bars (Beam drop) Kips/in.	per lin. in.	per sq. in.	
1022 D1	A	1.5	4.0		3.1	60		9.50	10.05	37.9	
2		1.2	4.0		5.0	100		6.39	12.57	59.1	
3		1.3	3.95		4.9	100		10.07	12.15	52.8	
M1	D	1.3	4.0		10.1	90		-	9.42	41.0	
2		1.4	4.0		10.7	80		5.75	8.56	34.5	
3		1.0	4.1		10.2	95			9.76	55.2	
1033 D1	A	1.1	4.07		4.1	100		13.13	13.35	45.8	
2		1.0	4.1		3.7	100		7.92	14.10	53.2	
3		1.2	4.0		4.0	100		13.50	14.76	46.4	
M1	D	1.2	4.1		6.7	55		3.94	6.29	19.8	
2		1.2	4.0		6.7	75		5.54	6.48	20.4	
3		1.1	4.0		6.8	65		-	6.34	21.7	
1044 D1	A	1.1	4.0		3.6	90		16.00	17.48	44.9	
2		1.1	4.1		3.7	100		9.64	18.25	46.8	
3		1.0	4.0		3.3	100		15.74	18.83	53.2	
M1	D	1.2	4.0		4.0	85		7.06	9.34	22.0	
2		1.1	3.9		4.1	90		6.50	13.97	35.8	
3		1.1	4.1		5.1	100		-	13.24	34.0	
1055 D1	A	1.1	4.0		2.9	100		-	16.72	34.4	
2		1.0	4.0		2.7	100		14.33	16.36	37.0	
3		1.1	4.0		2.5	100		-	17.87	35.7	
M1	D	1.1	4.0		3.6	100		6.94	16.30	33.5	
2		1.0	4.0		3.3	80		10.84	13.00	29.4	
3		1.1	4.05		3.4	90		-	13.97	28.7	
1066 D1	A	1.1	3.85		1.9	100		22.77	23.35	40.0	
2		.9	3.9		2.6	100		13.32	25.00	52.5	
3		1.0	3.95		2.3	100		-	21.53	40.6	
M1	D	1.0	4.0		3.1	90		12.85	15.40	29.1	
2		1.1	3.9		2.7	75		12.24	13.03	22.4	
3		1.0	4.1		3.4	80		-	16.82	31.7	

FABRICATION AND TEST DATA

SERIES 1600.

Table No.2

Mark	WELD DETAILS						Yield Point		Strength in Kips		Remarks
	Welder	Section Relative to Nominal	Length Normal in.	Length Parallel in.	Rate of Welding Ft/hr.	Fusion %	Welds (scale) Kips/in.	Bars (Beam drop) Kips/in.	per lin. in.	per sq. in.	
1633 D1	A	1.1	15.8		3.5	-		5.13	11.18	38.4	c bar failed
2		1.2	15.4		3.7	-		6.12	11.68	36.7	c bar failed
3a		1.1	8.0		4.0	-		7.42	11.76	40.4	c bar failed
3b		1.1	7.8		4.0	-		7.48	11.83	40.5	c bar failed
M1	D	1.0	15.0		10.0	80		5.99	9.02	34.0	
2		1.1	15.0		8.0	70		4.60	8.94	30.6	
3a		1.1	7.25		9.4	75		7.31	9.78	33.5	
3b		1.1	7.2		9.4	45		7.64	8.94	30.6	
1643 D1	A	1.2	15.7		3.4	-		6.72	12.35	38.9	c bar failed
2a		1.0	7.3		3.9	-		5.84	14.47	54.6	c bar failed
2b		1.0	7.3		3.9	85		-	14.37	54.2	
3a		1.10	7.8		3.7	100		8.38	14.23	48.8	
3b		1.1	8.0		3.7	-		9.50	14.25	48.9	c bar failed
M1	D	1.1	15.0		8.0	-		6.05	10.23	35.1	c bar failed
2		1.0	15.0		6.4	75		5.08	10.05	38.0	
3a		1.0	7.25		5.7	65		10.08	11.41	43.1	
3b		1.1	7.1		5.7	85		9.44	11.91	40.9	
1653 D1	A	1.2	15.7		3.3	100		8.36	13.77	43.3	
2a		1.0	7.6		3.6	-		8.88	14.22	53.7	b bar failed
2b		1.1	7.5		3.6	100		6.89	13.05	44.7	
3a		1.0	7.8		3.7	-		-	14.16	53.4	b bar failed
3b		1.1	7.8		3.7	-		-	14.51	49.8	b bar failed
M1	D	1.1	15.0		6.7	75		6.35	11.07	37.9	
2		1.0	15.0		7.6	65		5.65	11.51	43.4	
3a		1.0	7.1		6.7	90		-	12.30	46.4	
3b		1.0	6.95		6.7	65		-	11.64	43.9	
1663 D1	A	1.1	15.6		3.6	95		-	16.69	57.2	
2a		1.1	7.8		3.6	100		-	16.96	58.2	
2b		1.1	7.2		3.6	100		-	16.48	56.5	
3a		1.1	7.8		3.6	100		-	17.06	58.5	
3b		1.2	8.0		3.6	100		-	15.54	48.8	
M1	D	1.1	15.0		5.3	70		7.24	11.87	40.7	
2		1.0	15.0		6.4	60		5.93	10.15	38.4	
3a		.9	7.55		6.2	95		-	14.07	58.8	
3b		.9	7.1		6.2	60		-	11.40	47.8	

Note: Some specimens were cut in two before testing. The two halves are designated by subscripts a and b.

SERIES 1900

FABRICATION AND TEST DATA

Table No.3

Mark	WELD DETAILS						Yield Point		Strength in Kips		Remarks
	Welder	Section Relative to Nominal	Length Normal in.	Length Parallel in.	Rate of Welding Ft/hr.	Fusion %	Welds (scale) Kips/in.	Bars (Beam drop) Kips/in.	per lin.in.	per sq. in.	
1922 D1	A	1.3	8.0		3.5	100		9.45	14.04	61.0	
2		1.3	8.0		3.9	100		9.30	12.60	54.8	
3		1.3	8.0		3.8	95		9.50	14.06	61.0	
M1	D	1.5	8.1		6.2	95		7.37	10.38	39.1	
2		1.7	8.1		8.9	40		5.51	8.64	28.8	
3		1.4	8.1		7.4	100		9.38	9.99	40.3	
1933 D1	A	1.2	7.95		2.8	100		13.55	19.29	60.6	
2		1.1	8.0		3.6	80		13.22	19.80	67.9	
3		1.1	8.0		3.5	95		13.50	17.68	60.7	
M1	D	1.2	8.0		6.2	80		-	13.00	40.9	
2		1.2	8.0		6.7	65		6.67	11.94	37.5	
3		1.2	8.25		4.9	30		-	10.46	32.9	
1944 D1	A	1.1	7.95		2.2	100		16.50	24.35	62.5	
2		1.0	8.0		2.4	100		15.22	24.60	69.5	
3		1.0	8.0		2.7	100		16.64	24.56	69.3	
M1	D	1.2	8.1		3.4	95		10.54	16.95	39.8	
2		1.2	8.0		2.6	95		7.61	18.91	44.7	
3		1.4	8.0		1.7	95		15.00	18.77	37.8	
1955 D1	A	1.1	8.0		2.2	100		19.43	30.40	62.5	
2		1.0	8.0		2.0	100		18.95	29.18	66.0	o bar failed.
3		1.0	8.0		1.9	100		20.13	28.10	63.6	o bar failed.
M1	D	1.2	8.0		2.7	90		17.00	20.15	38.0	
2		1.1	8.2		2.1	90		9.36	19.74	40.5	
3		1.2	8.2		2.5	90		19.09	21.91	41.4	
1966 D1	A	1.1	7.85		1.9	70		23.95	33.81	57.9	
2		1.1	7.9		1.8	100		17.76	32.22	55.1	
3		1.0	7.95		1.8	-		24.95	31.79	60.0	o bar failed.
M1	D	1.1	8.0		1.9	90		16.91	23.01	39.4	
2		1.1	7.8		1.9	70		12.54	23.62	40.3	
3		1.1	8.0		1.9	50		22.26	22.39	38.3	

FABRICATION TEST DATA

SERIES 2100. Table No.4

Mark	WELD DETAILS						Yield Point		Strength in Kips		Remarks
	Welder	Section Relative to Nominal	Length Normal in.	Length Parallel in.	Rate of Welding Ft/hr.	Fusion %	Welds (scale) Kips/in.	Bars (Beam drop) Kips/in.	per lin. in.	per sq. in.	
2122 D1	A	1.2		3.9	2.9	100		-	8.19	38.1	
2		1.4		3.8	5.1	100		-	8.98	36.3	
3		1.2		3.9	5.0	100		8.46	9.10	42.8	
M1	D	1.3		4.2	3.2	80		7.55	7.75	33.7	
2		1.2		4.3	4.2	100		5.14	7.19	33.8	
3		1.3		4.1	4.0	95	6.58	-	7.56	32.9	
2133 D1	A	1.1		4.0	3.8	100		-	10.84	37.2	
2		1.0		4.0	4.0	100		4.69	11.31	42.7	
3		1.0		4.0	3.7	100		10.00	11.47	43.3	
M1	D	1.2		4.3	4.8	90		7.78	9.06	28.5	
2		1.3		4.0	4.1	100		4.51	9.99	29.0	
3		1.2		4.2	3.9	80	6.92	8.10	8.82	27.7	
2144 D1	A	1.1		4.0	2.6	100		-	14.29	36.6	
2		1.1		4.2	2.3	100		8.75	13.56	34.8	
3		1.0		4.0	2.3	70	8.50	12.50	14.22	40.2	
M1	D	1.1		4.0	3.6	75		7.38	10.79	27.7	
2		1.1		4.0	3.4	85		6.04	11.89	30.4	
3		1.2		4.1	3.6	60	8.30	-	12.36	29.0	
2155 D1	A	1.1		4.0	2.0	100		-	15.74	32.4	
2		1.0		3.9	2.1	100		-	16.83	38.1	
3		1.0		4.0	1.9	100	11.25	13.50	17.05	38.6	
M1	D	1.0		4.5	3.5	60		8.90	11.16	25.2	
2		1.1		4.2	3.3	90		-	13.65	28.0	
3		1.1		4.2	2.6	50	10.72	-	11.94	24.6	
2166 D1	A	1.0		3.9	1.5	100		-	17.72	33.5	
2		1.0		4.0	1.5	100		9.49	18.69	35.3	
3		1.0		3.85	1.5	100	13.52	-	18.60	35.1	
M1	D	1.1		4.4	1.9	40		10.26	11.85	20.3	
2		1.1		4.0	2.0	20		9.08	11.29	19.3	
3		1.1		3.9	1.9	70	11.03	-	14.11	24.2	

FABRICATION AND TEST DATA

SERIES 2300.

Table No.5.

Mark	WELD DETAILS						Yield Point		Strength in Kips		Remarks
	Welder	Section Relative to Nominal	Length Normal in.	Length Parallel in.	Rate of Welding Ft/hr.	Fusion %	Welds (scale) Kips/in.	Bars (Beam drop) Kips/in	per lin.in.	per sq.in.	
2322 D1	A	1.3		7.8	4.7	100		8.30	9.14	39.7	
2		1.5		8.0	5.6	100		5.76	8.88	33.4	
3		1.2		8.05	5.2	90	5.22	7.95	8.88	42.0	
M1	D	1.2		8.0	10.1	90		2.52	7.84	36.8	
2		1.2		8.3	7.6	80		4.32	7.30	34.4	
3		1.2		8.15	6.8	85		-	7.61	35.8	
2333 D1	A	1.2		8.0	3.3	100		10.26	13.00	40.8	
2		1.1		7.9	3.4	100		5.32	12.70	43.8	
3		1.1		8.0	3.5	100	7.50	10.62	12.96	44.4	
M1	D	1.1		8.5	7.9	80		-	10.28	35.3	
2		1.2		8.2	6.9	80		7.32	10.85	34.1	
3		1.2		8.6	4.5	95		9.08	9.57	30.0	
2344 D1	A	1.1		7.9	2.4	100		13.08	15.45	39.7	
2		1.1		8.0	2.8	100		6.26	15.53	39.8	
3		1.1		7.85	2.8	100	8.28	13.64	15.20	39.0	
M1	D	1.1		8.0	3.6	60		-	11.40	29.2	
2		1.1		7.9	3.7	70		8.34	14.37	36.8	
3		1.1		8.25	4.1	25		-	9.82	25.3	
2355 D1	A	1.1		8.0	2.1	100		13.75	17.90	36.9	
2		1.1		7.9	1.9	100		10.51	18.45	38.0	
3		1.1		8.15	2.0	100	9.45	14.11	17.30	35.6	
M1	D	1.1		8.4	2.0	80		11.32	15.12	31.1	
2		1.1		8.1	2.0	60		10.41	16.28	33.4	
3		1.3		8.05	1.7	50		13.66	15.24	26.5	
2366 D1	A	1.0		7.8	1.6	100		-	21.70	41.0	
2		1.1		8.4	1.9	100		10.80	19.70	33.8	
3		1.0		8.15	2.0	100	11.54	-	20.30	38.2	
M1	D	1.1		8.9	1.5	65		-	16.77	28.7	
2		1.0		8.2	1.4	30		12.28	15.05	28.4	
3		1.2		8.0	1.4	30		-	16.92	26.6	

FABRICATION AND TEST DATA

SERIES 2500 & 2700

Table No.6

Mark	WELD DETAILS						Yield Point		Strength in Kips		Remarks
	Welder	Section Relative to Nominal	Length		Rate of Welding	Fusion	Welds (scale)	Bars (beam drop)	per lin.in.	per sq. in.	
			Normal in.	Parallel in.	Ft/hr.	%	Kips/in.	Kips/in.			
2543 D1	A	1.0	4.25	4.35	3.1	100	-	-	11.18	42.2	
2		1.1	4.2	4.1	3.4	100	-	5.58	11.10	38.1	
3		1.0	4.2	4.15	3.5	100	-	6.70	11.25	42.5	
M1	D	1.4	4.0	4.0	4.2	100	-	5.70	11.53	31.1	
2		1.3	4.3	4.25	5.3	90	-	5.76	11.38	33.0	
3		1.5	4.1	4.15	5.4	95	-	-	11.30	28.4	
2563 D1	A	1.1	4.15	8.20	3.0	100		10.26	10.51	36.0	
2		1.2	4.05	8.10	3.0	100		-	10.66	33.5	
3		1.0	4.15	8.30	3.8	85		-	11.42	43.2	
M1	D	1.8	4.25	6.7	2.8	100	-	5.96	12.37	26.0	
2		1.7	4.2	6.2	3.5	100		7.74	13.02	29.0	
3		1.9	4.2	6.3	2.8	30	-	-	11.45	22.7	
2743 D1	A	1.1	8.3	8.5	4.5	100	-	7.74	13.75	47.1	
2		0.9	8.1	8.2	4.37	100	-	7.80	14.13	59.3	
3		1.1	8.4	8.45	4.2	-	-	7.95	13.68	48.2	Failure in b bar
M1	D	1.3	8.6	10.0	5.6		-	5.76	8.98	26.8	Failure in c bar
2		1.2	8.2	8.1	4.7	-	-	6.64	10.95	34.4	Failure in c bar
3		1.5	8.45	9.1	4.4	-	-	7.64	9.95	25.0	Failure in c bar
2763 D1	A	1.1	8.3	16.2	4.1	100	-	7.91	13.28	45.6	
2		1.0	8.2	16.1	4.4	-	-	6.47	-		Not broken
3		1.0	8.3	16.9	4.5	-	-	8.12	-		Not broken
M1	D	1.5	8.5	13.4	3.5	60	-	5.16	10.85	27.3	
2		1.5	8.5	11.6	2.7	-		5.89	11.83	29.8	Failure in c bar
3		1.8	8.45	11.8	3.1			9.96	11.95	25.0	Failure in c bar

Note: All c bars in 2743 M and 2763 M are below specified thickness.

FABRICATION AND TEST DATA

SERIES 2900, 3000 and 3100

Table No.7

Mark	WELD DETAILS						Yield Point		Strength in Kips		Remarks
	Welder	Section Relative to Pl.Thickn's	Length Normal in.	Length Parallel in.	Rate of Welding Ft/hr.	Fusion %	Welds (scale) Kips/in.	Bars (Beam drop) Kips/in.	per lin. in.	per sq. in.	
2922 D1	B	1.3	3.95		6.6	50		8.39	8.43	26.4	
2		1.3	3.90		6.5	60		4.10	9.34	28.3	
3		1.3	3.95		6.6	70		9.88	10.14	31.6	
G1	E	1.2	4.00		5.7	100		9.54	13.97	45.1	
2		1.6	3.9		4.8	100		-	13.29	32.4	
3		1.3	4.00		5.7	95		10.00	11.97	37.4	
2933 D1	B	1.1	4.0		3.6	90		11.92	12.75	30.3	
2		1.2	3.90		3.2	75		7.29	12.78	28.4	
3		1.2	3.95		3.6	80		12.28	12.85	28.5	
G1	E	1.2	4.00		5.0	90		13.17	14.47	31.8	
2		1.6	3.95		4.0	100		-	15.05	25.5	
3		1.2	4.00		5.0	100		15.75	19.89	43.2	
3044 D1	C	1.2	3.9		3.0	-		9.22	29.12	50.2	c bar failed.
2		1.1	4.0		2.9	-		-	27.68	49.4	c bar failed.
3		1.1	3.95		2.8	100		14.94	27.40	49.8	
G1	E	1.1	4.00		1.2	100		15.16	24.87	43.6	
2		1.2	3.90		1.4	100		10.68	21.80	37.6	
3		1.1	4.00		1.5	100		15.17	20.37	36.3	
3066 D1	C	1.1	3.9		1.6	100		17.82	35.42	43.7	
2		1.1	3.95		1.6	100		17.10	37.25	44.9	
3		1.1	3.95		1.8	100		-	37.20	44.3	
G1	E	1.1	4.00		1.0	100		18.81	34.62	40.7	
2		1.2	3.95		1.2	100		16.56	37.05	41.2	
3		1.1	4.00		1.1	95		20.75	31.66	37.3	
3144 D1	C	1.1	3.9		2.2	85		15.52	25.05	45.6	
2		1.1	3.95		1.4	85		14.12	25.05	44.7	
3		1.2	3.9		1.9	-		15.38	28.35	51.6	c bar failed
G1	E	1.3	4.0		2.9	90		13.98	23.12	35.6	
2		1.3	3.95		3.3	100		11.22	19.90	29.7	
3		1.3	4.0		2.5	100		-	26.40	40.6	
3166 D1	C	1.1	3.9		1.03	80		20.12	30.10	37.2	
2		1.1	3.9		1.2	85		17.33	36.70	42.6	
3		1.1	3.9		1.3	90		22.05	32.15	37.8	
G1	E	1.2	4.0		2.5	90		20.71	35.38	39.3	
2		1.3	4.0		1.7	90		15.85	35.80	36.9	
3		1.2	4.0		1.5	95		21.50	34.79	38.2	

FABRICATION AND TEST DATA

SERIES 5300, 5400, 5500, 5600

Table No.8

Mark	WELD DETAILS						Yield Point		Strength in Kips		Remarks
	Welder	Section Relative to Pl.Thickn's.	Length Normal in.	Length Parallel in.	Rate of Welding Ft/hr.	Fusion %	Welds (scale) Kips/in.	Bars (Beam drop) Kips/in.	per lin.in.	per sq. in.	
5322 D1	B	1.2		4.05	6.2	-		4.67	6.48	21.6	Bars failed
2		1.2		4.0	7.3	80		2.70	6.54	21.8	
G1	E	1.3		4.1	8.2	-		3.88	6.71	21.3	b bar failed
2		1.2		4.2	8.4	-		3.14	6.50	21.7	b bar failed
5333 D1	B	1.1		4.0	2.9	50		5.00	8.23	19.4	
2		1.2		4.0	3.2	-		3.72	9.82	22.5	Bars failed
G1	E	1.2		4.2	6.0	-		4.40	8.99	20.0	c bar failed
2		1.1		4.05	4.5	-		3.50	9.49	23.7	b bar failed
5444 D1	C	1.0		4.0	1.6	-		4.30	11.85	22.4	Bars failed
2		1.1		4.05	1.9	-		3.70	12.15	21.7	b bar failed
G1	E	1.4		4.5	2.1	-		5.58	9.84	13.7	Bars failed
2		1.3		4.6	2.3	-		3.75	8.71	13.0	b bar failed
5466 D1	C	1.1		4.05	1.5	-		6.35	16.42	19.5	Bars failed
2		1.0		4.0	1.7	-		6.08	17.05	21.6	b bar failed
G1	E	1.4		4.55	2.0	-		9.86	14.55	13.5	b bar failed
2		1.4		4.75	2.0	-		7.84	13.00	12.1	b bar failed
5522 D1	B	1.2		6.2	4.6	-		5.35	5.55	18.5	b bar failed
2		1.2		6.0	5.7	100		3.87	5.85	19.5	
3		1.2		5.9	4.4	100		5.60	5.94	19.8	
G1	E	1.2		6.2	4.1	90		5.07	6.00	19.3	
2		1.2		6.15	4.4	-		-	6.16	19.9	b bar failed
3				6.20	4.4	100		6.29	6.59		
5533 D1	B	1.1		6.0	3.3	60		-	7.35	17.3	
2		1.2		6.10	3.7	50		4.01	8.19	18.4	
3		1.2		6.05	3.2	50		-	7.68	17.1	
G1	E	1.3		6.10	3.6	80		6.58	8.96	18.3	
2		1.3		6.3	5.3	100		-	9.30	19.0	
3				6.2	5.6	90		-	9.46		
5644 D1	C	1.0		6.2	1.3	-		9.94	14.66	30.5	Bars failed
2		1.2		6.15	1.7	-		8.94	13.70	22.1	Failure in c bar to
3				6.3	1.6	-		13.01	13.65	—	Failure by tearing
G1	E	1.2		7.0	2.3	-		8.96	13.80	22.3	Bars failed
2		1.4		7.05	1.8	-		8.87	12.93	18.5	c bars failed
3				7.2	2.2	-		10.55	12.19		Failure by tearing
5666 D1	C	.9		5.95	1.3	-		12.71	18.75	26.4	c bar failed
2		1.3		6.20	1.7	-		10.95	19.60	20.0	c bar failed
3				6.15	1.7			19.70	20.00		Bars failed
G1	D	1.3		7.0	1.1	-		15.24	17.90	17.9	Bars failed
2		1.0		7.0	1.0	-			17.45	23.0	c bars failed
3				7.0	1.2	-		16.57	17.70		Bars failed

FABRICATION AND TEST DATA

Table No.9

SERIES 6000.and 6100

Mark	WELD DETAILS						Yield Point		Strength in Kips		Remarks
	Welder	Section Relative to Nominal	Length Normal in.	Length Parallel in.	Rate of Welding Ft/hr.	Fusion %	Welds (scale) Kips/in.	Bars (Beam drop) Kips/in.	per lin. in.	per sq. in.	
6023 D1	B	1.1		8.00	4.0	-		1.94	5.05	17.3	Bars failed
2		1.0		8.30	3.6	-		3.02	5.24	19.8	Bars failed
3		-		-	-	-		-	-		
G1	B	1.1		8.15	6.3	-		4.64	6.20	21.3	b bar failed
2		1.2		8.0	5.3	-		2.84	6.26	19.7	b bar failed
3		1.0		8.0	5.9	-		4.50	6.21	23.8	b bar failed
6043 D1	C	1.0		15.95	3.5	30		3.46	5.77	21.8	
2		1.0		16.05	3.6	50		3.26	5.32	20.0	
3		1.1		16.5	3.4	70		5.09	5.72	19.6	
G1	E	1.2		15.80	8.8	-		5.40	5.96	18.8	
2		1.2		15.8	7.5	100		3.18	6.08	19.7	
3		1.2		15.95	8.4	100		-	6.18	19.5	
6063 D1	C	1.1		23.8	4.5	30		4.00	6.68	2[illegible].0	
2		1.1		24.4	3.6	25		2.87	6.36	21.7	
3		1.5		24.25	4.9	20		5.36	5.42	13.6	
G1	E	1.1		24.0	7.5	100		6.16	6.58	22.6	
2		1.1		23.95	7.3	-		4.64	6.36	21.9	
3		1.1		23.85	8.2	95		-	6.63	22.7	
6123 D1	C	1.0		4.05	3.7	100		2.57	6.08	23.0	
2		1.0		4.1	3.2	-		3.86	6.73	25.4	Bars failed
3		1.2		4.4	3.7	40		-	6.38	20.0	
G1	E	1.1		4.25	8.5	-		4.93	6.41	22.0	b bar failed
2		1.0		4.0	4.4	-		3.92	6.52	24.6	b bar failed
3		.9		4.0	5.7	-		-	6.37	26.6	b bar failed
6143 D1	C	1.0		12.05	1.1	60		3.81	7.02	26.5	
2		1.0		12.0	3.9	60		4.96	6.92	26.1	
3		1.4		12.1	4.2	60		-	6.94	18.7	
G1	E	1.1		11.6	12.9	90		5.28	5.94	20.4	
2		1.0		12.6	1[illegible].0	100		3.24	5.74	21.6	
3		1.0		11.85	10.7	100		5.47	6.04	22.8	
6163 D1	C	1.0		20.2	4.0	40		2.73	5.93	22.4	
2		1.0		20.5	4.3	50		3.40	6.74	25.4	
3		1.2		20.0	4.3	40		-	6.68	21.0	
G1	E	1.1		19.7	8.6	100		6.79	7.92	27.2	
2		1.2		19.9	9.0	100		5.52	7.00	22.0	
3		1.1		19.7	9.4	95		6.36	8.11	27.8	

SERIES 6300 and 6700

FABRICATION AND TEST DATA

Table No.10

Mark	WELD DETAILS						Yield Point		Strength in Kips		Remarks
	Welder	Section Relative to Nominal	Length		Rate of Welding Ft/hr.	Fusion %	Welds (scaled) Kips/in.	Bars (Beam drop Kips/in.	per lin.in.	per sq. in.	
			Normal in.	Parallel in.							
6323 D1	C	1.2	7.9		5.8	10		2.57	5.64	17.7	
2		1.0	8.0		5.9	25		1.96	5.98	22.6	
3		1.1	8.0		7.3	5		4.75	5.10	17.5	
G1	E	1.0	8.05		7.3	100		4.35	6.55	24.7	
2		1.1	8.05		7.3	90		3.40	7.00	24.0	
3		1.0	8.1		7.4	85		4.81	6.90	26.0	
6343 D1	C	1.0	7.85		5.6	20		3.11	8.12	30.7	
2		1.0	7.85		5.2	35		2.56	8.60	32.5	
3		1.2	7.9		4.9	40		7.72	8.49	26.5	
G1	E	1.0	7.9		7.9	100		-	8.92	33.7	
2		1.1	7.8		7.1	100		-	8.86	30.4	
3		1.0	7.9		7.2	90		7.72	9.04	34.1	
6363 D1	C	1.4	7.8		4.9	30		4.96	9.88	26.6	
2		1.1	8.0		4.2	20		5.00	10.26	35.2	
3		1.2	8.05		5.0	30		10.44	10.80	34.0	
G1	E	1.0	8.0		7.3	80		7.09	10.26	38.8	
2		1.1	7.95		6.6	95		5.82	10.51	36.0	
3		1.0	7.9		6.1	95		-	10.15	38.4	
					Mins/Weld			Kips/Weld	Kips/Weld		
6725 D1	B & C				8.3	100		-	43.1		
2					8.3	80		29.8	31.6		
3					6.0	-			44.3		
G1	E				7.0	100		-	65.2		
2					5.5	-		27.2	62.8		
3					4.5	-		-	63.7		
6735 D1	B & C				6.5	100		-	43.7		
2					7.0	80		30.2	33.5		
3					6.0	-		-	26.9		
G1	E				5.5	90		-	65.2		
2					6.5	-		33.8	59.3		
3					6.5	-		-	69.7		

SERIES 6800 and 6900

FABRICATION AND TEST DATA

Table No. 11

Mark	WELD DETAILS						Yield Point		Strength in Kips		Remarks
	Welder	Section Relative to Nominal	Length Normal in.	Length Parallel in.	Rate of Welding Ft/hr.	Fusion %	Welds (scaled) Kips/in.	Bars (Beam drop) Kips/in.	per lin.in.	per sq. in.	
6821 D1	B	.9		4.0	10.0	100	-	-	4.67	40.6	
2		1.2		4.0	10.0	100	3.45	3.08	5.39	34.8	
3		1.0		3.8	9.5	100	3.68	-	5.22	41.8	
G1	E	1.4		4.0	6.7	100	-	3.94	6.20	35.4	
2		1.2		4.1	6.8	100	4.22	3.40	6.76	43.3	
3		1.2		4.0	5.0	100	-	6.25	6.71	43.0	
6842 D1	B	.7		5.65	3.8	100	6.30	7.43	9.11	49.2	
2		.9		6.0	4.0	100	5.27	4.55	9.48	43.5	
3		.9		6.0	4.0	100	-	-	8.88	40.7	
G1	E	1.0		6.2	3.9	100	-	5.74	8.85	35.4	
2		1.0		6.0	3.3	100	6.23	5.28	8.86	35.4	
3		1.1		6.2	3.9	100	-	-	8.79	31.1	
6921 D1	B	.8		8.0	11.2	95	3.03	3.18	4.77	47.7	
2		1.2		8.0	8.9	100	3.25	2.98	4.86	31.3	
3		.8		8.0	8.9	100	3.75	3.32	4.72	49.7	
G1	B	1.4		8.0	8.0	100	-	2.71	4.78	27.3	
2		1.2		8.0	10.0	-	3.42	3.14	4.72	30.3	o bar failed
3		1.5		8.35	8.3	-	4.07	3.23	4.65	24.9	o bar failed
6942 D1	B	.7		12.0	4.8	-	4.33	-	6.41	36.6	o bar failed
2		1.0		12.0	4.1	-	3.54	3.38	6.48	25.9	o bar failed
3		.9		11.95	4.1	-	3.85	3.89	6.42	29.5	o bar failed
G1	E	1.1		12.1	2.7	100	-	2.59	6.30	22.9	
2		1.0		12.0	2.8	-	3.41	3.32	6.24	25.0	o bar failed
3		1.2		12.65	3.7	-	-	-	5.99	19.2	o bar failed

CONSTANTS FOR CALCULATION OF "SECTION OF WELD RELATIVE TO NOMINAL"

GAS WELDS

Table No.12

SPECIMEN NUMBER			Vol. per lin. in. of Weld Cu. in.	Wt. per lin. in. of Weld Lb.	Length of Wire per lin. in. of Weld Size of Wire 1/8	5/32	3/16	1/4
Fillet Welds								
10 22, 19 22, 21 22, 23 22,			.0312	.00884	2.54	1.63	1.13	.636
33, 16, 33, 33, 33, 25, 27, 60, 61, 63			.0703	.0199	5.72	3.66	2.55	1.42
44, 44, 44, 44,			.125	.0354	10.2	6.51	4.53	2.54
55, 55, 55, 55,			.195	.0552	15.9	10.1	7.06	3.97
66, 66, 66, 66,			.281	.0795	22.9	14.6	10.2	5.72
Butt Welds								
Straight	Single V	Double Vee						
29 22,			.0375	.0106	3.05	1.95	1.30	
33,			.0842	.0238	6.86	4.39	3.05	
	30 44,		.2725	.0771			9.87	5.55
	66,		.6045	.1711			21.90	12.30
		31 44, 54 44, 56 44,	.2093	.0592			7.58	4.26
		66, 66, 66,	.435	.1232			15.8	8.87
53 22,			.0375	.0106	3.05	1.95		
33,			.0844	.0239	6.87	4.40	3.06	
55 22,			.0375	.0106	3.05	1.95		
33,			.0844	.0239	6.87	4.40	3.06	

Rivet Welds	Vol. of Weld Cu. in.	Wt. of Weld Lb.	Length of Wire per Weld 1/8	5/32	3/16	1/4
67 25,	.721	.204	57.8	37.0	25.8	14.5
35,	1.193	.3375	97.0	62.1	43.2	24.3

Band Welds	Vol. per in. of Weld Cu. in.	Wt. per in. of Weld Lb.	Length of Wire per lin. in. of Weld 1/8	5/32	3/16	1/4
68 21, 57 21,	.0625	.0177	5.08	3.25	2.26	1.27
42, 69 42,	.250	.0707	20.3	13.0	9.06	5.09

CONSTANTS FOR CALCULATION OF "SECTION OF WELD RELATIVE TO NOMINAL"

ARC WELDS

Table No. 13

SPECIMEN NUMBER			Vol. per lin. in. of Weld Cu. in.	Wt. per lin. in. of Weld Lb.	Length of Wire per lin. in. of Weld Size of Wire			
					1/8	5/32	3/16	1/4
Fillet Welds								
10 22, 19 22, 21 22, 23 22,			.0312	.00884	2.54	1.63	1.13	.636
33, 16. 33, 33, 33, 25.27.60.61.63.			.0703	.0199	5.72	3.66	2.55	1.42
44, 44, 44, 44,			.125	.0354	10.2	6.51	4.53	2.54
55, 55, 55, 55,			.195	.0552	15.9	10.1	7.06	3.97
66, 66, 66, 66,			.281	.0795	22.9	14.6	10.2	5.72
Butt Welds								
Straight	Single V	Double Vee						
29 22,			.0375	.0106	3.05	1.95	1.36	
33,			.0842	.0238	6.86	4.39	3.05	
	30 44,		.2929	.0828			10.6	5.97
	66,		.6097	.1725			22.1	12.4
		31 44, 54 44, 56 44,	.2201	.0623			7.98	4.49
		66, 66, 66,	.4230	.120			15.3	8.62
53 22,			.0531	.0150	4.33	2.76		
33,			.1078	.0305	8.79	5.62	3.91	
55 22,			.0688	.0194	5.60	3.58	2.49	
33,			.1078	.0305	8.79	5.62	3.91	

	Vol. of Weld Cu. in.	Wt. of Weld Lb.	Length of Wire per Weld			
			1/8	5/32	3/16	1/4
Rivet Welds						
67, 25,	.596	.1686	48.5	31.0	21.6	12.1
35,	1.006	.2850	81.8	52.4	36.5	20.5

	Vol. per in. of Weld Cu. in.	Wt. per in. of Weld Lb.	Length of wire per lin. in. of Weld.			
			1/8	5/32	3/16	1/4
Band Welds						
68 21, 69 21,	.0625	.0177	5.08	3.25	2.26	1.27
42, 69 42,	.250	.0707	20.3	13.0	9.06	5.09

Table No.14

Mark	Lowest Recorded Fusion, %											Texture and Position of Rupture						Remarks
												75-100% Silky				75-100% Crystalline On Ends of b Bars	Silky and Crystalline	
												Thro'	On face of bar					
	0	10	20	30	40	50	60	70	80	90	100	Throat	b	c	b & c			
Gas 1022							●				●●	1,2,3						
33											●●●					1,2,3		
44										●	●●	1,2				3		
55											●●●					1,2,3		
66											●●●					1,2,3		
Arc 1022									○	○	○					1,2,3		
33						○		○	○							1,2,3		
44									●	○	○	2				1, 3		
55									●	○	○					1,2,3		
66								○	○	○				2		1, 3		
Gas 1922										●	●●	3					1,2	
33									●	●	●					1,2	3	
44											●●●			1	2,3			
55											●					1		2 &3 broke in c bar.
66								●			●							3 broke in c bar.
Arc 1922					○					○	○	1,3				2		
33				○				○	○			1,2	3					
44												1,2,3						
55										○○○		1,2						
66						○		○		○		1,2,3						
Gas 2122											●●●	1,2,3						
33											●●●	1,2,3						c bars 3/8 in. thick.
44								●			●●		2	1	3			
55											●●●			1,2,3				Tore into c bar.
66											●●●			1,2,3				Tore into c bar.
Arc 2122									○		○○				1,2,3			Welds porous.
33									○	●	○				1,2,3			c bars 5/8-in. thick.
44							○	○	○					1,3	2			Welds porous.
55						○	○			○				1,2,3				
66			○		○			○						1,2,3				
Gas 1633																		Failed in c bar.
43									●		●			2b,3a				1,2a,3b failed in c bar.
53											●●						1,2b	2a,3a,3b failed in b bar.
63										●	●●●●					1,2a,2b,3a,3b		
Gas 2322										●	●●	1,2,3						
33											●●●	1,2,3						
44											●●●			1,2,3				Tore into c bar.
55											●●●			1,2,3				Tore into c bar.
66											●●●			1,2,3				Tore into c bar.
Arc 2322									○●	○		1.		2,3				
33									○○		○	1,2,3						
44				○			○		○					1,2,3				
55							●	○	○					1,2,3				
66				○○				○						1,2,3				
Gas 2543											●●●	2p,1n,3p				2n,3n		n-normal weld p-parallel weld
2563										●	●●	3,1p,2p				1n,2n		n-normal weld p-parallel weld
	0	10	20	30	40	50	60	70	80	90	100							

Table No.13

Mark	Lowest Recorded Fusion, %											Texture and Position of Rupture						Remarks
												75-100% Silky				75-100% Crystalline	Silky and Crystalline	
												Thro'	On face of bar			On Ends of b Bars		
	0	10	20	30	40	50	60	70	80	90	100	Throat	b	c	b & c			
Arc 2543										○	○○	1,2,3						
63				○							○○	1,2		3				
Gas 2743											●●●	1,2						3 failed in b bar.
63											●	1p		1n				2 & 3 not broken.
Arc 2743																		All broke crystalline in c bar.
63							○							1				2 & 3 failed in c bar.
Gas 2922						●	●	●								2	1,3	
33									●●	●						1,2		
Arc 2922										○	○○	1,2,3						Part failure in base metal.
33										○	○○					1,2,3		
Gas 3044											●●●					3		1 & 2 failed in c bar.
3066											●●●	3				2	1	Part failure in base metal.
Arc 3044											○○○					1,2,3		
3066										○	○○					1,2,3		
Gas 3144									●	●	●					1		2 & 3 failed in plate.
3166									●	●●						1,2	3	
Arc 3144										○	○○					1,2,3		
3166										○○	○					1,2		
Gas 5322									●			2						
5333						●						1						
Arc 5322																		
5333																		
Gas 5444	Failed in b or c bar by tearing, except as noted.																	
5466																		
Arc 5444																		
5466																		
Gas 5522											●●	2,3						1 failed in plate
5533						●●●								1,2,3				
Gas 5644																		
5666																		
Arc 5644	Failure due to tearing of bars.																	
5666																		
Arc 1633						○		○	○○					1,2,3a,3b				
43							○	○	○			3b		2,3a				1 failed in c bar.
53							○○	○		○				1,2,3a,3b				Rupture partly crystalline
63							○○	○		○		3a,3b		1,2				
	0	10	20	30	40	50	60	70	80	90	100							

Table No.16

Mark	Lowest Recorded Fusion, %											Texture and Position of Rupture						Remarks
												75-100% Silky				75-100% Crystalline On Ends of b Bars	Silky and Crystalline	
												Thro' Throat	On face of bar					
	0	10	20	30	40	50	60	70	80	90	100		b	c	b & c			
Gas 6023																		All failed in b bar.
43				●		●		●						1,2,3				
63			●	●	●									1,2,3				
Arc 6023																		All failed in b bar.
43											○○○	1,2,3						
63											○○○	1,2,3						
Gas 6123					●						●●	1		3				2 failed in b bar.
43							●●●					1,2,3						
63					●●	●								1,2,3				
Arc 6123																		All failed in c bar.
43											○○○	1,2,3						
63											○○○	1,2,3						
Gas 6323		●●	●											1,2,3				Partly crystalline.
43		●		●●														
63		●	●●															
Arc 6323									○	○	○	1,2						2 failed in c bar.
43										○	○○	1,2,3						Partly crystalline.
63									○	○○○		1,2,3						Partly crystalline.
Gas 6725																		
35																		
Arc 6725																		
35																		
Gas 6821											●●●	1,2,3						
42											●●●	1,2,3						
Arc 6821											○○○	1,2,3						Welds porous.
42											○○○	1,2,3						Welds porous.
Gas 6921											●●●	1,2,3						
42																		All failed in plate.
Arc 6921																		All failed in plate.
42																		All failed in plate.
Arc 5522										○	○	1,3						2 failed in plate.
5533									○	○	○			1,2,3				
	0	10	20	30	40	50	60	70	80	90	100							

SHEAR STRESS IN CERTAIN THIN FILLETS

PETER GILLESPIE, C. A. HUGHES, K. B. JACKSON, J. H. FOX

Two elastic metallic bars are lapped and united by a fillet weld along their edge of contact, the fillet having a direction parallel with the axes of the united members. When these members are stressed in either tension or compression, the fillet resists by shear, and a consideration of its special function points to the probability that this shear is not of uniform intensity from end to end, but variable with its greatest value at either extremity. This phenomenon is suggested by the fact that the members joined and the fillet also are elastic and that at those sections where there is greatest disparity between the stresses which the members sustain, there the angular deformation and the shear stress in the fillet are greatest also. It is quite analogous indeed to the old problem of rivets in series. If the members were perfectly rigid, and the fillet were elastic, the shear stress would be of uniform intensity throughout.

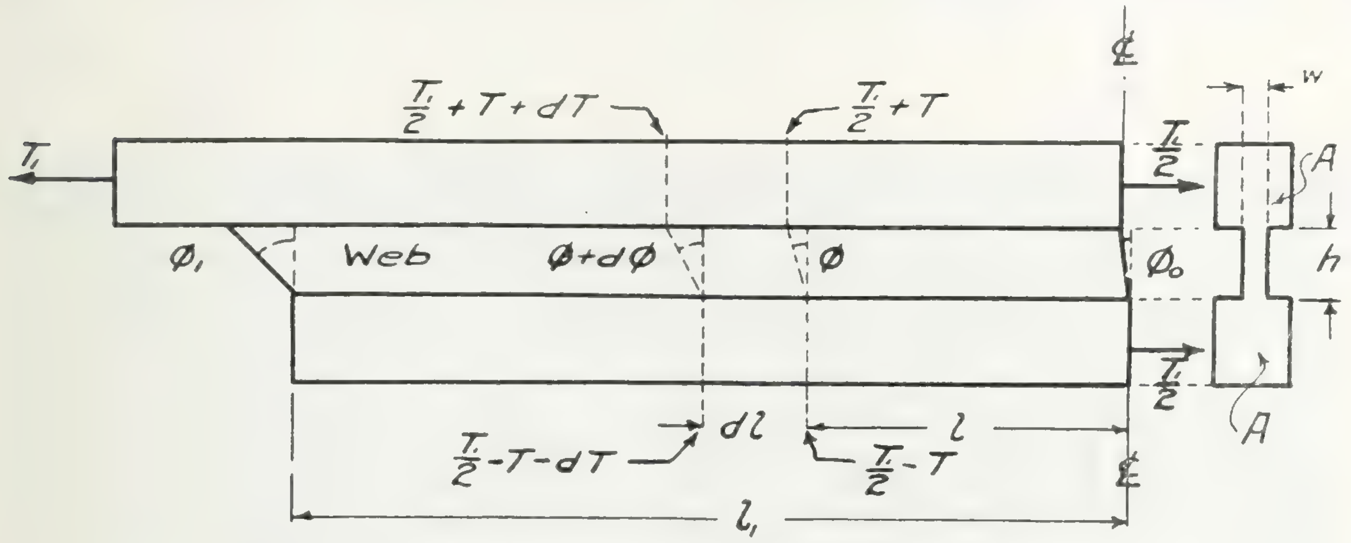


Fig. 1. Shear Stress in Parallel Fillet. (Diagram giving notation.)

If in Fig. 1, the web representing a fillet,

w = width of fillet,

h = depth of fillet,

A = cross-section of either member,

$2l_1$ = length of fillet,

C = shearing Modulus of Elasticity and

E = Young's Modulus,

$$d\phi=\frac{2T\,dl}{AEh} \qquad (1)$$

and $dT = C\phi w dl$ .... (2)

It can then be shown that

$$T = \sqrt{\frac{CwAEh}{2}} \sqrt{\phi^2 - \phi_0^2} \qquad (3)$$

where T is tension increment in distance l,

and $$\frac{T_1}{2} = \sqrt{\frac{CAwEh}{2}} \sqrt{\phi_1^2 - \phi_0^2} \qquad (4)$$

From (1) and (3)

$$\log\{\phi + \sqrt{\phi^2 - \phi_0^2}\} = \sqrt{\frac{2Cw}{AEh}} . l + \log \phi_0 \qquad (5)$$

from which

$$\phi = \phi_0 \cosh\left(\sqrt{\frac{2Cw}{AEh}} . l\right) \qquad (6)$$

and $$q = C\phi_0 \cosh\left(\sqrt{\frac{2Cw}{AEh}} . l\right) \qquad (7)$$

where q is shear stress in the fillet.

It then follows that

$$T = \phi_0 \sqrt{\frac{CwAEh}{2}} \sinh\left(\sqrt{\frac{2Cw}{AEh}} . l\right) \qquad (8)$$

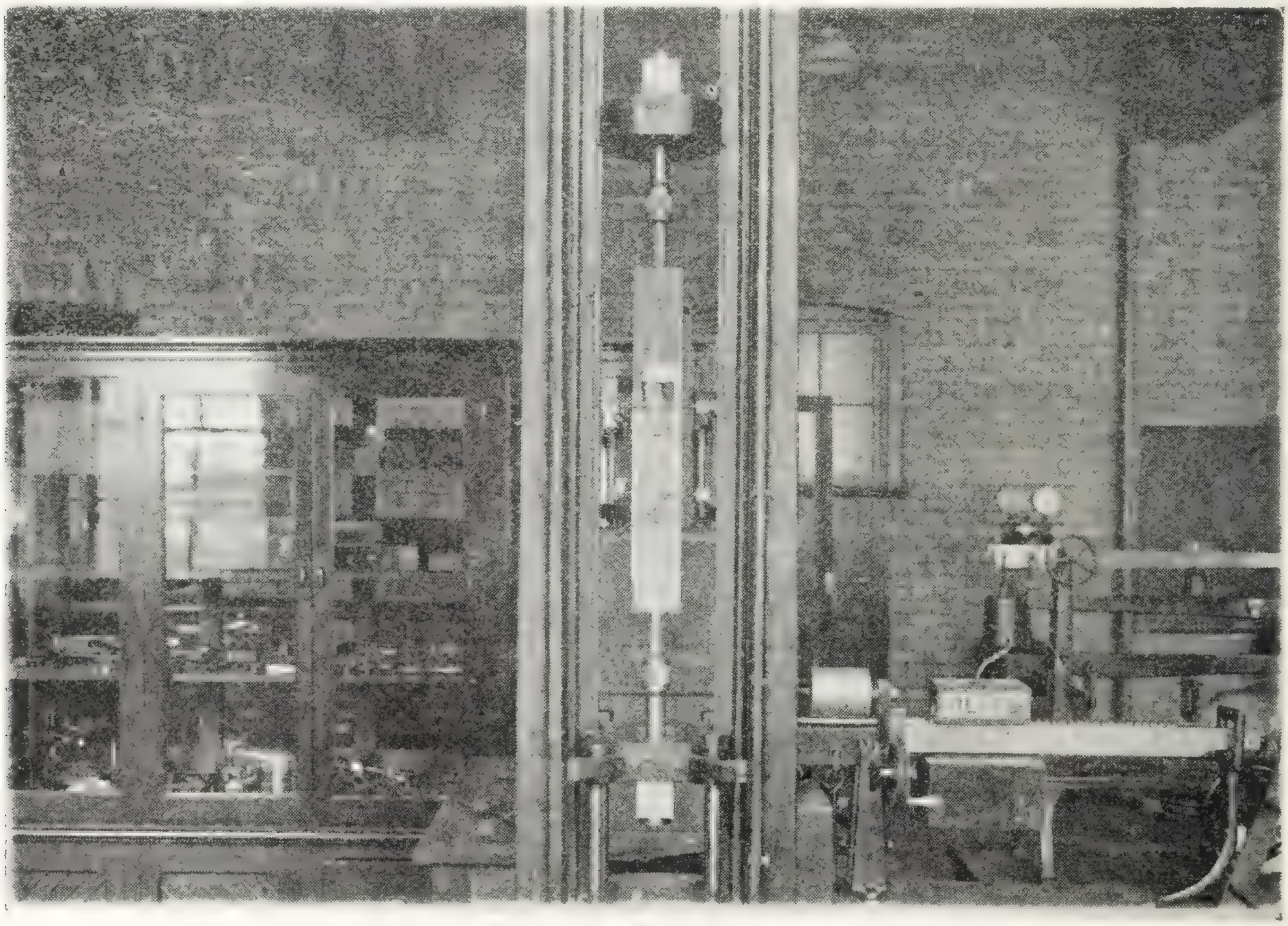


Fig. 2. Set-up for Investigating Shear Stress in Parallel Fillets.

and

$$T_1 = 2\phi_0\sqrt{\frac{CwAEh}{2}}\sinh\left(\sqrt{\frac{2Cw}{AEh}}\,l_1\right) \qquad (9)$$

From (9) may be obtained ϕ_0, knowing which (7) gives q for any assumed value of l. If that part of the tension denoted by T is desired it may be obtained from (8).

Fig. 2 is a photograph of the set-up for an experiment whereby, by means of short-length Martens extensometers, an effort was made to determine the stress in a tension member secured to two other tension members, one on either side of it, by thin web-like fillets. These fillets were not weld fillets but were made by milling the superfluous metal out of a 3 in.×1 in. bar of mild steel. The test specimen differed from that indicated in Fig. 1, chiefly in the fact that the shear was transmitted symmetrically between the central member and the outside members, there being two of the latter as the photograph shows. The following were either assumed or determined by measurement:

$w = .166$ in.,
$h = .50$ in.,
$A = .95$ sq. in. (The area of each outside member was .475 sq. in.)
$2l_1 = 13$ in.,
$C = 12{,}000{,}000$ lb. per sq. in.,
$E = 30{,}000{,}000$ lb. per sq. in., and
$T_1 = 15{,}000$ lb.

The following is a portion of the log of the experiment:

CENTRAL MEMBER

Distance from Loaded End	Observed Tension
1.1 in.	12,550 lb.
2.0 in.	11,490 lb.
2.9 in.	9,520 lb.
3.8 in.	8,970 lb.
4.7 in.	8,280 lb.
5.6 in.	8,125 lb.
6.5 in.	7,465 lb.
7.4 in.	7,180 lb.
8.3 in,	6,790 lb.
9.2 in.	6,330 lb.
10.1 in.	5,380 lb.
11.0 in.	3,880 lb.
11.9 in.	2,330 lb.

Knowing the manner in which the tension in the central member varied from point to point it was an easy matter to determine the shear stress in the fillet. This was done, and the result is shown in Fig. 3, the dotted line representing the experimental determination. Here is also shown the manner in which the tension in the central member varied.

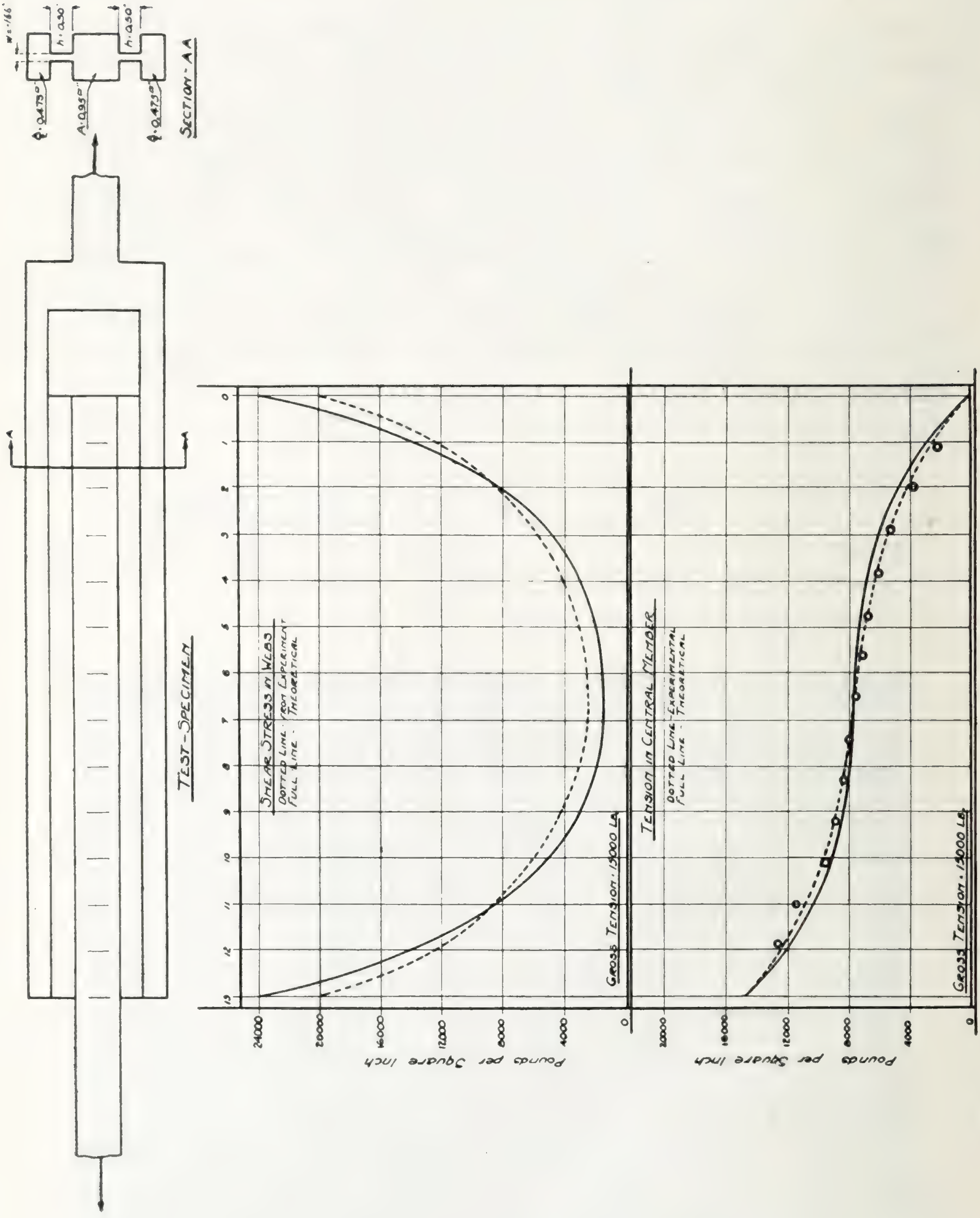


Fig. 3. Theoretical and Experimental Results of Investigation of Shear Stress in Parallel Fillets.

The value of ϕ_0 was obtained from equation (9) and was found to be .000128 radian. The following table gives a few values of both shear and tension as obtained from the formulae, T_1 being 15,000 lb.

COMPUTED VALUES OF T AND q

l	T	$T_1/2+T$	q
0 in.	0 lb.	7,500 lb.	1,540 lb. per sq. in.
1 in.	270 lb.	7,770 lb.	1,750 lb. per sq. in.
2 in.	610 lb.	8,110 lb.	2,480 lb. per sq. in.
3 in.	1,130 lb.	8,630 lb.	3,900 lb. per sq. in.
4 in.	1,980 lb.	9,480 lb.	6,450 lb. per sq. in.
5 in.	3,400 lb.	10,900 lb.	10,900 lb. per sq. in.
6 in.	5,800 lb.	13,300 lb.	18,400 lb. per sq. in.
6.5 in.	7,500 lb.	15,000 lb.	24,000 lb. per sq. in.

It will be apparent that the variation in shear stress intensity depends on the relative elastic deformations of the united members and the fillet. This then becomes a question of dimensions primarily. It may with approximate truth be said that if A, h and w are small and l_1 large, the variation in q is very pronounced, but if A, h and w are large and l_1 small, something approaching uniformity in the values of q results. The two graphs shown in Fig. 4 are intended to illustrate this phenomenon, it being recognized that about the mid-point of the length of the fillet, shear conditions are symmetrical. The study points out that in parallel welds the critical places may be at the extremities and that long parallel welds may sometimes be objectionable.

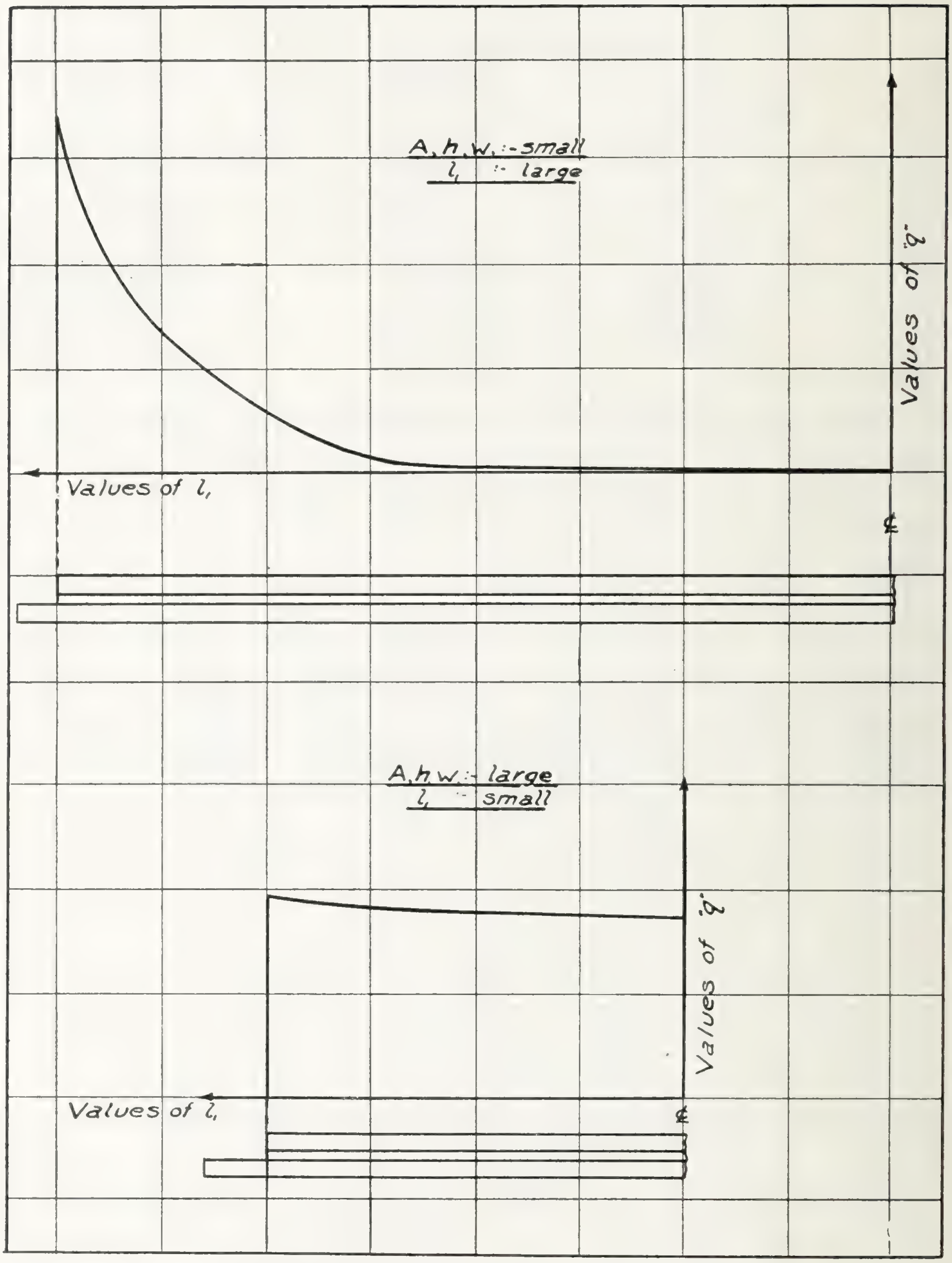


Fig. 4. Shear Stress Variation in Parallel Fillets for Hypothetical Cases.

Paper read before semi-annual meeting of American Society of Heating and Ventilating Engineers, June, 1927, and published in their Journal for July, 1927.

EFFECT OF TURBULENCE ON THE EFFICIENCY OF HOT BLAST HEATERS

By ROBERT W. ANGUS, *Professor of Mechanical Engineering,*

Recently the writer was asked to make up commercial tables for several new types of heater, and in carrying out the necessary experiments some information was obtained which does not appear to have been published before, and which is, therefore, presented herein.

All of the tests were carried out in the Mechanical Laboratory of the University of Toronto, Toronto, Canada, under the supervision of the author. The method of testing was the same as has been used by other experimenters, and the equipment is very similar to that described by the late Mr. John R. Allan in a paper presented to the American Society of Heating and Ventilating Engineers in January, 1917. A photograph of the equipment is shown on Fig. 1, this photograph showing the box *A* in which the heaters were placed, the receivers *B* for collecting the condensation, and the tank *C* for cooling the condensate so as to allow of its easy weighing. It will be noticed that the fan was placed in direct contact with the heater box instead of some distance away from it, as in Mr. Allan's tests, and the discharge from the fan took place through about 12 ft. of 24 in. round pipe, not shown in the illustration, the pitot tube measurements being made near the discharge end of this piping. Steam entered the heaters through the pipe *D*, shown near the top of the figure, and after passing through a separator *S* and reducing valve *V*, was delivered to the heater sections.

Arrangements were made for taking the temperature and pressure of the steam entering the sections, and the illustration shows a place at *E* for thermometer cups where the condensate left the sections. At this point, also, petcocks *F* were used so as to clear the air from the system, these being shown on the illustration and, as will be seen, the entire apparatus was well lagged with magnesia. The various air velocities were obtained by means of a variable speed motor, which enabled most of the control to be obtained, but in addition to that it was necessary to use an orifice for some of the lower velocities, this orifice being placed near the discharge end of the pipe, and no throttling took place between the heater and the fan. The illustration shows the arrangement for four stacks, but in the series of tests run experiments were made on one, three and four stacks separately.

The illustration shows a permanent set-up in the laboratory, and the galvanized work in the box was, therefore, specially constructed so as to be tight. The pressure carried in the heater throughout was approximately 5 pds. per sq. in., this being obtained by means of the reducing valve *V* shown, to which valve steam was supplied at about 25 pds. pressure. This reducing valve produced superheated steam, since the supply steam was nearly dry, but the amount of superheat was controllable and was kept below 12° F., and it was thought that some superheat was preferable to an uncertain amount of moisture.

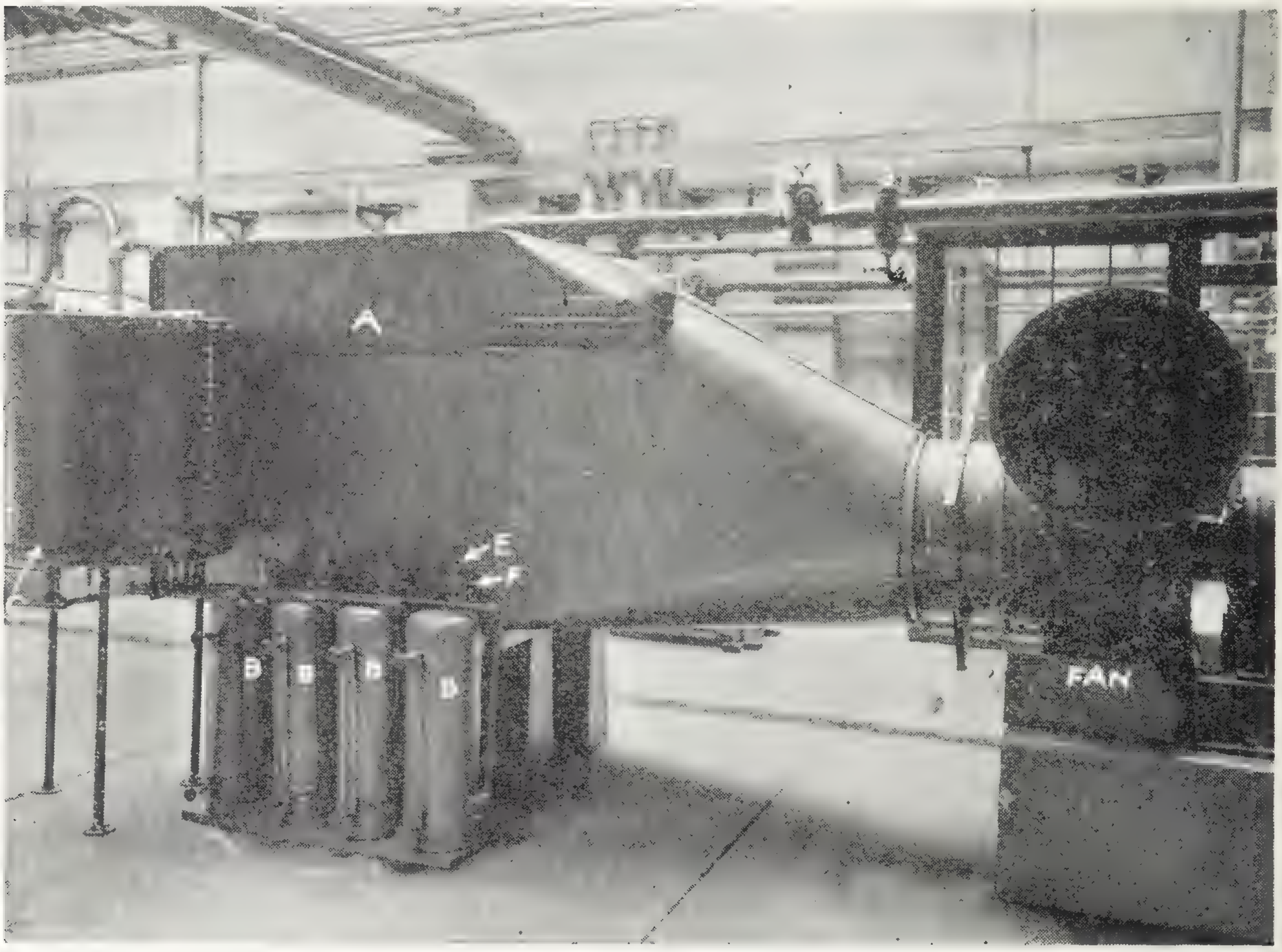


Fig. 1.—Laboratory Set-up of Testing Equipment

The steam condensed in each heater section was drawn off separately in a receiver, and the petcocks, attached to each discharge, were kept slightly open during all the tests, and no tests were used where the temperature of the condensate differed more than one degree from that corresponding to the steam pressure in the heater. The condensate passed from the receiver through a half inch pipe and globe valve controlled by hand, and during the tests the level of the water in each receiver was kept approximately at the middle of the water glass, and at the beginning and end of each test very great care was taken to see that the levels were exactly the same.

The temperatures of the air were taken both with thermocouples and with thermometers, and these agreed quite closely. It was found that by taking the temperature on the discharge side of the fan a uniform reading was obtained at all points in the 24 in. pipe, this not being the case when the temperature was taken on the suction side of the same fan.

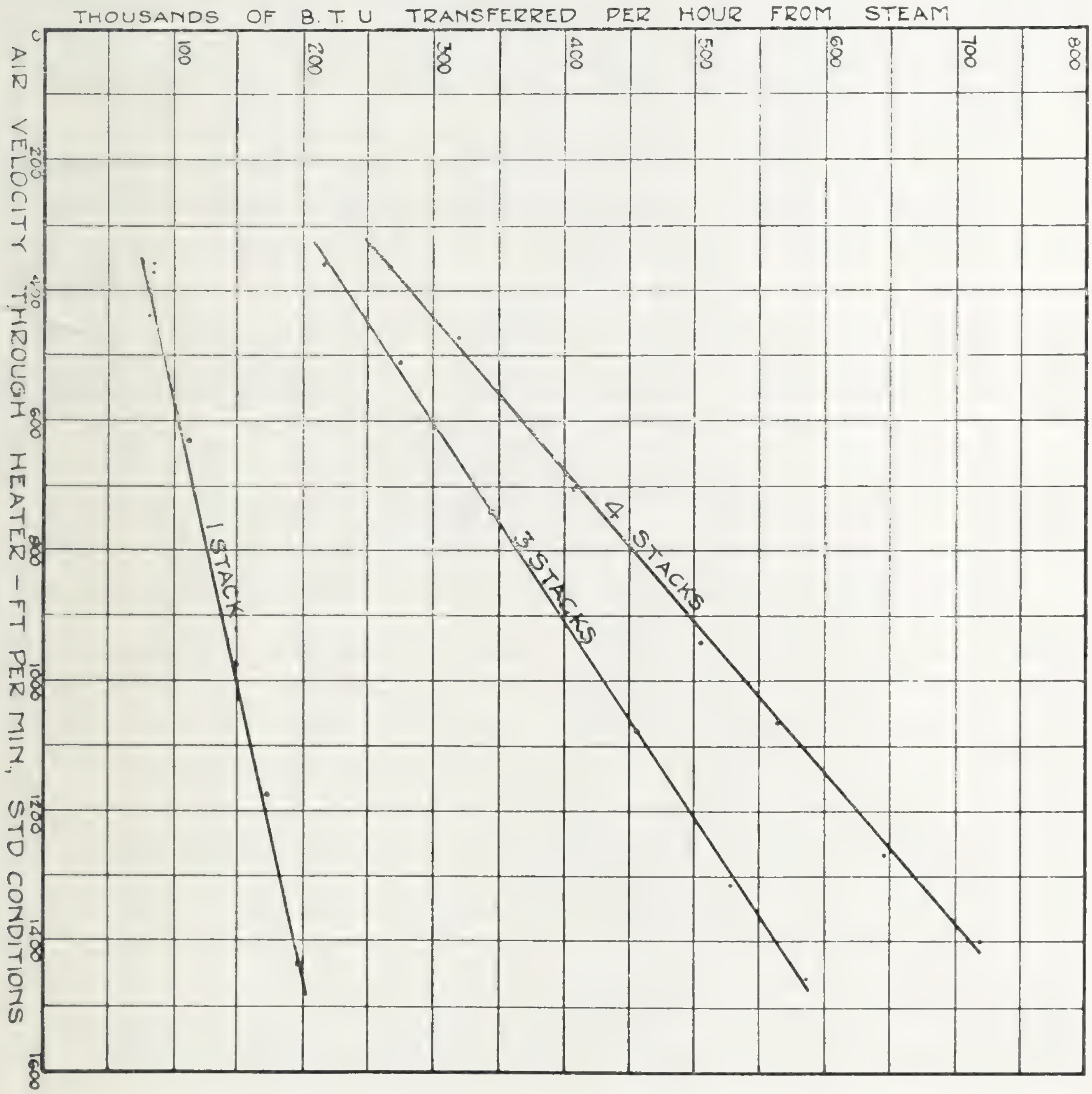


Fig. 2.—Amount of Heat Transferred per Hour in Tests of 1, 3 and 4 Stacks

The volume of air used was measured in exactly the same way as shown in Mr. Allan's paper, forty readings being taken for each test, ten across a vertical diameter, ten across a horizontal, and the same number across each of the diagonals, and in many of the tests two sets of traverses were made, so that the discharge should be accurate. The velocity head was read on a sloping alcohol gauge, on which readings could easily be obtained so as to give the velocity head to one one-hundredth of an inch of water for each point.

The friction loss in the heater was also accurately determined by inserting pressure tubes at the centre of the suction and discharge sides of the heater, the pressure openings in these tubes being about six inches away from the heater on each side, and a sensitive differential gauge was employed which enabled readings to be taken to one one-thousandth of an inch quite easily.

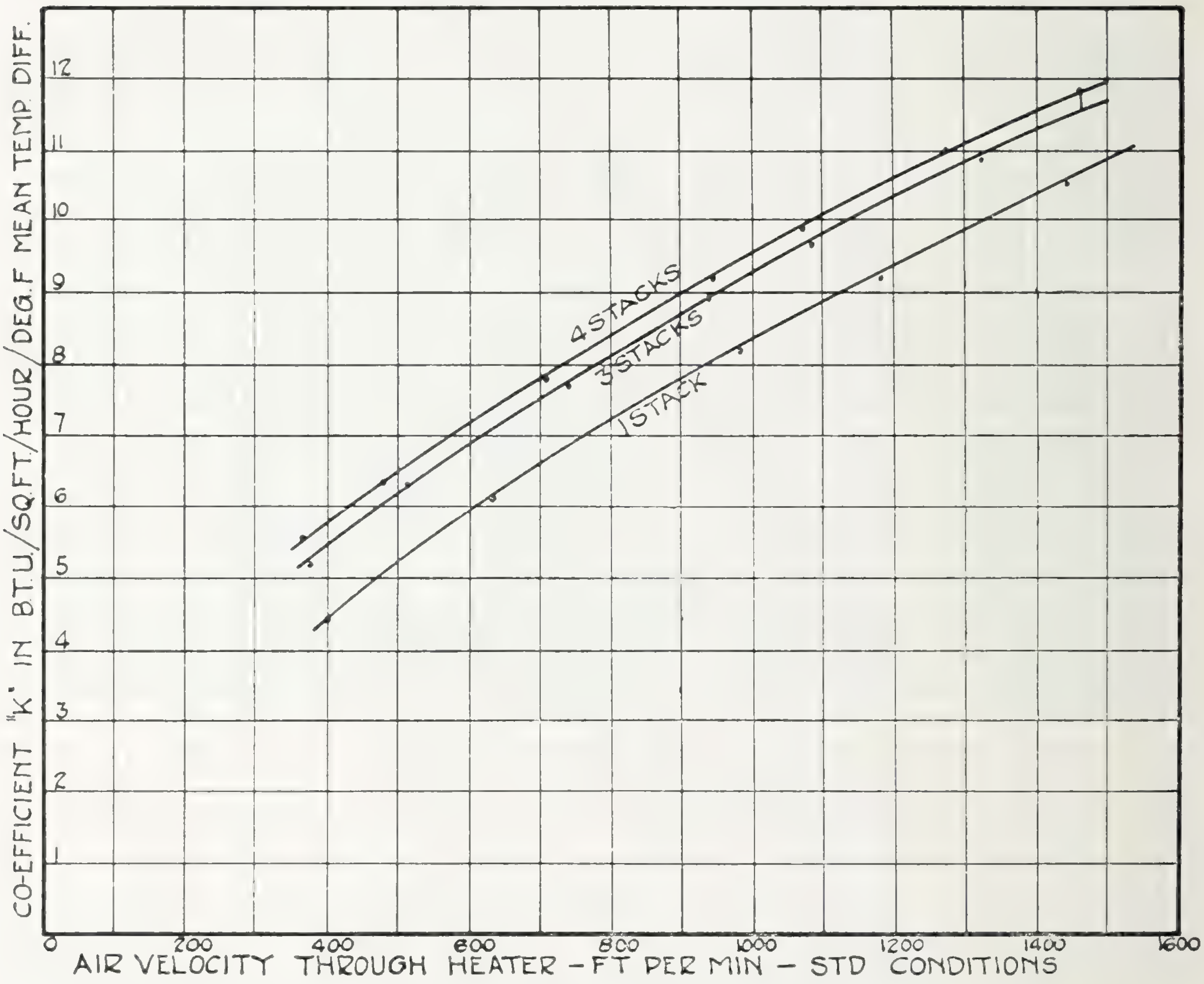


Fig. 3.—Coefficients of Heat Transfer K for Different Air Velocities

After experimenting with the special heaters on which information was sought, four stacks of 50 in. Vento heaters were placed in the box, each of these stacks being composed of nine sections placed on 5 in. centres. The heating surface of the Vento was measured and found to be 120.7 sq. ft. for the 9 sections, including the pipe connections, which corresponds very well with the value of 121.5 sq. ft. given in the maker's catalogue.

As the tables are all based on the velocity of the air through the heater, it was necessary to measure the clear area, and on this point there appears to be some difference of opinion. The method employed in arriving at this area was to take a vertical section passing through the centre of the supply opening on a given section and calculate the plane area of the outside of this section. Multiplying this area by the number of sections in the front stack and subtracting the product from the area of the box in which the heater was placed, gave the clear area used in the calculations, allowance having been made for the pipe connections. This figured out at 7.87 sq. ft. for the nine section stack, and is evidently very much higher than the 6.91 sq. ft. given in the maker's catalogue. The box containing the heaters was 47 1/8 in. wide and 51 in. high.

TABLE I

TYPICAL WEIGHTS OF STEAM CONDENSED IN EACH STACK OF A VENTO HOT BLAST HEATER WITH MORE THAN ONE STACK

Test No.	Number of Stacks in Heater	Velocity of Air Through Heater ft. min.	Condensation per hour, lbs.			
			Stack 1	Stack 2	Stack 3	Stack 4
1	4	1399	196.5	208.5	177.7	161.3
2	4	1265	177.7	188.3	159.0	141.7
3	4	1063	155.2	165.0	137.3	123.7
4	4	940	140.3	147.0	122.2	111.0
8	3	1455	201.7	218.3	187.5	
9	3	1311	182.3	198.0	168.0	
10	3	1073	157.5	170.3	143.2	
11	3	933	126.7	155.3	147.7	
12	3	733	121.5	128.2	105.0	
22	2	1469	210.0	225.7		
23	2	1221	180.0	194.3		
24	2	955	151.5	162.0		
25	2	659	123.0	130.5		

As already stated, the condensation in each stack was measured separately by the method already indicated, and the amounts of the condensation are given in Table I herewith. This table only includes a few of the tests actually made, and only those where the heat transferred from the steam to the air, as computed from measurements on the air, was the same as that computed from the condensation of the steam, within about one per cent. Care was taken that the sections were clear of air, and observations were frequently made on the front stack to check this up.

The complete details of the tests will probably not be of interest here, as similar information has been published already, but the results have been plotted on Figures 2, 3, 4, 5, and 6.

All of the curves have been drawn on a base showing the velocities of the air through the clear area, measured as above, the air having been reduced to the standard conditions corresponding to 70° F. and a barometer reading of 29.92 inches of mercury, this condition having been used in a number of tests which have been reported previously.

On Fig. 2 are given the B.T.U.'s of heat transferred per hour as computed from the steam, although, as already stated, the result agrees almost exactly with that computed from the air measurements. It will be noticed that the lines on this diagram are all straight, and if produced to the left it will be found that they all intersect the axis of velocities close to the same point. The experimental points lie so close to the lines in each case that no doubt exists as to the position of the latter, and check tests which were taken confirm the accuracy of these observations. Experiments were continued so as to include lower velocities and all of the lines bend down towards the origin, but such experiments are not shown on the figure because they contained some slight inconsistencies.

The coefficients of heat transfer K in B.T.U.'s, per square foot, per hour, per degree Fahrenheit mean temperature difference, for the different air velocities, are shown on Fig. 3. The points used in locating these curves are also shown. These values of K have been computed from Table I and the information shown on Fig. 2, using the formula

$$\log_{10}\left(\frac{T-t_1}{T-t_2}\right)=\frac{K\ S}{2.3026 \times 60 \times 0.2375\ W}$$

where T represents the temperature of the steam, t_1 that of the entering air, and t_2 the temperature of the leaving air, all in degrees Fahrenheit. The heating surface of the heater in square feet is denoted by S, and W is the weight of air passing through the heater in pounds per minute, the number 0.2375 corresponding to the specific heat of air at constant pressure. If the clear area through the heater in square feet is represented by A, and V is the velocity of the air through the heater reduced to standard conditions, then $W=0.075\ A\ V$, since the weight of one cubic foot of air under standard conditions is 0.075 pounds. The above formula then reduces to

$$\log\left(\frac{T-t_1}{\mathrm{T}-t_2}\right)=\frac{K\ S}{2.461\ A\ V}$$

In the heater experimented on, the heating surface per stack of nine sections was 120.7 sq. ft., and the clear area 7.87 sq. ft., so that the value of $\frac{S}{A}$ per stack was $\frac{120.7}{7.87} = 15.34$, and hence, for one stack the above relation becomes

$$\log\left(\frac{T - t_1}{T - t_2}\right) = 6.234\,\frac{K}{V}$$

For any other number of stacks the constant is to be multiplied by the number of stacks.

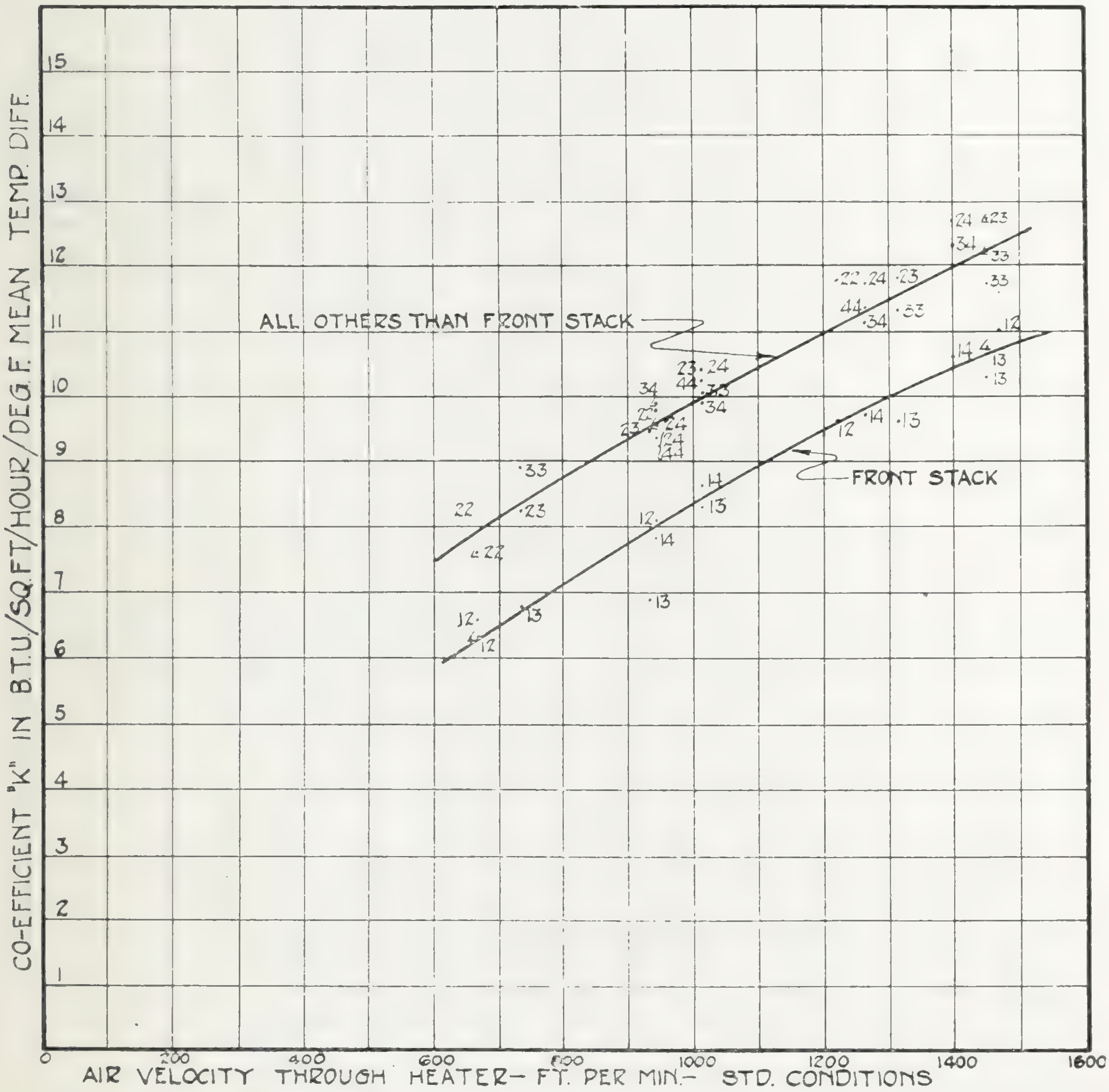


Fig. 4.—Values of K for the Different Stacks

The values of *K* plotted on Fig. 3 locate three curves, all concave to the axis of velocities, and all roughly parallel, the lower one corresponding to a one stack heater and the upper two to a three stack heater and a four stack heater, respectively. They show that the one stack heater is relatively much less efficient than the other, since it has a much lower value of *K*, and further, that the increased efficiency of the stacks behind the first one is great enough to make the *K* for the three stacks approximate quite closely to that for the four stacks, and the probability is that the coefficients for a two stack heater would, in the same way, differ little from those for the three stacks. This confirms the conclusion that the one stack heater is not as good for the transfer of heat as the multi stack arrangement, and that relatively greater heat transfer will be obtained from a given heating surface, if it is arranged in more than one stack.

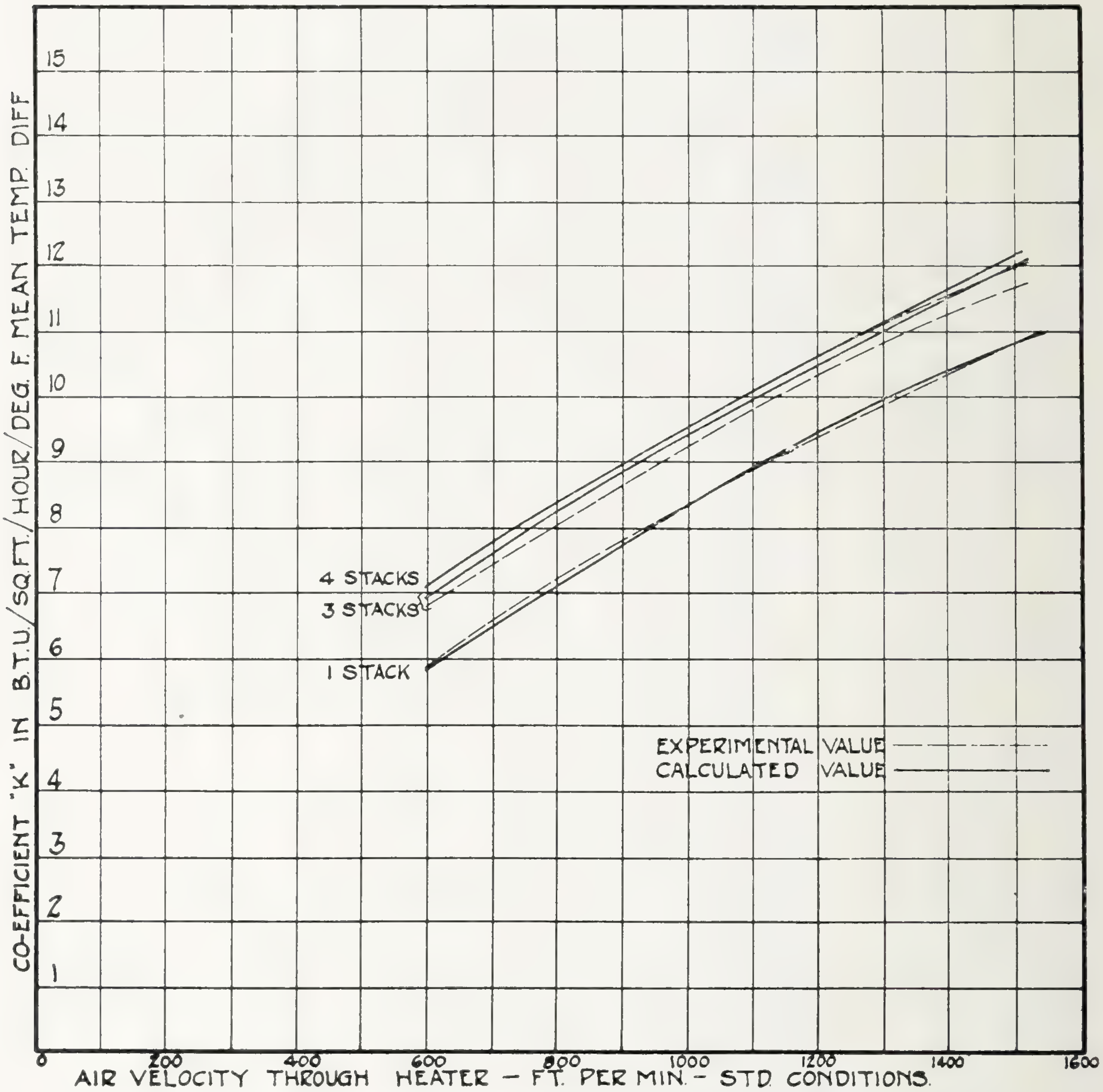


Fig. 5.—Comparison between Experimental and Calculated Values of K

The results on the condensation shown on Table I further bring the above facts out in a very striking way, for the table shows that the condensation in the first stack was invariably less than in the second one, although, since the first stack receives the colder air, it should have had the highest condensation if its efficiency were as good. In other words, while the first stack has the best opportunity to transfer heat, its efficiency in this respect is so low that it actually transfers less heat than the one behind it. From a practical point of view it would mean that if these stacks are taken care of separately, the second stack, and not the first one, should have the larger steam trap.

These facts are of considerable interest, and it was thought worth while to analyse them a little more fully. By the use of Table I it is possible to find the rise in temperature for each stack separately, and knowing this rise in temperature it is possible, by the formulas given, to calculate the value of K for each stack in a heater containing a number of stacks. This calculation has, therefore, been made and the results of the calculation are plotted on Fig. 4, for velocities above 600 ft. per min., the numbers at the points indicating to which stack each point belongs, the last figure indicating the number of stacks in the heater, and the first figure the stack to which the K belongs. Thus, point 34 represents the value of K calculated for the third stack on a four stack heater.

These calculations again separated the first stack from the others, and a check is given on the method of separation in this way by the fact that the value of K computed for the first stack agrees almost exactly with the test results of a single stack heater. While the other points are to some extent scattered, there would appear to be only one curve corresponding to them, and this curve has been drawn on the figure located as accurately as possible from the points shown, and they indicate that all stacks but the first one appear to have the same value of K.

In order to examine this matter further, a set of curves has been plotted, in Fig. 5, for one stack, three stack and four stack heaters, these curves being only approximately determined in this case, since the assumption has been made that the K to be used for a multi stack heater would be obtained by averaging arithmetically the results given on the curves on Fig. 4. Thus, for a four stack heater, with an air velocity of 1000 feet per minute, it has been assumed that the average K would be $\frac{1}{4}(8.35+3\times 9.90)=9.51$. On the same figure, Fig. 5, the results of tests on the four stack heater, the three stack heater, and the one stack heater, are shown dotted, and they agree fairly well with the theoretical curves obtained on the above assumption. This investigation would, therefore, appear to indicate very distinctly that the losses are primarily in the first stack, and apparently the only real difference between the stacks is the condition of the air approaching the different ones, that coming to number one being more or less a straight line flow, whereas that approaching the others is very turbulent, due to the disturbance from the stacks in front of them.

At the higher velocities on the multi stack heaters the computed and experimental results differ somewhat, and the matter is open for further investigation. The difference is not very great, however not exceeding three per cent. in any case, although the two sets of curves are rapidly diverging.

Two practical results arising from this investigation are:

1. That the action of a heater may be very materially improved by placing in front of the first stack baffles of some kind which will produce marked turbulence of the air reaching it. If sufficient care is taken in the design of these baffles, it should be possible to gain enough from the heating to counter-balance the loss due to the extra friction.

2. The results show that in preparing rating tables for heaters of this kind the experiments can be done by connecting a single stack to the supply steam and doing all of the experiments on it, the first set with the single stack alone in the test box, aud the second set with another stack of the same kind simply set in its proper position in the test box in front of the test stack in such a way as to produce the necessary amount of turbulence.

Heaters are rarely over six stacks deep and the computed curve for six stacks would not be far above that for four stacks. The average value of K for the three and four stack heaters at a velocity of 600 ft. per min. does not differ by more than three per cent. from the K for the three or the four stack heater, and at 1200 ft. per min. this average does not differ by two per cent. from the extreme curves. In the construction of tables, therefore, only two values of K need be used at a given velocity, that for a one stack heater, and the other for more than one stack, the error being within three per cent. for the usual combinations.

The two curves of Fig. 6 show the friction loss in the four stack heater, and are given partly to show a check on the accuracy of the velocity measurements, and partly to enable the experiments to be compared with others on the same make of heater where a different clear area has been used. The figure on the left gives the logarithms of the velocity and friction loss, while the right hand figure gives the direct relation between friction loss and velocity.

PIPE HEATERS

During the same investigation tests were also conducted on modified forms of pipe heaters, but the indication here is that the first stack behaves similarly to the others, within quite narrow limits except at the lower velocities. At velocities above 900 ft. per min. the values of K for

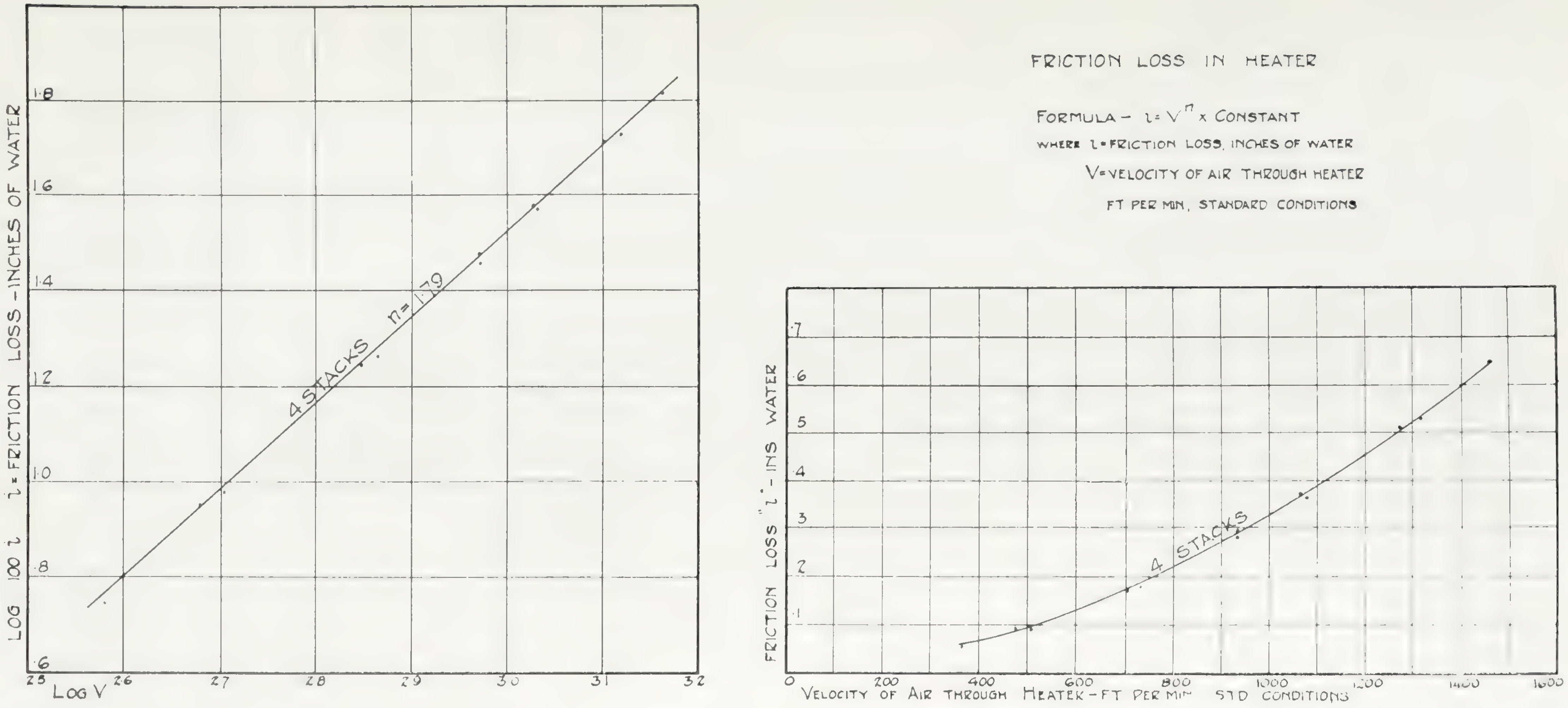


Fig. 6.—Friction Loss in Four Stack Heater

a single stack heater were found to be within five per cent. of the value of K for a multi stack heater with the same velocity, but at the lower velocities the difference is much more marked. In all cases examined the condensation in the first stack was always the greatest, and there was a decrease toward the back; the first stack always showed about 20 per cent. greater condensation than the one behind it, and the decrease was about the same per cent. per stack.

The same results were found by other experimenters whose results have been already reported. In these cases the value of K was found to be nearly constant per stack. The author's tests indicate a similar condition, but it is impossible to analyse them fully at the present time.

LOSSES IN SUCTION PIPES OF FIRE ENGINES

Robert W. Angus, *Professor of Mechanical Engineering.*

Fire engines often draw water from hydrants under pressure, in which case the loss on the suction side of the pump is of no very great moment, although this should always be reduced to a minimum. But in case the engine draws from a well or lake, there is a vacuum on the suction, and this vacuum may become high enough to interfere with the operation of the engine at full discharge, since the output of fire engines decreases for higher suction lifts.

A short time ago, when the author was conducting some acceptance tests on fire engines drawing from the lake, he decided to determine the loss in the suction line as soon as the engine tests were finished. Only one type of rubber hose with strainer was available, this hose being 6 in. internal diameter with a coil of wire imbedded in the rubber to keep the hose from collapsing. The inside of the hose was quite smooth, but corrugations due to the wire were visible, and could easily be felt, although the extreme depth of the corrugations was quite small.

Two lengths of suction hose were used, each 9 ft. 7½ in. long exclusive of the couplings, and for the purpose of the tests the two lengths were screwed together in series.

As the couplings had a net length of 6½ in. and the two piezometer connections were respectively 3½ in. and 2 in. from the end of the coupling, the total length from one piezometer connection to the other was 20 ft. 3 in., of which the rubber hose occupied 19 ft. 3 in. The exact inside diameter of the hose was hard to measure and it varied from about 6 in. to 6-1/16 in., but as it was nominally 6 in. hose, the latter diameter was used in the calculations, and would be very nearly correct.

The form of strainer used is shown in Fig. 1, and it had an opening 9¼ in. diameter in the casting which was covered with a strainer made of 1/8 in. wire, there being 14 wires across one diameter and 15 across the other, the mean pitch of the wires being 0.615 in., centre to centre. The set up of the apparatus is shown in Fig. 2, where the strainer end of the hose is shown inside a large tank and two piezometer rings are shown in the line, one at each end of the rubber hose. Four pressure connections were made on each piezometer and the suction hose under test was bent into a half circle so as to include the loss from curvature that occurs in practical operation.

RESULTS OF TESTS ON SUCTION HOSE AND STRAINER AS USED ON FIRE ENGINES

Test No.	Discharge Q c.f.s.	Velocity v ft. sec.	Velocity Head h_v ft.	H_1 ft.	STRAINER				HOSE				Total Loss $H_1-H_3-h_v$ ft.
					H_2 ft.	H_2+h_v ft.	Strainer Loss $h_{r_{1-2}}$ ft.	Resistance coeff. c_r	H_3 ft.	H_3+h_v ft.	Hose loss $h_{r_{2-3}}$ ft.	Friction factor f	
1	.512	2.61	.11	21.21	21.08	21.19	.02	.23	21.05	21.16	.03		.05
2	.622	3.17	.156	21.18	20.98	21.14	.04	.28	20.93	21.09	.05		.09
3	1.136	5.80	.523	21.27	20.59	21.11	.16	.30	20.32	20.85	.26		.42
4	1.543	7.87	.963	21.21	20.03	20.99	.22	.23	19.35	20.31	.68	.0174	.90
5	2.043	10.43	1.69	21.15	19.06	20.75	.40	.24	17.86	19.55	1.20	.0175	1.60
6	2.519	12.84	2.54	21.19	17.98	20.52	.67	.25	16.12	18.66	1.86	.0181	2.53
7	2.651	13.52	2.83	21.18	17.74	20.57	.61	.22	15.67	18.50	2.07	.0181	2.68
8	2.947	15.03	3.51	21.15	16.95	20.46	.69	.20	14.26	17.77	2.69	.0189	3.38
9	3.378	17.24	4.64	21.16	15.61	20.25	.91	.20	12.20	16.84	3.41	.0181	4.32
10	3.452	17.61	4.82	21.18	15.27	20.09	1.09	.23	11.66	16.48	3.61	.0185	4.70
							Average . .	.238	Average of last seven . .			.0181	

The water discharged was caught in large measuring tanks, graduated so that the volume was easily measured, and the time was taken with a stop watch. The rate of flow was controlled by the valve shown, and the water was delivered to the piping from the large tank shown on Fig. 2, the level of the water in this tank being kept constant by means of an overflow.

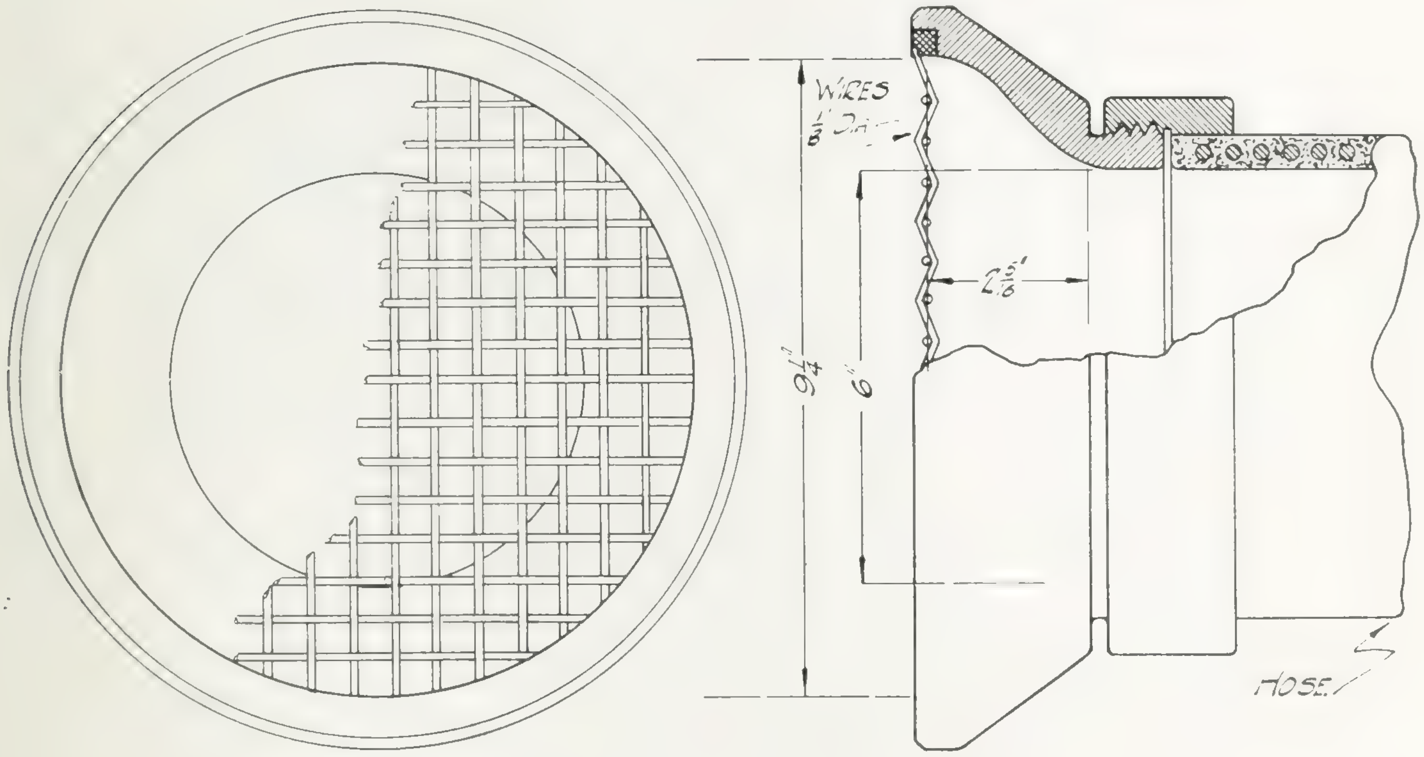


Fig. 1.—Strainer for Fire Hose

Altogether ten experiments were made, at discharges varying from 0.512 cu. ft. sec. to 3.452 cu. ft. sec., the latter corresponding to a velocity of 17.61 ft. per sec. in the 6 in. hose. The piezometers enabled the loss to be measured separately in the strainer and also in the suction hose and this was done. The results of the tests are given in the table, in which Q is the discharge in cu. ft. sec., H_1 is the height of the water in the tank above datum level, and H_2 and H_3 are, respectively, the heights of water column above the datum in the tubes attached to the piezometer ring between the strainer and entry end of the suction hose, and to the ring between the discharge end of the hose and the control valve respectively. The velocity v in the hose, in ft. per sec., was calculated on the basis of a diameter of 6 in. for the hose.

The velocity head h_v is equal to $v^2/2g$ and is the same at sections 2 and 3. The loss of head in feet in the strainer is $h_{r_{1\text{-}2}} = H_1 - (H_2 + h_v)$ and in the hose is $h_{r_{2\text{-}3}} = H_2 - H_3$ while the loss in the strainer and hose combined is $h_{r_{1\text{-}3}} = H_1 - (H_3 + h_v)$. In order to get the friction coefficients for the strainer and hose, the loss $h_{r_{1\text{-}2}}$ has been divided by h_v giving the coefficient c_r for the strainer, or $c_r = h_{r_{1\text{-}2}}/h_v$; and for the hose the value

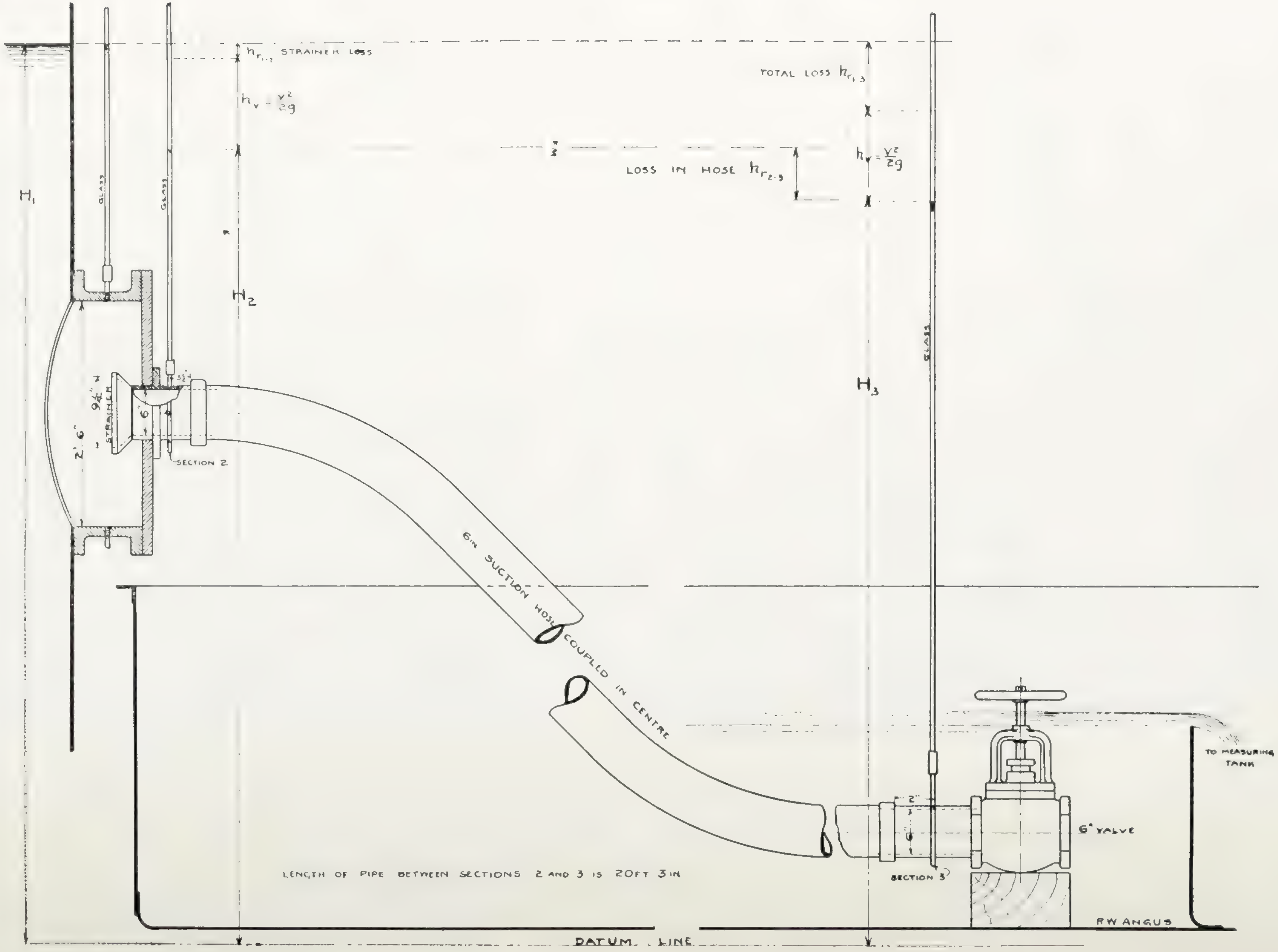


Fig. 2.—Arrangement of Test Apparatus

of f has been found in the formula $h_{r_{2-3}} = f\frac{l}{d}h_v = f\frac{20.25}{.5}h_v$ based on the 20 ft. 3 in. between piezometer connections.

The results of the tests are also plotted on the curve sheet, Fig. 3, and indicate a consistent set of experiments. The loss in the strainer remains very close to the average of 24 per cent. of the velocity head in every case, a loss which is extremely small and amounts to only 1.09 ft. in Test 10 where the hose velocity is 17.61 ft. per sec. The friction factor shows very great uniformity, more so than might be expected. In the first two tests the loss was so small that quite serious errors are possible in f, but at all velocities of 8 ft. per sec. or above, f differs very little from the average of 0.0181 and is much more constant than for iron pipe of the same size. Its value, however, differs very little from Merriman's factors for clean iron pipes.

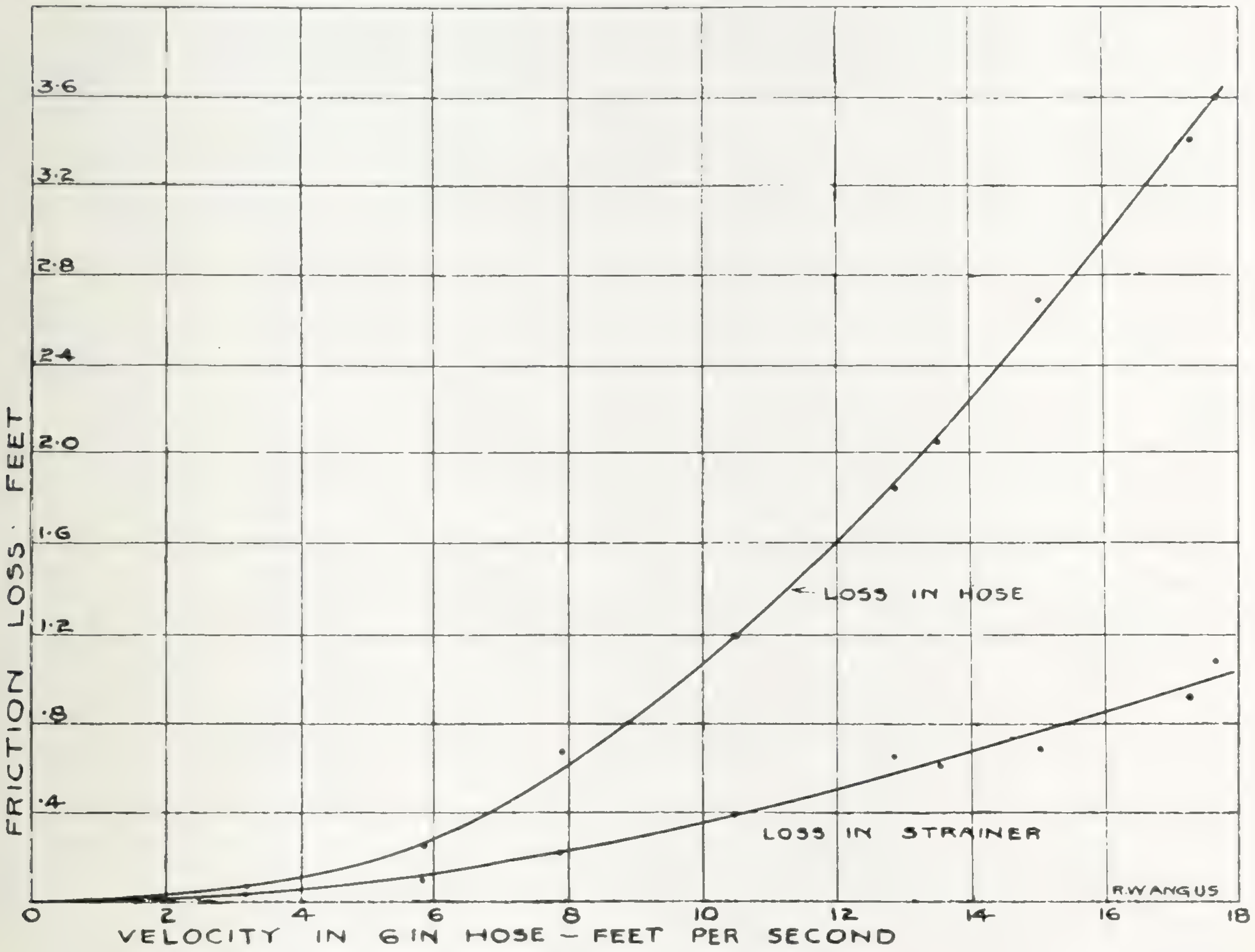


Fig. 3.—Loss in Strainer and Suction Hose

It is interesting to note the conditions in the suction pipe. In the case of Test 10 the loss of head is 4.70 ft. and the velocity head is 4.82 ft. making a total of 9.52 ft., which means that if the suction connection of the engine were at water level the vacuum at the engine would be 9.52 ft. for the draft of 3.45 c.f.s. or 1295 Imp. galls. per min. If it were desired to keep the vacuum at the engine down to 12 ft., it is evident that the engine could not draw this volume of water if it were placed over 2.5 ft. above the water surface.

Velocities of about 18 ft. per sec. are used at times on suction lines, in which case the engine should be set close to the suction water level.

These tests were made in the Hydraulic Laboratory of the University of Toronto in March 1927 and the writer acknowledges the assistance of Professor R. Taylor and of R. M. Robertson, and also the co-operation of the Bickle Fire Engine Company of Woodstock, Ontario, who kindly supplied the piezometer sections and loaned the hose and strainer.

VENTURI METER COEFFICIENTS AND FRICTION LOSS IN A 14 INCH TEE

By Robert W. Angus, *Professor of Mechanical Engineering*

It is well known that when a pipe flowing full of water is gradually reduced in size there is, with certain shapes of reducer, an almost complete conversion of velocity energy into pressure energy, so that by measuring the change in pressure head the velocity of water in the pipe may be determined.

Many years ago Clemens Herschel first turned this principle to practical use by inventing the Venturi water meter, one of the most accurate devices for water measurement ever discovered. For this meter the best angle for the reducing section was determined by experiment, and in order to reduce the energy loss in the device to a minimum, the best angle for the enlarging section has also been accurately determined. The result is that the ordinary Venturi meter, consisting of a reducer, a throat piece and an enlarger, causes a very small loss of energy in the line, and the loss in the upstream cone is so small that the energy at the throat differs only about two per cent. from that at the entrance to the meter.

The formula for the meter is so well known that it is stated here without proof:—

$$Q = A_2 v_2 = \frac{A_2\, c\, \sqrt{2g\, H}}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}}$$

where Q is the discharge in cubic feet per second, A_1 and A_2 are the areas, in square feet, of the entrance end, and of the throat of the meter, respectively, H is the drop, in feet of water column, in the hydraulic gradient, and is measured by differential gauge connected between the entry end of the meter and its throat, and c is the meter constant.

Experience and theory both show that the coefficient c is always less than unity, provided the differential gauge is so connected that it is not influenced by the velocity at either point of connection. As a matter of fact its value is close to 0.99 in ordinary meters properly installed and used, which indicates how small the energy loss in the meter is, and the fact that this coefficient is nearly constant for great ranges of velocity and sizes of meter has been one of the greatest factors in establishing the device on its present permanent footing.

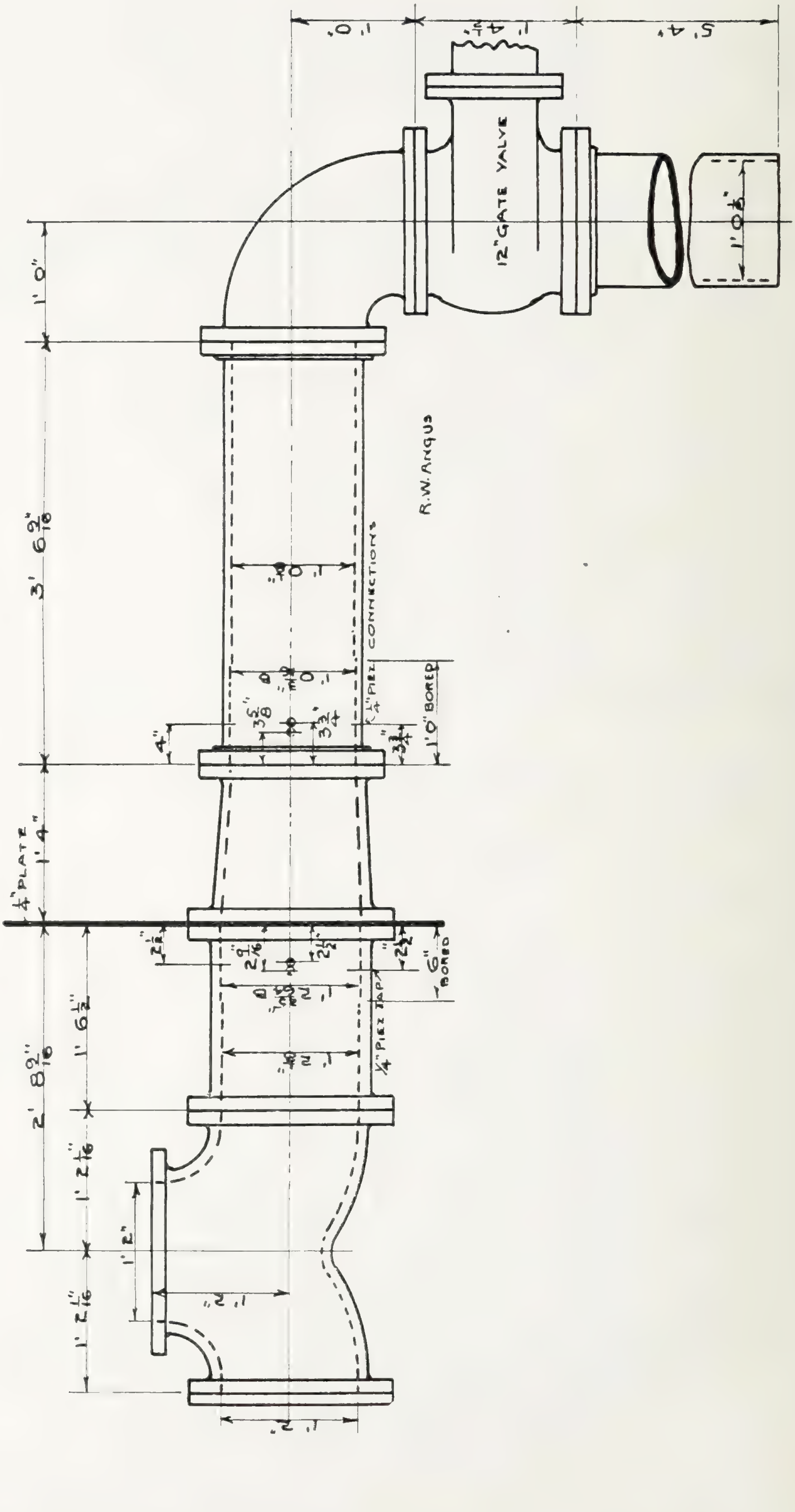


Fig. 1.—Detail of Piping Used in Tests

That there is a much broader application of the principle has recently been recognized. Hydraulic power engineers have desired a continuous and convenient record of the water used by turbines, etc. but ordinary methods of measurement are expensive, cumbersome and frequently inaccurate. The Venturi meter as usually proportioned is relatively quite long, and in the large penstocks of water turbines it is often impracticable to instal the device. Further, the loss of head, though very small, is, particularly in many low head plants, serious from a power standpoint, and relatively few power plants have used the Venturi meter in its commercial form.

The penstocks in plants usually carry water at high velocity and at certain points in them there are reductions in area which cause sufficient increase in velocity to produce a marked decrease in pressure. Frequently, however, whirling takes place in the penstock, and it is impossible to select a measuring section in such a way that the differential gauge will not be affected by the velocity, and it is often found that the tapering section is so long that there is an appreciable friction loss in it.

If, therefore, the standard formula is applied to the case just stated, the value of *c* sometimes falls to nearly 0.90, but on the other hand it may exceed unity, depending of the effect of velocity on the differential gauge reading. These unusual values of the coefficient do not affect the application of the method of measurement, but they do show that in such a case the value of *c* must be determined in the plant for the various conditions of flow, that is to say, the pipe must first of all be rated by some accurate method which will give the constant with accuracy, for each differential reading.

Not long ago the writer was asked to make a test of a turbine pump on which it was impossible to connect a commercial Venturi meter. The discharge piping, temporarily connected to this pump, was made up of the fittings shown in Fig. 1, and it was decided to use the tapering reducer as a meter, after the value of *c* had been determined by direct measurement. Before making the pump test, therefore, this piping was set up in the Hydraulic Laboratory of the University of Toronto, in the manner shown in Fig. 2, and discharges up to 10.89 c.f.s. were passed through it. The volume of water was measured by a 5 ft. suppressed weir, the coefficients for which had been previously determined, and the head on the weir was measured by hook gauge. The rate of discharge was controlled by the 12 in. gate valve.

Piezometers above and below the reducer were connected at the points shown. In each case four piezometer connections 90° apart were made, at the points shown in Figs. 1 and 2, and a separate glass tube was lead out from each point of connection, so that information could be obtained on the variations in pressure at a given section. There were

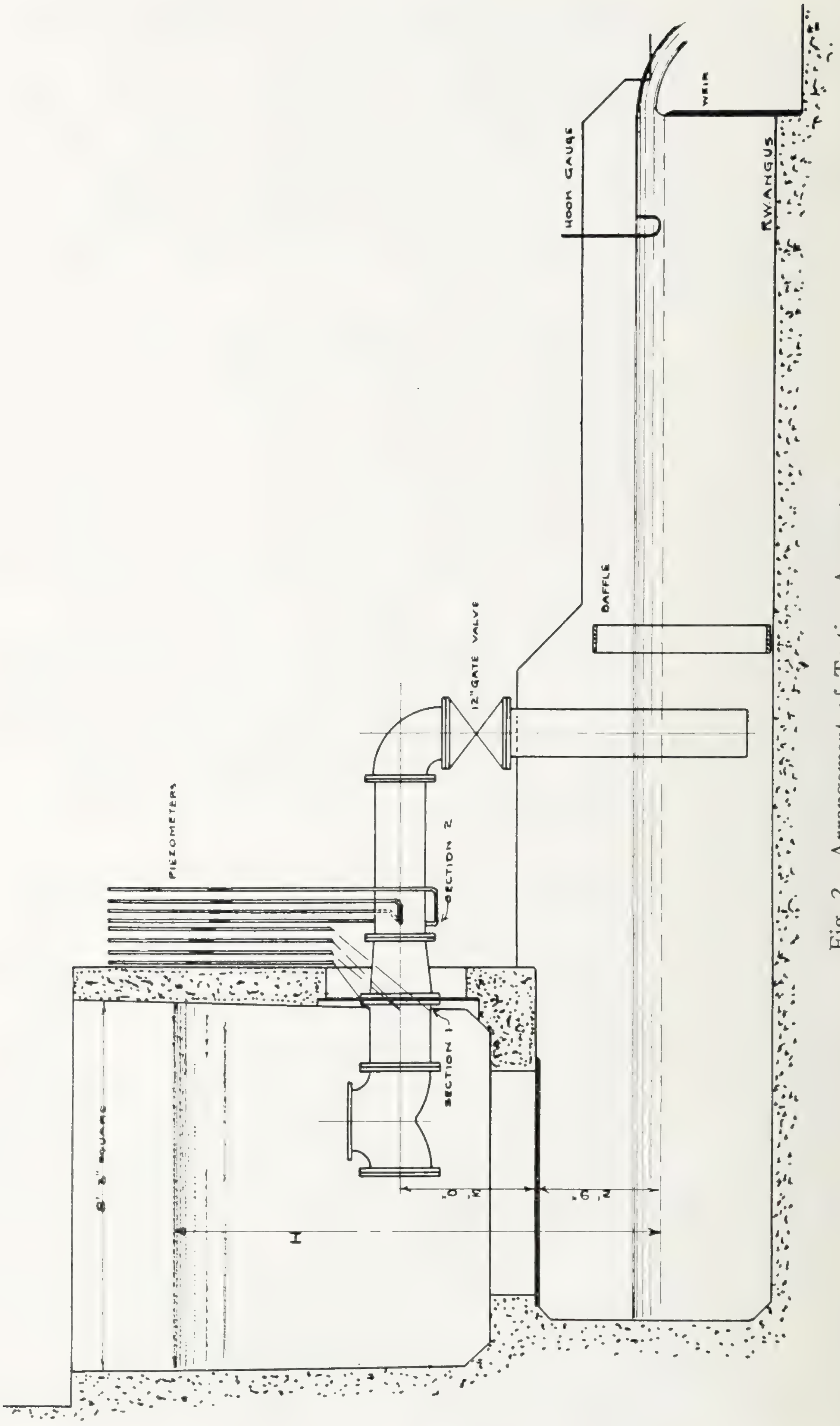


Fig. 2.—Arrangement of Testing Apparatus

TABLE I

TESTS FOR COEFFICIENTS FOR VENTURI METER AND OF FRICTION LOSSES

OBSERVATIONS TAKEN DURING TESTS, FEB. 2ND AND 3RD, 1928

Test No.	Elevation of water H in.	Piezometers on 14 25/64 in. pipe Section 1. Inches of water above the weir crest.					Piezometers on 12 3/16 in. pipe Section 2. Inches of water above the weir crest.					Weir	
			On sides of pipe					On sides of pipe					
		On top of pipe in.	East side in.	West side in.	On bottom of pipe in.	Average in.	On top of pipe in.	East side in.	West side in.	On bottom of pipe in.	Average in.	Head in.	Discharge c.f.s.
1	129.13	—	83.13	83.38	83.88	83.46	67.37	69.13	65.88	71.28	68.43	8.87	10.89
2	129.13	82.88	82.55	82.80	83.13	82.84	66.87	68.34	65.55	71.34	68.02	8.83	10.81
3	128.38	93.25	92.38	93.38	93.50	93.13	82.25	81.50	79.38	84.13	81.82	8.10	9.49
4	128.13	99.13	98.25	99.38	98.88	98.91	89.25	89.25	87.38	90.88	89.19	7.62	8.66
5	130.63	116.50	116.00	116.38	116.63	116.38	111.75	111.75	111.00	112.50	111.75	6.01	6.08
6	132.75	126.38	126.38	126.50	126.38	126.41	124.38–	124.38	124.13	124.63	124.38	4.60	4.10

thus eight tubes giving the pressure heads at sections 1 and 2, and in addition to this the height of water in the tank was measured. The experiments, therefore, enabled the losses in the different parts of the piping to be determined, as well as the coefficient for the reducer.

The results of the tests are given in the tables herewith, Table I giving the actual observations taken and showing how the pressures vary at the different points on the same ring. The pressure heads recorded throughout are in inches of water above the weir crest.

LOSS IN THE 14 INCH TEE

It is interesting to note that the pressures on the four piezometers on the large section agree very closely, quite as closely indeed as the measurement of head could ordinarily be made, since the water in the tubes surged slightly, even though the head H was kept constant by an overflow in the large tank. On the smaller pipe, however, the pressures indicated by the four piezometers were not the same, the one on the bottom of the pipe always being the highest, but it is to be noted that they get closer together as the discharge decreases, and in Test No. 6 they are almost the same. Thus the disturbances due to the tee and to the entry conditions affect only the smaller section.

In Table II the results have been worked out from the observations. The velocity at the smallest or "throat" section varied from 13.46 ft. per sec. to 5.07 ft. per sec. and for each test the average pressure heads from Table I and the velocity heads at Sections 1 and 2 have been found, and by using these along with the level of the water in the tank the loss of head in the tee and short piece of straight pipe is readily found. This loss was divided by the velocity head at Section 1 and the result is set down in column 9, which indicates that this quotient is very constant and averages 1.66, that is to say, the loss of head at entrance to the tee and also in the tee and short piece of straight pipe is 1 66 times the velocity head in the pipe. This is on the assumption that the manometers at Section 1 correctly measure the pressure at that point.

COEFFICIENT FOR THE REDUCER USED AS A METER

A calculation of the loss between Sections 1 and 2 indicates that this is always negative, beginning with 0.12 ft. for Test No. 1 and decreasing steadily till it reaches a minimum of 0.03 ft. for Test No. 6, this negative result indicating that the piezometers at either or both of Sections 1 and 2 are slightly affected by the velocity. The loss in this section must, however, be very small. Using Sections 1 and 2 the coefficient c in the Venturi meter formula has been worked out and is given in column 12, and this coefficient is greater than unity, as is to be expected from the apparently negative friction loss in the section. The coefficient is remarkably constant, having an average value of 1.051, the maximum difference between this and any of the values being only two per cent., while in four of the six tests the difference from the average is only about one-third of one per cent.

When being used as a meter in the pump test a mercury differential gauge was connected between Sections 1 and 2 and the discharge in cu. ft. sec. was taken from the formula:

$$Q = \frac{A_2 \, c \, \sqrt{2g \, H}}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}}$$

where H is the differential in feet of water.
Since the diameters at Sections 1 and 2 are 14 25/64 in. and 12 3/16 in. respectively, the area $A_1 = 1.129$ sq. ft. and $A_2 = 0.809$ sq. ft. and $c = 1.051$, so that

$$Q = \frac{1.051 \times .809 \, \sqrt{2g \, H}}{\sqrt{1 - \left(\frac{.809}{1.129}\right)^2}}$$

As the discharge in millions of Imperial gallons per day was required, this is given by

$$M = .539 \, Q = \frac{.539 \times 1.051 \times .809 \times 8.02 \, \sqrt{H}}{.698} = 5.261 \, \sqrt{H}$$

If the differential in inches of mercury m is used

$$\text{then } H = \frac{12.6}{12} \times m = 1.05 \, m \text{ and hence}$$

$$M = 5.391 \, \sqrt{m}$$

gives the discharge in millions of Imperial gallons per day.

The investigation gives some coefficients for losses in a large tee and also shows that when such an unusual piece of piping is available it may be used as a meter if a careful calibration is made to determine the proper coefficient. The latter clearly cannot be calculated from the dimensions of the piping, owing to the close proximity of the tee.

TABLE II

1 Test No.	2 Discharge c.f.s.	3 Velocity at Section 1 v_1 ft. sec.	4 Velocity at Section 2 v_2 ft. sec.	5 Elevation of water in tank H ft.	6 Piezometer reading at Section 1 ft.	7 Velocity head at Section 1 h_{v_1} ft.	8 Loss of head up to Section 1 $h_{r_{0-1}}$	9 Coefficient of resistance for the tee $c_r = \frac{h_{r_{0-1}}}{h_v}$	10 Piezometer reading at Section 2 ft.	11 Velocity head at Section 2 h_{v_2} ft.	12 Coefficient c in Venturi meter formula for pipe 1-2 c
1	10.89	9.65	13.46	10.76	6.96	1.45	2.35	1.63	5.70	2.81	1.048
2	10.81	9.59	13.36	10.76	6.90	1.43	2.43	1.70	5.94	2.78	1.047
3	9.49	8.40	11.73	10.70	7.76	1.10	1.84	1.67	6.82	2.14	1.051
4	8.66	7.67	10.70	10.68	8.24	.91	1.53	1.68	7.43	1.78	1.035
5	6.08	5.39	7.52	10.89	9.70	.45	.74	1.64	9.31	.88	1.053
6	4.10	3.63	5.07	11.06	10.53	.20	.33	1.65	10.36	.40	1.073
							Average	1.66			1.051

THE EFFECT OF AIR LEAKAGE ON TURBINE PUMP CHARACTERISTICS

By Robert W. Angus, *Professor of Mechanical Engineering* and
C. V. Armour, *Demonstrator in Hydraulics.*

All persons using turbine and centrifugal pumps are well aware of the serious nature of an air leakage on the suction side of the unit. Figures are, however, lacking to connect in any way the volume of air leaking in with the decrease in capacity and efficiency, and it was for the purpose of getting such information that the investigation described herein was made.

The experiments were made in the Hydraulic Laboratory of the Department of Mechanical Engineering, University of Toronto; and of the several pumps in the laboratory the most convenient one for the purpose was a 3 in., four stage, Rees pump, made and presented by Goldie and McCullough, Galt, Ontario. This pump was selected purely because of the convenience with which it could be experimented on. A longitudinal section of this pump is shown in Fig. 1 and it is direct connected to an electric motor. To the 3 in. horizontal suction branch of this pump there was attached a nipple 6½ in. long screwed into the companion flange bolted to the suction branch and also screwed into a 3 in. long radius elbow from which elbow a vertical 3 in. pipe, 5 ft. 6 in. long extended down to the well. The general arrangement of the apparatus is shown in Fig. 2.

Ordinarily this suction pipe has a foot valve for convenience in priming, but the resistance of such valves is not constant, and to prevent its effect from vitiating the points under examination, the foot valve was removed and the water, therefore, entered the open end of the 3 in. suction pipe direct, in all the experiments. The discharge of the pump was also 3 in. and carried the water to a calibrated measuring tank. A valve in the discharge pipe enabled the discharge to be controlled.

The suction and discharge pressures were taken on gauges tapped into the suction and discharge flanges, both of which were at the same level. The air admitted in the course of the experiments was let in through a single ¼ in. pipe, tapped into the side of the suction flange, and by tightening up the glands and smearing them and all suction pressure joints with vaseline it is believed that air could not enter the pump in

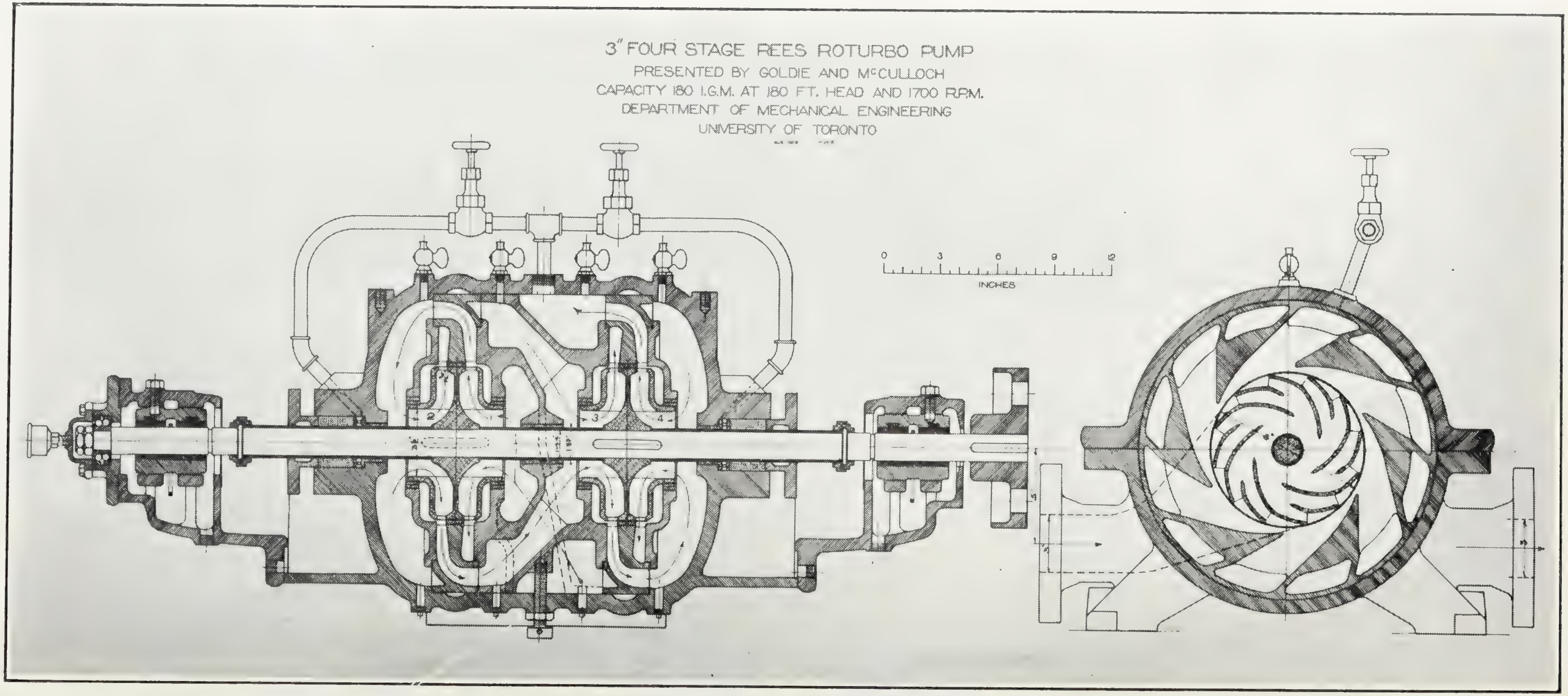

3" FOUR STAGE REES ROTURBO PUMP
PRESENTED BY GOLDIE AND McCULLOCH
CAPACITY 180 I.G.M. AT 180 FT. HEAD AND 1700 R.P.M.
DEPARTMENT OF MECHANICAL ENGINEERING
UNIVERSITY OF TORONTO
0
3
6
9
12
INCHES
2
3
4

Fig. 1

any other way than through this ¼ in. opening. The suction inlet was always so far below the water surface that direct entry of air through the bottom of the suction pipe was impossible.

A considerable problem presented itself in the regulation and measurement of the air admitted. For two months, work was conducted with a large displacement gas meter for measuring the air, and by means of a cock between the meter and suction pipe the flow of air was regulated. This had two objections, the first being the pulsations in flow caused by the movement of the meter bellows, and the second, and more serious difficulty, that the rate of flow could not be predetermined, and, therefore, it was almost impossible to admit any specified volume of air per second, and it was also impossible to keep the rate of flow constant for different experiments. After nearly two months work the method was abandoned.

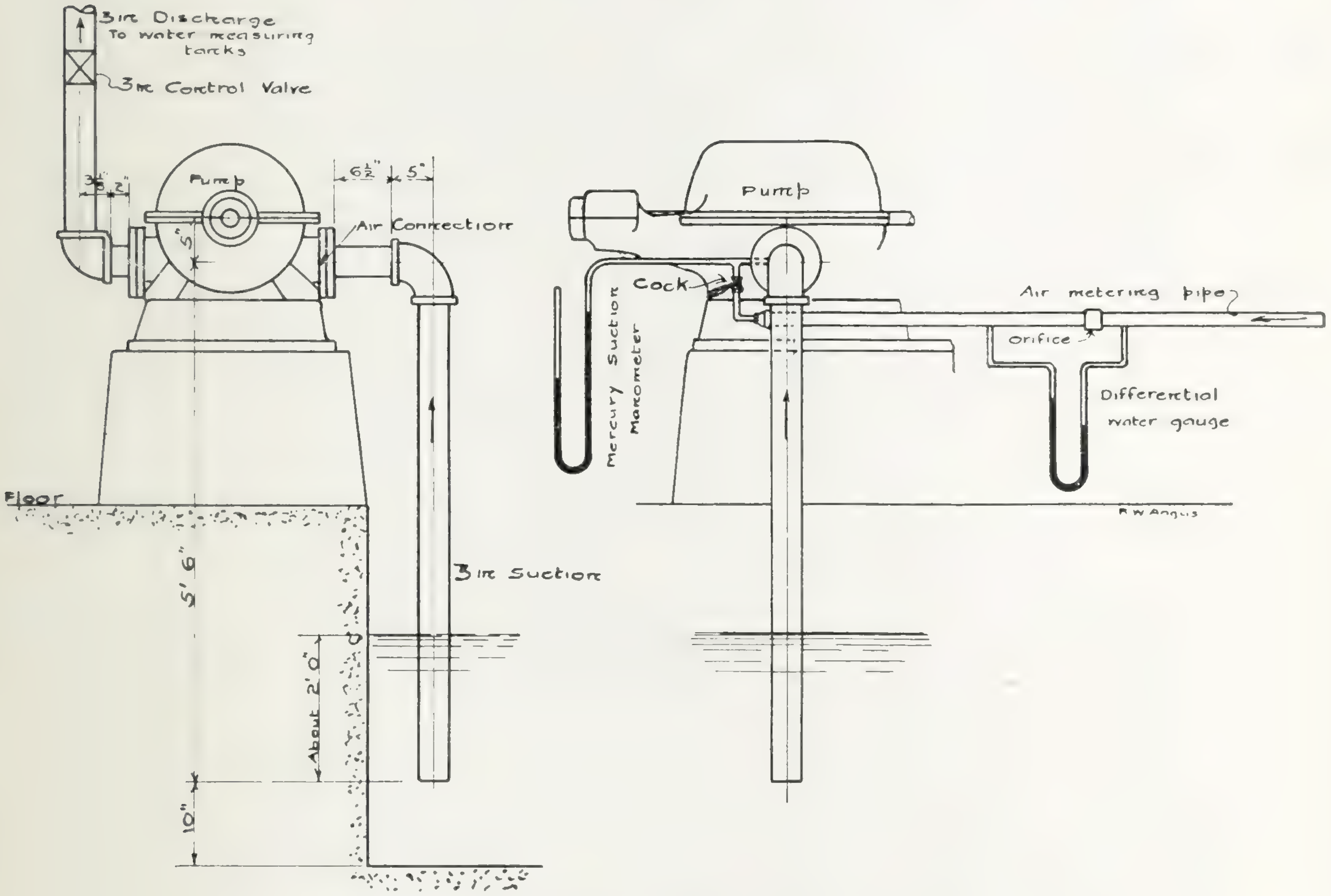


Fig. 2.—General arrangement of equipment

It was then decided to use a thin plate orifice in a pipe line and the apparatus was set up as shown in Figs. 2 and 3. The measuring tube was 1½ in. pipe about 5 ft. 4½ in. long, and the orifice was placed 2 ft. 3 in. from the place where the air entered the 1½ in. pipe, and the pipe was continued over 3 ft. below the orifice before it was reduced to ¼ in. for connection to the pump suction. The differential gauge across

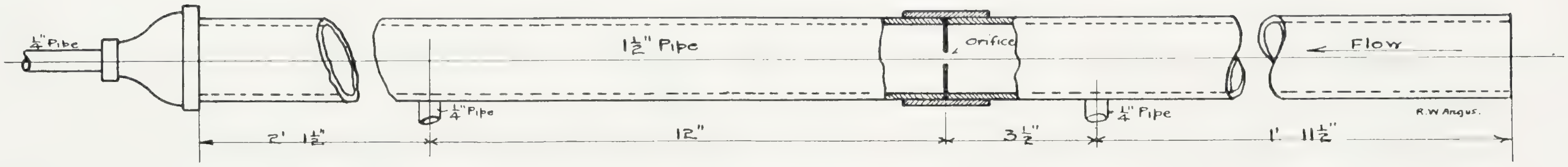


Fig. 3.—Air measuring orifice

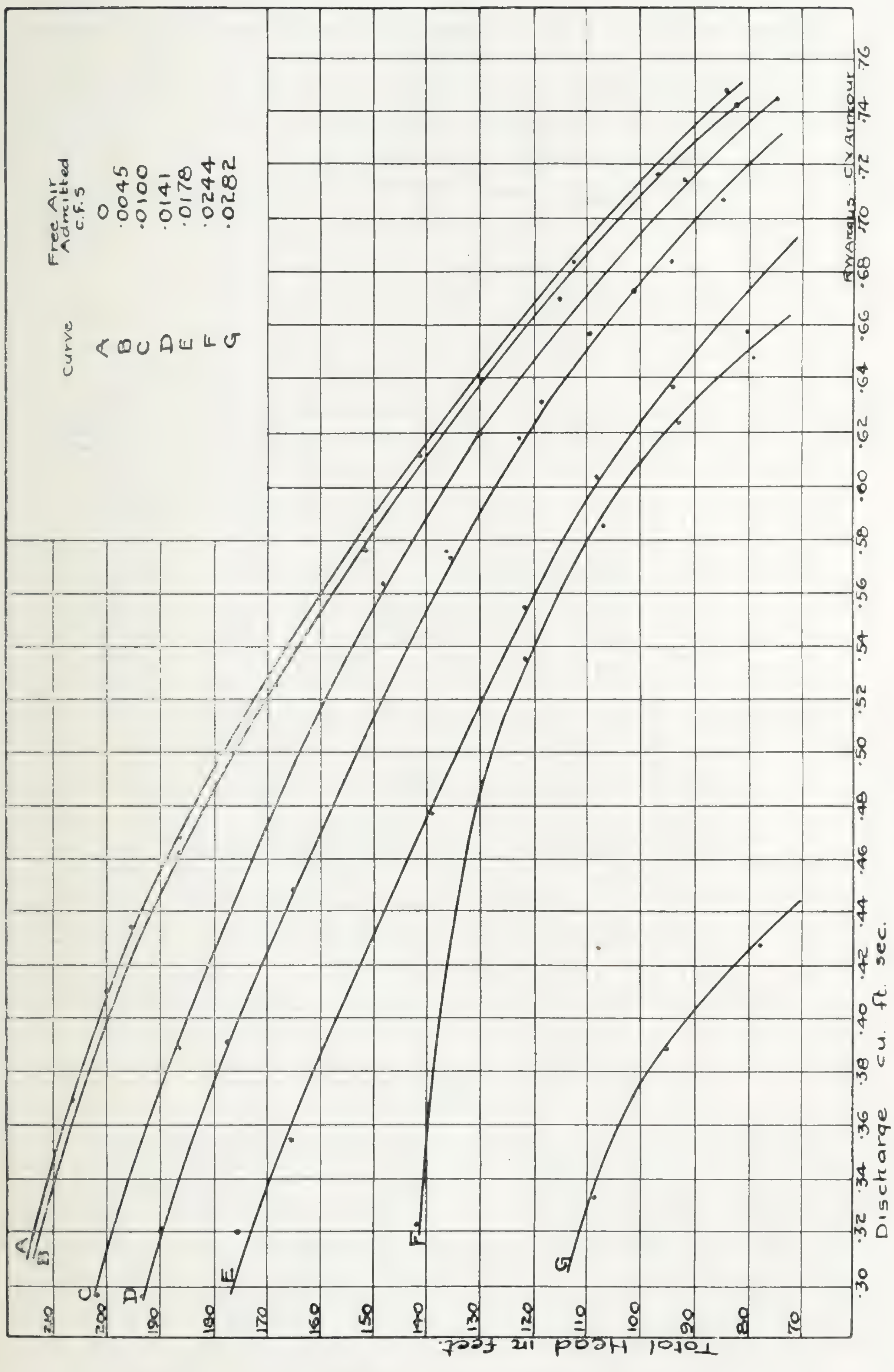


Fig. 4.—Head Discharge Curves

the orifice was connected to two ¼ in. pressure taps, the one 3½ in. above the orifice and the other 12 in. below the orifice, care being taken that the pressure taps just came flush with the inside of the pipe. Only two sizes of orifice were used, *viz.*, ¼ in. and ½ in., the differential being controlled by the small cock placed between the 1½ in. pipe and the point of connection to the suction pipe of the pump.

The volume of air passing through the orifice was calculated from the formula ordinarily used with such a device

$$Q = A_2 v_2 = \frac{A_2 c \sqrt{2gH}}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}}$$

where Q is the discharge in cu. ft. sec., A_1 and A_2 are the areas, in sq. ft., of the pipe and of the orifice, respectively, H is the differential head in feet of air column and c is a coefficient. No allowance was made for the expansion of the air after passing the orifice and the approximate value of the coefficient c was taken as 0.62 throughout. This introduced some error, but it was very slight and the method of conducting the experiments was such as to minimize it.

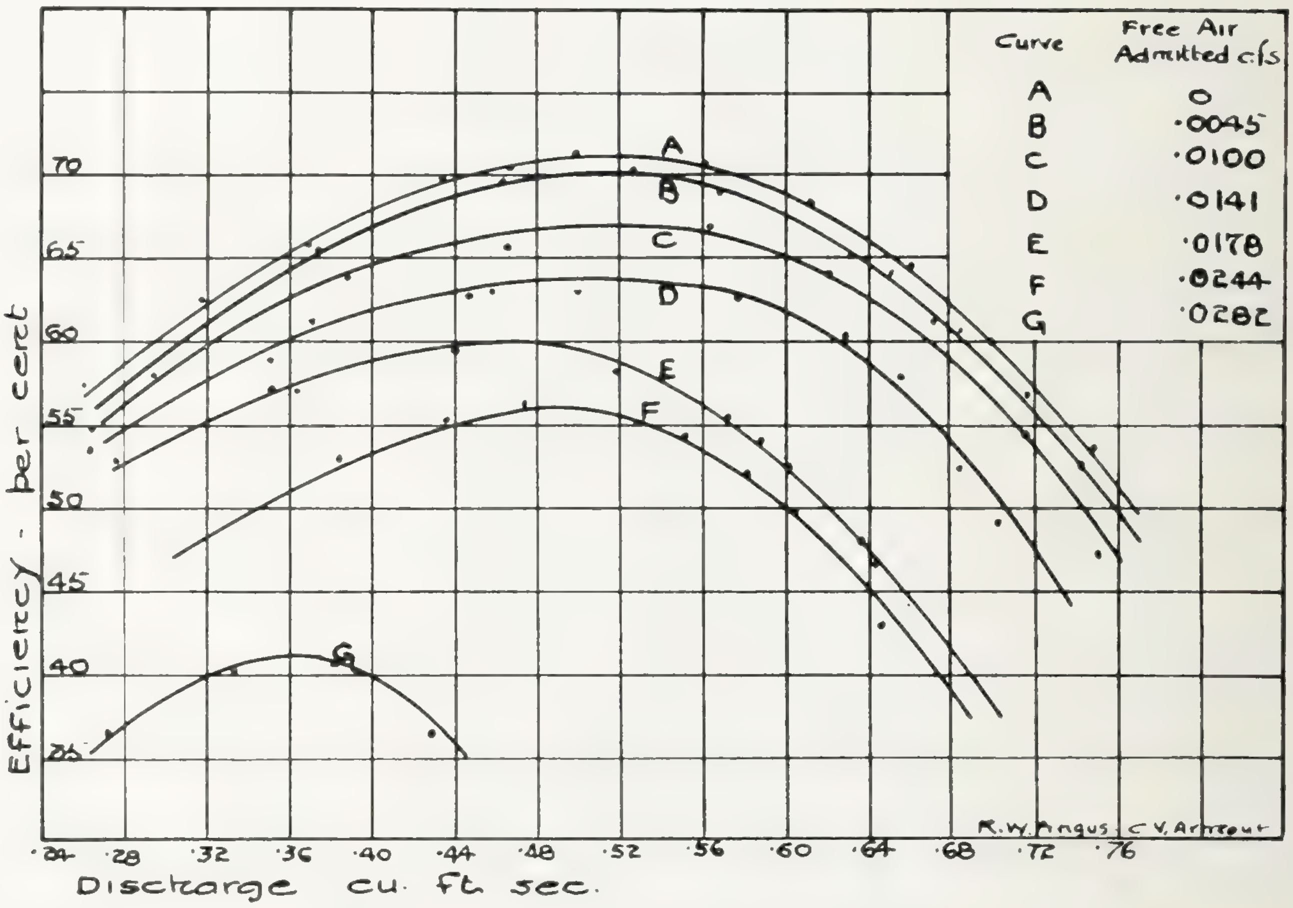


Fig. 5.—Efficiency Discharge Curves

An examination of the curves in Fig. 4 shows the method of procedure. The speed was kept constant at 1700 r.p.m. and the first test was run with no air admitted to the suction and the glands and all joints closed

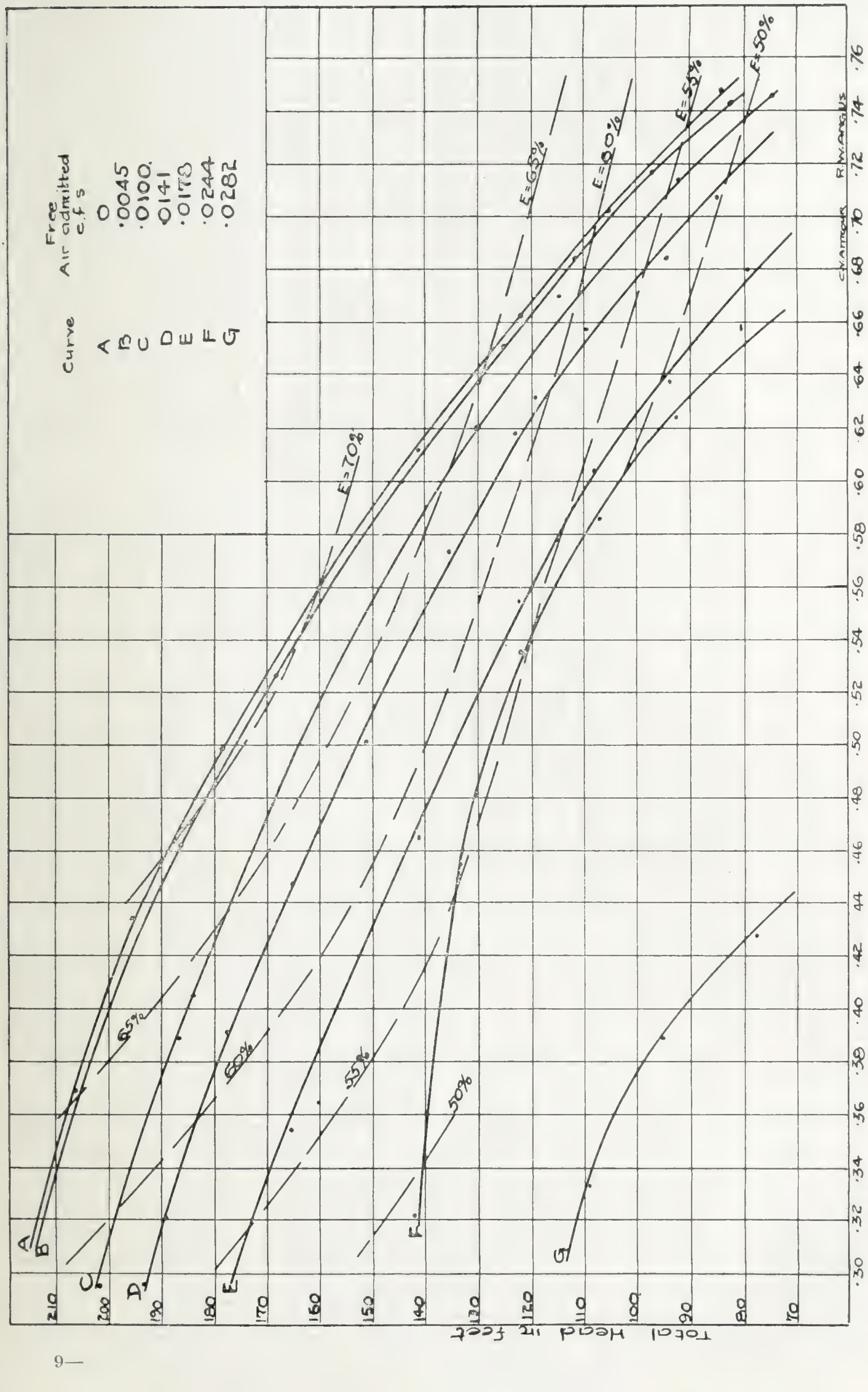


Fig. 6.—Combined Efficiency and Head Discharge Curves

with vaseline as already described, which gave the curve *A*, Figs. 4 and 5. The next curve B was then made by fixing on a definite differential on the orifice and adjusting the cock between it and the suction pipe to give the same differential at each discharge. The other curves *C*, *D*, *E*, *F*, *G* were obtained similarly. Throughout the work measurements were taken to determine the efficiency, and these efficiencies are plotted on a discharge base on Fig. 5, while the head discharge curves are shown on Fig. 4. For convenience in comparison the curves in Fig. 4 and 5 are combined to give Fig. 6.

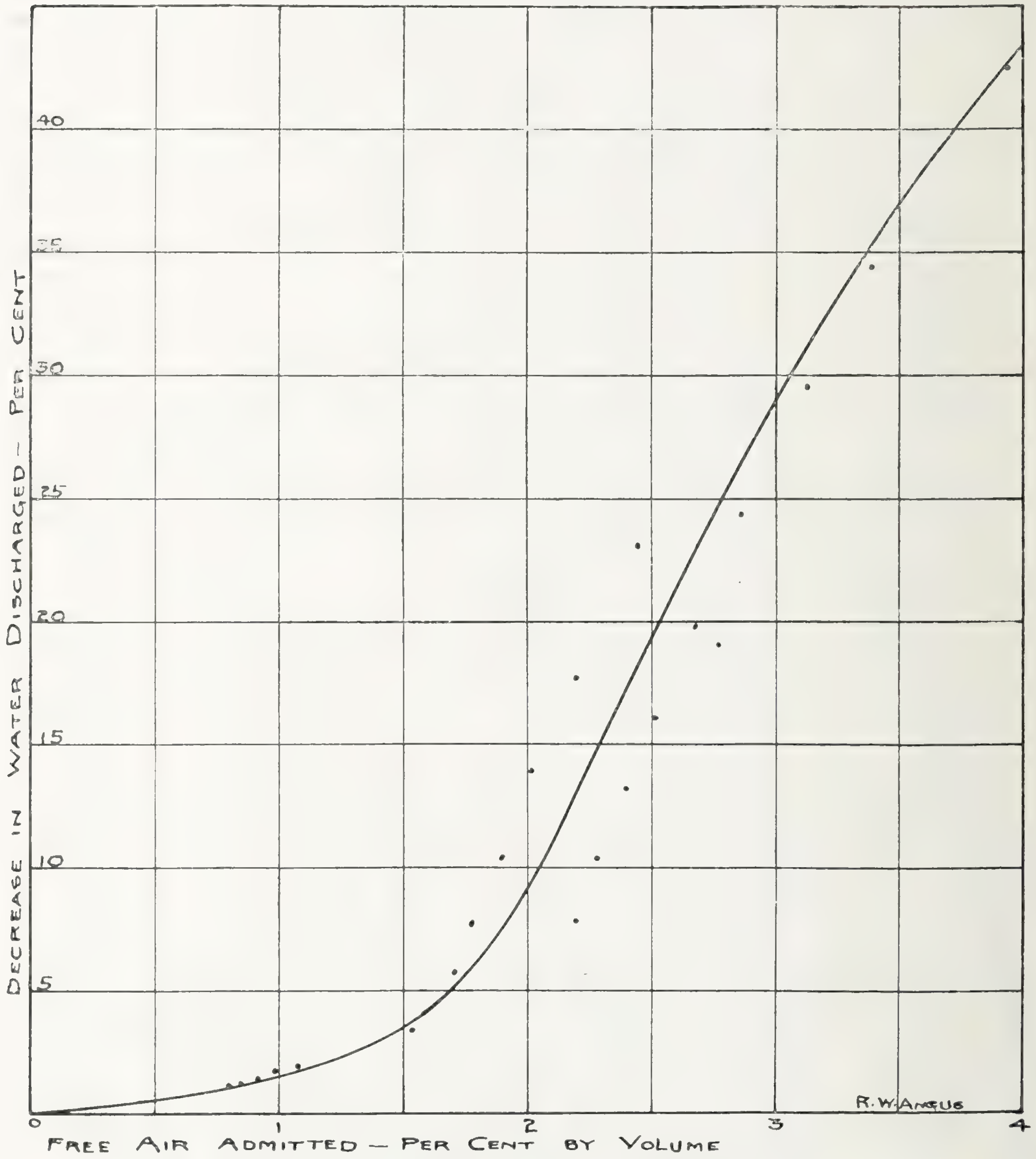


Fig. 7

TABLE I

Effect of Air in the Suction Pipe of a Centrifugal Pump

Suction lift ft.	Total head ft.	Discharge no air admitted c.f.s.	Free air admitted c.f.s.	Water discharged air admitted c.f.s.	Free air admitted %	Loss of discharge %
5.65	200	.408	.0100	.314	2.45	23.02
6.50	200	.408	.0045	.400	1.09	1.96
6.20	190	.454	.0141	.320	3.12	29.50
6.40	190	.454	.0100	.374	2.20	17.60
7.15	190	.454	.0045	.446	0.99	1.76
6.20	180	.493	.0141	.373	2.87	24.35
7.10	180	.493	.0100	.424	2.02	14.00
7.75	180	.493	.0045	.486	0.91	1.42
5.05	170	.527	.0178	.334	3.39	36.65
6.90	170	.527	.0141	.422	2.68	19.93
7.80	170	.527	.0100	.472	1.90	10.45
8.40	170	.527	.0045	.520	0.85	1.33
5.85	160	.560	.0178	.383	3.18	31.60
7.55	160	.560	.0141	.470	2.52	16.08
8.45	160	.560	.0100	.516	1.78	7.86
9.05	160	.560	.0045	.554	0.80	1.07
6.70	150	.589	.0178	.430	3.03	26.95
8.25	150	.589	.0141	.512	2.40	13.08
9.10	150	.589	.0100	.555	1.70	5.77
9.60	150	.589	.0045	.584	0.76	0.85
6.75	140	.616	.0244	.352	3.96	42.80
7.40	140	.616	.0178	.477	2.89	22.58
8.95	140	.616	.0141	.553	2.29	10.22
9.70	140	.616	.0100	.590	1.62	4.25
10.18	140	.616	.0045	.611	0.72	0.81
7.50	130	.642	.0244	.483	3.81	24.78
8.05	130	.642	.0178	.520	2.78	19.00
9.60	130	.642	.0141	.590	2.20	7.95
10.30	130	.642	.0100	.620	1.55	3.43
10.75	130	.642	.0045	.638	0.70	0.62

The two sets of curves show in a very marked way how objectionable air is in the suction pipe. For each curve many more points were obtained than are shown on Figs. 4 and 5. For greater convenience in comparing the effects of the air, Table 1 has been constructed, where for eight heads the results have been taken from the curves. This table is self explanatory, but it is to be noted that the air admitted is computed under atmospheric conditions, whereas the volume entering the suction pipe is greater owing to the suction lift. Full data are given for those desiring to determine the results under this basis.

In the table all percentages are given in terms of the volume of water discharged at the given pressure when no air is admitted to the suction pipe. Apparently there is no relation whatever between the percentage of air admitted and the percentage reduction in discharge, but for the larger volumes of air the percentage decrease in discharge was approximately ten times the percentage of air present, and this ratio falls to about unity with the smaller percentages of air. The percentage of loss in discharge has been plotted against percentage of free air admitted in Fig. 7 and the curve is a fairly definite one although possibly it might be fairer to draw two limiting curves.

The curve in Fig. 7 shows in a very striking way the effect of the air. Up to about 1.5% the loss in the pump discharge is very slight but beyond that the curve rises very steeply almost in a straight line, and for 4% air leakage the discharge loss is over 40%. In practice the leakage of air occurs largely at the glands and flanges and is directly dependent on the suction lift; if the pump is submerged the lift is zero and there is no air leakage. As the pump ls raised above the suction well the suction lift is increased, more particularly where the velocity in the suction pipe is high, and the air leakage greatly augmented, resulting in the serious losses indicated in Fig. 7. For example, if the suction velocity is 8 ft. sec. then a pump placed at the suction level will have a net pressure of one foot of water causing air leakage, but if it is raised 8 ft. above the suction well the net head causing air leakage will be 9 ft. and the volume of free air leakage would be three times as much as in the former case.

Centrifugal pumps of other types may give somewhat different results, but the experiments bring the information forward in a marked way.

ORIFICE STEAM METER COEFFICIENTS

By Robert W. Angus, *Professor of Mechanical Engineering*

The value of the thin plate orifice as a fluid meter has been established beyond all question, and many very successful commercial applications of it are on the market. There are, however, not many tables of actual orifice coefficients, these being apparently retained by the manufacturers. Recently, the author was conducting some tests on a steam turbine of about 100 H.P., which was not equipped with surface condenser, and as it was connected to the main header from a large battery of boilers, the feed water method of measuring the steam consumption was not available. An orifice meter was then decided on, and after making the turbine tests, the orifice was set up in the steam laboratory of the Department of Mechanical Engineering, of the University of Toronto, and calibrated.

As far as the test was concerned, the coefficients under the test conditions were all that were necessary, but it was decided to enlarge the scope of the work, and to determine the coefficients under a wide range of conditions, and this paper is the result of such work.

The orifice used in the test had a diameter of 1.838 in. and thickness of .015 in. It was set concentric with a 4 in. extra heavy pipe having an internal diameter of 3.823 in. The set up in the laboratory was, therefore, based on these dimensions and a drawing of the experimental equipment is shown in Fig. 1. The 6 in. main steam pipe from the experimental boilers is shown at S and, by means of a 6 in. × $1\frac{1}{2}$ in. tee, connection was made to the experimental piping shown on Fig. 1. The 4 in. extra heavy pipe started 1 ft. $10\frac{3}{4}$ in. from the centre of the 6 in. main, and the 4 in. section had a total length of 7 ft. 10-3/16 in., with a pair of companion flanges in the centre for insertion of the experimental measuring orifice. The pipe was again reduced to $1\frac{1}{2}$ in., which in turn delivered steam to a small steam turbine in the laboratory.

The entire piping was lagged with $1\frac{1}{4}$ in. of 85% magnesia, and 3/8 in. drains were placed at the two points shown, to prevent the collection of water. A throttling calorimeter was connected at the point indicated, and the orifice plates were inserted between the two companion flanges, and very carefully centred. Only one pressure tap was used on the upstream side for all experiments, this being placed $6\frac{3}{4}$ in., or 1.76 times the internal pipe diameter above the orifice. Four taps were used on the down-stream side, being placed in the positions shown in Fig. 1, and all of the pressure taps were made of 3/8 in. pipe, screwed into the side of the 4 in. pipe so that they just came flush with the inside of the latter.

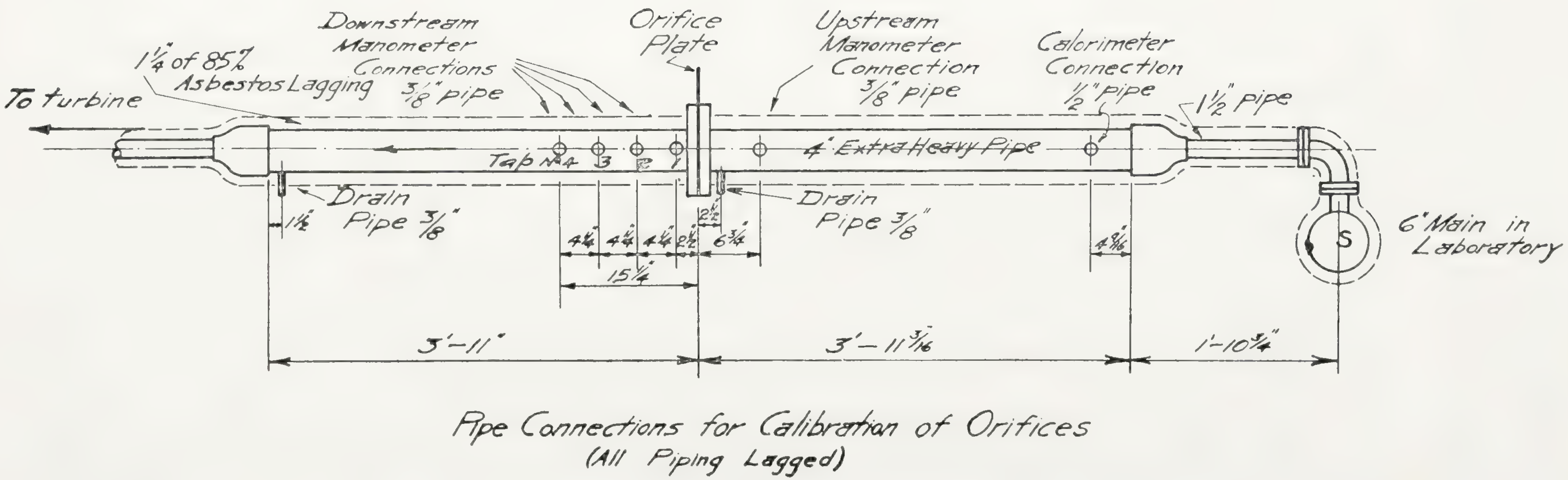

To turbine
1¼" of 85% Asbestos Lagging
Downstream Manometer Connections 3/8" pipe
Orifice Plate
Upstream Manometer Connection 3/8" pipe
Calorimeter Connection 1/2" pipe
1½" pipe
4" Extra Heavy Pipe
Tap No 4
3
2
1
Drain Pipe 3/8"
Drain Pipe 3/8"
1½
4¼
4¼
4¼
2½
2½
6¾
15¼
4 9/16
6" Main in Laboratory
S
3'–11"
3'–11 3/16
1'–10¾"
Pipe Connections for Calibration of Orifices
(All Piping Lagged)

Fig. 1.—Arrangement of Piping used in obtaining Orifice Coefficients

The differential pressures across the orifices were measured by a differential mercury manometer, reading to 0.05 in. mercury, and very great care was taken in every case to see that the connecting pipes were free of air and contained water up to the point of connection with the 4 in. pipe. Frequent readings were taken on this manometer for each test.

The orifice plates were made of galvanized iron .016 in. thick, the orifice being very carefully bored out in the lathe and afterwards calipered, when hot, to thousandths of an inch. Each plate contained a 9/32 in. hole, as well as the orifice, the latter hole being placed so that its lower side was flush with the bottom of the pipe. Three orifice plates were used, with orifice *A* of diameter 2.66 in., *B* of 1.838 in. and *C* of 1.33 in. respectively, as indicated in Fig. 2.

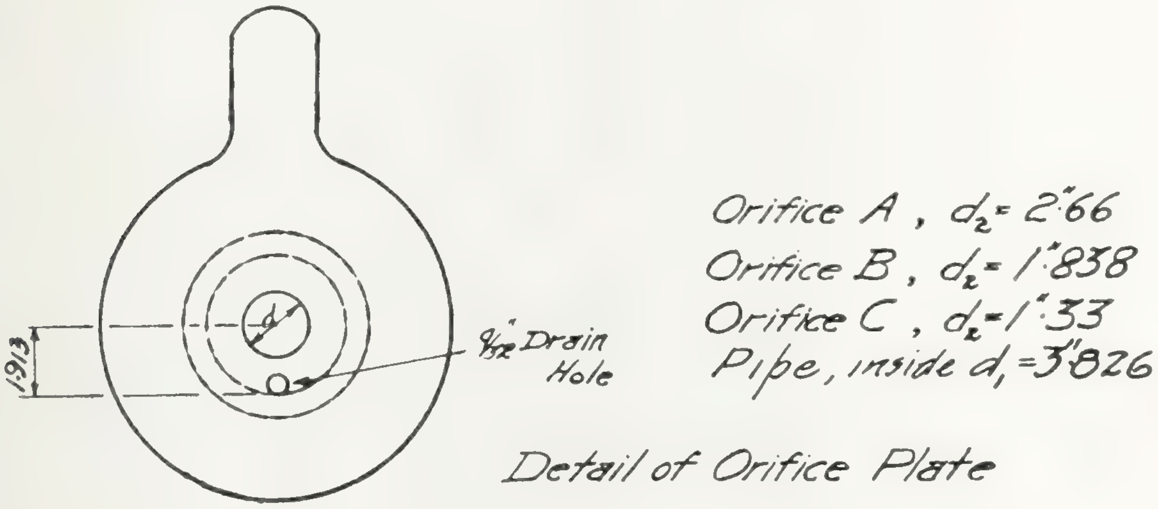


Fig. 2.—Orifice Plate

A large number of experiments was made with the four down-stream taps and the three orifices, each experiment lasting for from thirty to forty minutes. In all cases the steam pressure at the orifice was kept as close to 125 pds. per sq. in. gauge, as possible, and was very steady during any given test. The moisture of the steam entering the 4 in. pipe was also maintained very uniform by keeping the drip on the 6 in. main partly open, and the quality remained uniformly constant at 98.5% and this quality was used in all calculations. Before each test the 3/8 in. drains on the bottom of the 4 in. pipe were opened, and after they were closed the brake load on the turbine at the end of the 1½ in. pipe was adjusted, and this load was kept constant, within a fraction of a per cent., during a test, thus producing a constant differential across the orifice. The steam passing through the orifice and turbine was then condensed and weighed, the average of at least eight weighings being taken in each case, and no test was used where the several weighings showed any material variation.

ORIFICE COEFFICIENTS FOR STEAM METER

1 Test No.	2 Steam discharged lbs./sec.	3 Absolute steam pressure pds.sq.in.	4 Volume of steam per lb. cu. ft.	5 Orifice area sq. ft.	6 Velocity through orifice V ft./sec.	7 Differential in. mercury	8 Head H ft.	9 Constant c
No. 1 Downstream pressure tap.								
1	.1454	139.6	3.187	.0100	46.3	.49	102.3	.567
2	.2106	142.8	3.117	.0100	65.6	1.03	210.4	.560
3	.2996	141.3	3.149	.0100	94.3	2.06	425.1	.566
4	.3545	141.0	3.156	.0100	111.9	2.80	579.0	.575
5	.4147	139.6	3.186	.0100	132.1	3.85	803.5	.576
6	.4615	139.0	3.199	.0100	147.6	4.80	1006.0	.576
7	.4186	138.8	3.203	.0188	71.3	1.08	227.5	.573
8	.5135	139.8	3.182	.0188	86.9	1.57	327.3	.582
9	.5756	141.3	3.133	.0188	95.9	1.96	403.3	.579
10	.601	139.3	3.192	.0188	102.0	2.12	443.4	.587
11	.6571	141.3	3.133	.0188	109.5	2.58	529.2	.577
12	.673	138.3	3.214	.0188	115.0	2.55	538.0	.601
No. 2 Downstream pressure tap.								
13	.1417	143.0	3.121	.0100	44.2	.48	98.2	.552
14	.2237	142.9	3.115	.0100	69.7	1.06	215.9	.587
15	.3017	140.9	3.158	.0100	95.3	1.95	403.8	.586
16	.4037	140.4	3.169	.0100	127.9	3.05	633.2	.629
17	.4323	140.5	3.167	.0100	136.9	3.69	765.6	.612
18	.5039	140.3	3.185	.0100	160.5	4.95	1033.0	.618
19	.2798	140.8	3.160	.0188	47.0	.39	80.7	.635
20	.4258	140.2	3.174	.0188	71.9	.93	193.4	.626
21	.5313	140.2	3.174	.0188	89.7	1.45	301.6	.626
22	.618	138.8	3.203	.0188	105.3	2.0	419.7	.623
23	.4343	141.0	3.156	.0390	35.1	.13	26.9	.738
24	.6307	138.9	3.197	.0390	51.7	.28	58.7	.735
No. 3 Downstream pressure tap.								
25	.1552	142.3	3.128	.0100	48.6	.50	102.5	.593
26	.2313	140.3	3.171	.0100	73.3	1.03	214.0	.620
27	.3328	140.2	3.174	.0100	105.6	2.08	432.6	.628
28	.4122	139.6	3.186	.0100	131.3	3.23	674.2	.626
29	.4652	136.8	3.248	.0100	151.1	4.21	895.8	.624
30	.4968	136.9	3.246	.0100	161.2	4.75	1010.0	.627
31	.3213	140.3	3.171	.0188	54.2	.49	101.8	.651
32	.4620	138.6	3.208	.0188	78.8	.99	208.1	.662
33	.5633	139.6	3.186	.0188	95.5	1.45	302.6	.665
34	.6543	139.0	3.199	.0188	111.3	1.95	408.7	.667
35	.4757	137.6	3.230	.0390	39.4	.15	31.7	.761
No. 4 Downstream pressure tap.								
36	.3122	141.5	3.145	.0188	52.2	.47	96.8	.643
37	.4610	140.2	3.174	.0188	77.8	.93	193.4	.678
38	.5400	137.4	3.235	.0188	92.9	1.33	282.7	.668

The maximum weight of steam that could be put through the equipment was 2423 lbs. per hour, and the discharge varied from this amount down to a minimum of 510 lbs. per hour, and these ranges corresponded to differentials of from 0.13 in. to nearly 5 in. of mercury and to orifice velocities of from 32.8 ft. per sec. to 161.2 ft. per sec. Unfortunately the steam available did not permit the use of these velocity ranges with each orifice.

The table attached hereto gives a summary of the results of the tests, which were made in February, March and April, 1928. The calculations are based on Goodenough's Steam Tables, and the formula employed in getting the coefficient c was

$$V = \frac{c\sqrt{2gH}}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}}$$

where V denotes the velocity through the orifice in feet per second, A_1 and A_2 are the areas, in square feet, of the inside of the 4 in. pipe and of the orifice, respectively, the area A_2 including that of the small hole in the plate. H is the head, in feet of steam, corresponding to the differential mercury gauge reading, and c is the coefficient sought. The velocity V was found by taking the weight of steam discharged per second and multiplying by the volume of the actual steam per pound, and then dividing by the orifice area, while the head H was given by the formula

$$H = \text{in. mercury} \times \left(\frac{13.6-1}{12}\right) \times 62.4 \times \text{vol. of 1 lb. steam.}$$

For the lower differentials the expansion of the steam through the orifice may be neglected, and the above formula treats the fluid as incompressible. The higher differentials, however, were nearly 5 in. of mercury, which was slightly under two per cent. of the steam pressure, and in this case the expansion of the steam would have a slight effect. In the table, column 9 gives the value of the coefficient without making any allowance for expansion, as the latter should not affect the coefficients over about one-half per cent. in the worst cases, and the effect would be negligible in the greater part of the work.

The results of the tests are plotted on the curve sheets, Figs. 3 to 7 inclusive. On Fig. 3 the base is velocity V through the orifice, in feet per second, and three sets of curves have been plotted, one for each orifice. Taking the upper set of three curves these all belong to the smallest orifice, of diameter 1.33 in., and the three curves show how the coefficient varies with the position of the down stream tap, the numbers referring to the position of the tap as indicated on Fig. 1. Unfortunately, it was not possible in the time available to get a curve for the fourth tap, but the middle group of curves for the 1.838 in. orifice gives the complete results for the four taps. The lower group, for the 2.66 in. orifice was very limited owing to the steam capacity available.

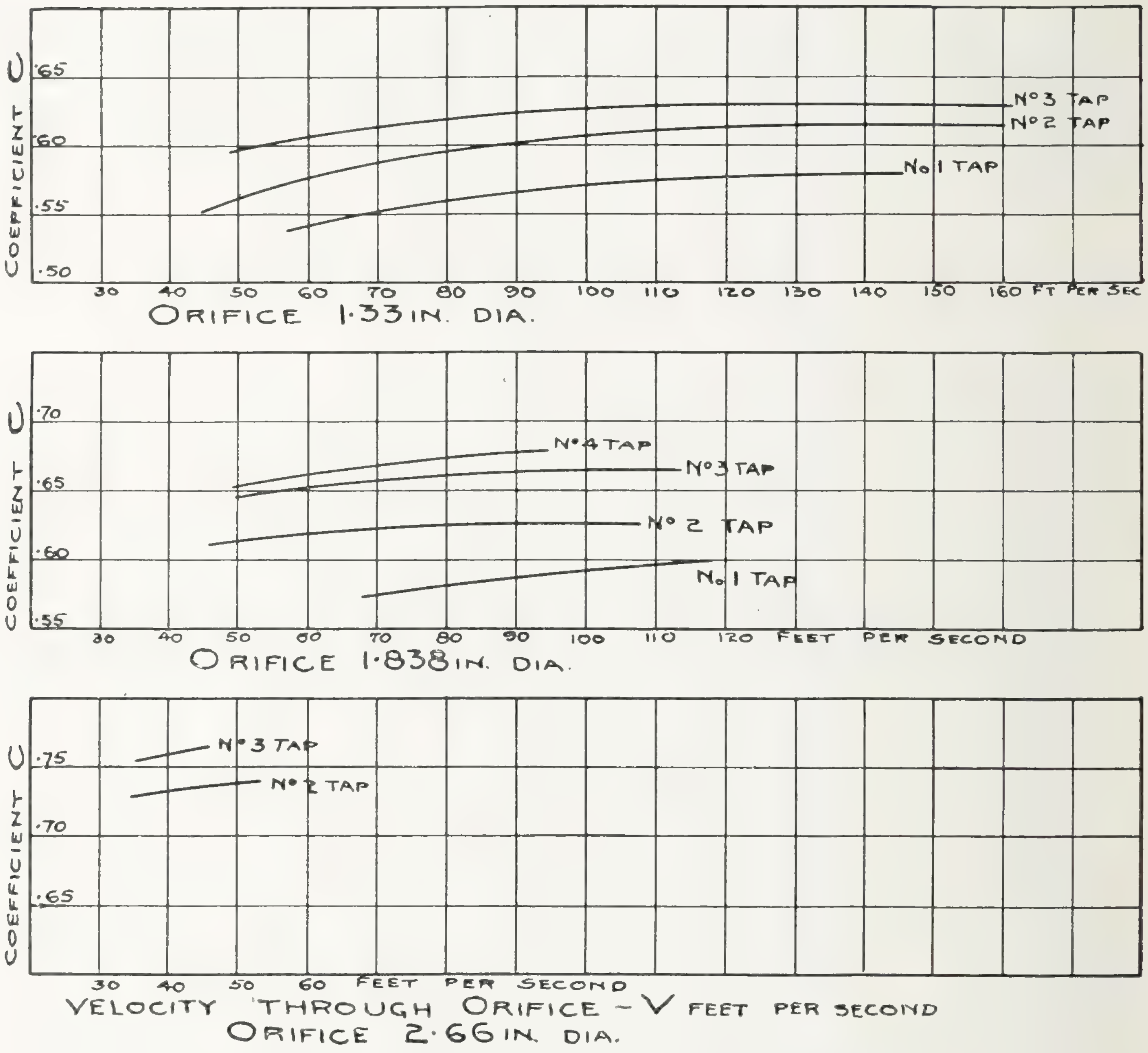


Fig. 3.—Coefficients for the Orifices separately

The curves in Fig. 3 are very useful in showing how the position of the down-stream tap affects the coefficients for a given orifice. All of these curves slope upward at first, indicating that the coefficient for a given orifice, and a fixed pair of pressure taps, varies with the velocity through the orifice, that is, with the differential manometer reading. The curves clearly flatten off and tend to become horizontal beyond a certain velocity, for example, above 110 to 120 ft. per sec. velocity the coefficients for the 1.33 in. orifice are constant.

The curves for the 1.838 in. orifice are not as evenly spaced as one would desire and there may be some errors, even with the extreme care used in the work.

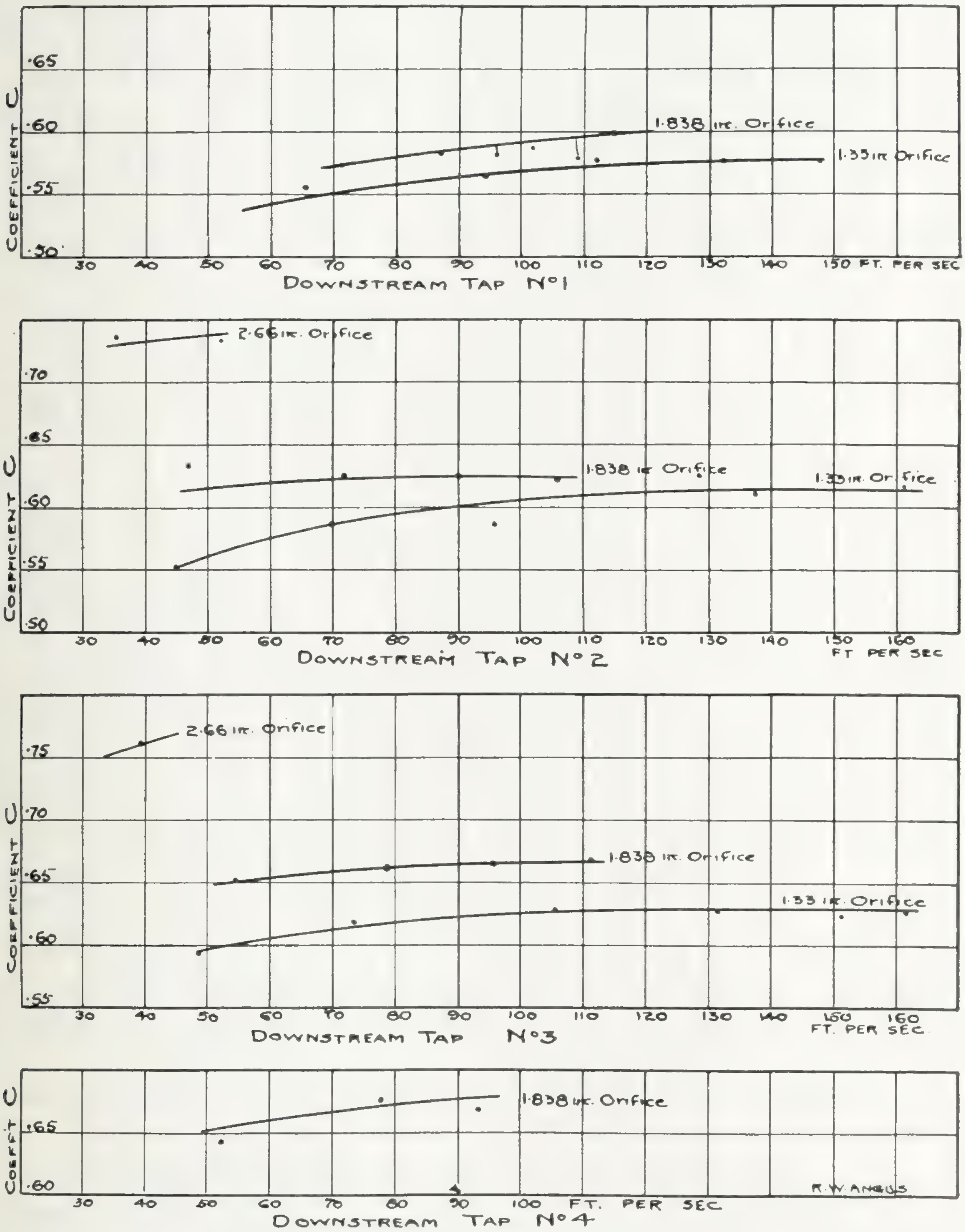


Fig. 4.—Coefficients for the Pressure Taps separately

The curves on Fig. 4 are copies of those on Fig. 3 but grouped so as to show the effect of the size of the orifice on the coefficients, for each given pair of taps. In this case the experimental points used in plotting the curves are shown. The upper group of curves belong to No. 1 down-stream tap and show a marked change in the coefficient with the size of the orifice, for a given velocity in the orifice; and the same thing is true of each of the other three groups, which were taken with the second, third and fourth down-stream taps respectively.

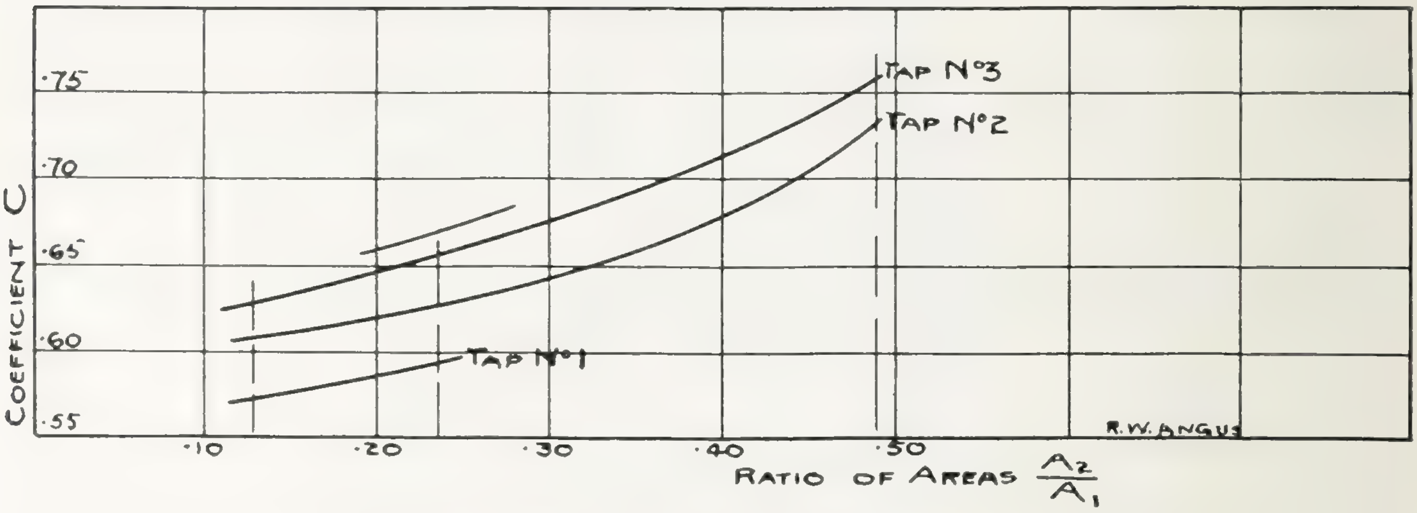


Fig. 5.—Variation of coefficient with the ratio of areas

The curves in Fig. 5 show the relation between the coefficient *c* and the ratio A_2/A_1 of the area of the orifice to the area of the pipe, these coefficients varying, as in the other curves, with the position of the down-stream pressure tap.

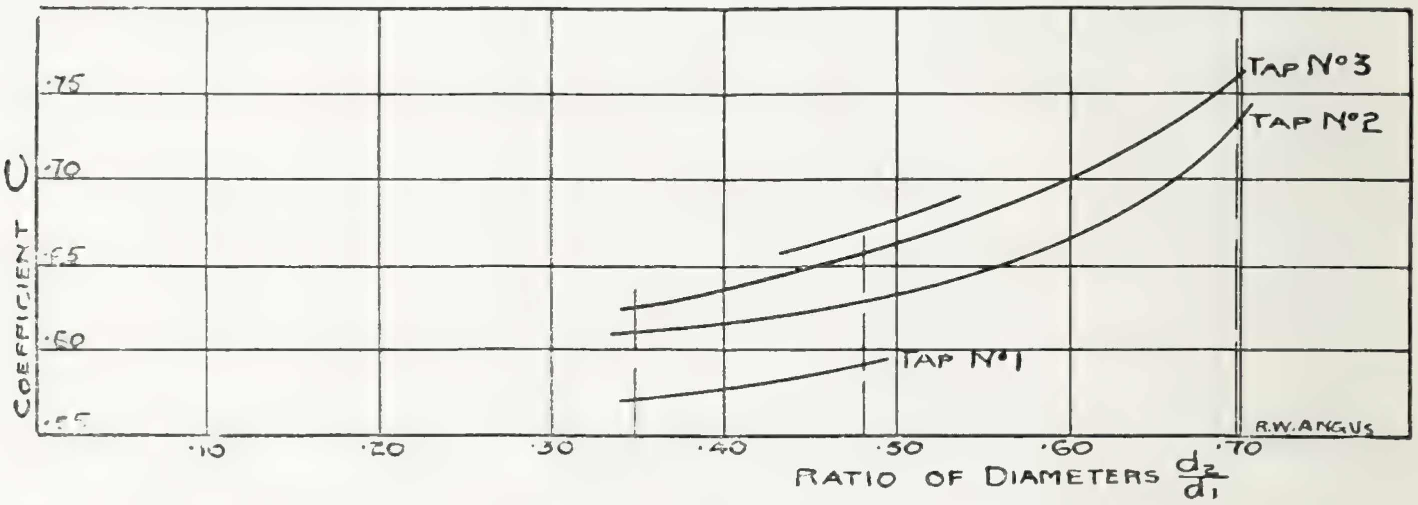


Fig. 6.—Variation of coefficient with the ratio of diameters

The curves in Fig. 6 give the same information but referred to a base showing the ratio d_2/d_1 of the diameters of the orifice and of the pipe respectively. It should be noted that the ratio A_2/A_1 takes account of the small 9/32 in. hole in the orifice plate, whereas the ratio d_2/d_1 does not. In plotting these curves approximate average values of the coefficients were used as obtained from Figs. 3 and 4.

The final curves in Fig. 7 give the coefficients for the three orifices in relation to the position of the down-stream pressure tap. As in the case of Figs. 5 and 6, the shape and position of the curves in Fig. 7 will vary with the adopted coefficient, as obtained from Figs. 3 and 4. The curves shown are based on approximate average values for each of the curves, although it might have been better to have used the coefficients corresponding to the horizontal parts of the curves.

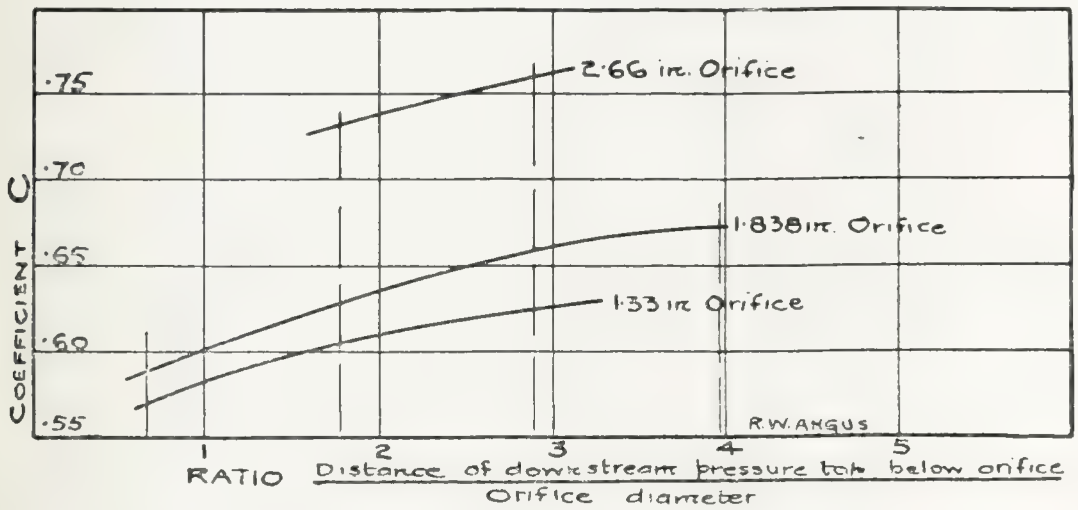


Fig. 7.—Variation of coefficient with the location of the down-stream tap

A fairly complete discussion of the orifice meter is found in "Fluid Meters, their Theory and Application", Part I, Report of the American Society of Mechanical Engineers Special Research Committee on Fluid Meters. This report was published by the A.S.M.E. in 1927 in a revised edition and gives a complete bibliography on Fluid Meters, so that references are not necessary in the present paper.

The author's coefficients differ from those shown in this report, as none of the experiments discussed there take account of the variation of the coefficient with the orifice velocity and the author's experiments apparently establish such a variation. The author's up-stream tap was 1.76 pipe diameters above the orifice, whereas the curves of Fig. 14, in the report referred to, were obtained with the tap one pipe diameter up-stream, but tap No. 1 used by the author is very close to the vena contracta and hence the author's coefficients should agree closely with those in the report.

On Fig. 14, the report gives a value $c = .608$ for a ratio of diameters $d_2/d_1 = .48$, a ratio corresponding to the 1.838 in. orifice used by the author with tap No. 1. For this case the author's values varied from .57 to .60 although the curve suggests that the value would be about .61 at the higher orifice velocities. The author's smaller orifice had a ratio $d_2/d_1 = .348$ and with tap No. 1 the coefficients vary from .54 to .575 the latter value corresponding to the horizontal part of the curve. For this case Fig. 14 shows a value of about .602, although for this case the author's pressure tap was probably not at the vena contracta.

The experimental work described herein was carried out in the Steam Laboratory, of the Department of Mechanical Engineering, University of Toronto, Canada, and the author acknowledges the work of Messrs. DeLaplante and Saunders, Demonstrators in the Laboratory who did the actual experimental work.

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AIRFLOW ABOUT WING TIPS

By J. H. Parkin, M.E., F.R.Ae.S., G. J. Klein, B.A.Sc., and
G. L. Roberts, B.A.Sc.

Summary

In connection with the subject of lateral control at and above the stall, it was desired to determine the direction of the airflow, both lateral and in the vertical plane, in the neighbourhood of the tips of aircraft wings.

The airflow in the region above and below the wing tip region of aerofoils of three typical types, namely, thin, medium and thick sections, was investigated by means of a yawmeter, at the three principal angles of incidence, no lift, maximum L/D and maximum lift.

The results are given in both tabular and graphical form.

The observed influence of the aerofoil on the air flow at the wing tips is generally the same for the aerofoils studied.

At all the incidences tested, the changes in the direction of the air flow, both lateral and vertical, is most pronounced in the plane of the wing tips and at or in the rear of the trailing edge.

The lateral flow is outward below and inward above the aerofoil, is most marked in the plane of the tip, increases with the incidence of the aerofoil, and appears to be not greatly influenced by the aerofoil thickness.

Deflections of the air flow in vertical planes increase with the incidence (lift), but for any particular condition, as no lift or maximum lift/drag, appear to be virtually independent of the aerofoil section. At and beyond the tip there is an upwash, which is most marked in the plane of the tip and in the rear of the trailing edge. Above and below the aerofoil the flow follows fairly closely the profile.

The observed airflow directions indicate the presence of tip vortices.

Introduction

The control of aircraft at low speeds has, in recent years, been recognized as of vital importance, and as a consequence this subject at the present time is receiving a great deal of attention. Efforts are being directed principally toward the improvement of lateral control at and above the stall. In this connection an inquiry was received in 1927 regarding the direction of the airflow at the wing tips at the stalling point. As there appeared to be no experimental information on this subject, the present brief investigation was made.

SCOPE OF RESEARCH

That the information obtained might be of a general nature, three types of wing section were studied, namely, the RAF-15, a thin section; the USA-27, a section of medium thickness; and the Gottingen-387, a thick, high-lift section.

Observations of the direction of the airflow, both in the horizontal plane (parallel to plane of span and chord), and in the vertical plane (normal to the span), were made at a number of points distributed over the whole of the region at the wing tips, both immediately above and below the aerofoil.

In addition to the measurements at the stalling angle for each aerofoil, readings were made at the incidence corresponding to maximum lift/drag and at the angle of no lift.

MODELS AND APPARATUS

The aerofoils were machine cut on a Nichols generating machine from duralumin, and of standard size, 3″ x 18″. The wing tips were square. The models were mounted in the chuck of an NPL balance on the usual end spindle, so that the protractor of the balance could be used for setting the incidence of the aerofoils.

The direction of the airflow was determined by means of the direction and velocity meter, or "yawmeter," used in a previous research (A.R.P. No. 10—Downwash). The yawmeter is essentially a five-point Pitot tube, four of the openings, arranged in pairs, being used for determining the direction of flow in two planes at right angles, and the fifth opening being used for determining the magnitude of the velocity. The instrument has been described in R. & M. 156—IV Report for 1914-1915, page 178.

In use, the yawmeter is placed at the point in the wind tunnel at which the measurements are to be made, and, with the aerofoil removed, is adjusted in the planes of its two pairs of openings until two manometers (one connected to each pair of openings) indicate zero. With the aerofoil in place, the angles through which the yawmeter has to be turned to retain zero indication on the manometers, are the angles through which the airflow is deflected in the two planes by the aerofoil.

In this investigation the two pairs of direction tubes of the yawmeter were connected through small bore rubber tubing to two Chattock micromanometers. The velocity opening of the yawmeter was not used.

In order that all the adjustments of the yawmeter required in the investigation might be made from outside the tunnel and with the air flowing, the mounting shown in Photographs 1 and 2 was devised. This arrangement permitted not only adjustment of the yawmeter to align it with the air stream and so determine the direction of the latter, but also adjustment to locate the meter at the point relative to the aerofoil where the measurements were to be made, from outside the tunnel. The mounting was arranged to interfere as little as possible with the airflow about the aerofoil. It should be noted that the rubber tubing, which appears in the photographs to be quite close to the aerofoil, was blown well back with the air flowing. While the photographs show the apparatus rather clearly, a detailed description is given in the Appendix to this report.

OBSERVATIONS

For each aerofoil the direction of the airflow, in the horizontal and vertical planes, was determined at some 24 points in each of two horizontal planes, A located ¼ inch (1/12 chord) above the upper surface of the aerofoil, and B located ¼ inch (1/12 chord) below the chord of the aerofoil. (See Plate 1.) The points were located in six rows, L, M, N, X, Y and Z, parallel to the span and one inch apart (1/3 chord), the first row, L, being one inch ahead of the leading edge, and the last row, Z, one inch behind the trailing edge. In each row, measurements were made at four points, 1, 2, 3 and 4, located one inch outside the wing tip, at the tip, one inch inside the tip, and 3 inches (1 chord) inside the wing tip, respectively.

The horizontal and vertical planes in which the angles were measured (and tabulated) refer to the aerofoil in normal flight attitude with span horizontal. Horizontal angles were those in planes parallel to that containing the span and chord, and vertical angles those in planes normal to the span. Vertical angles were measured with reference to the geometric chord of the aerofoil.

PRESENTATION OF RESULTS

The measured angles are tabulated in Tables 1, 2 and 3 and plotted on Plates 2-10. On the plates the directions of the airflow are shown by small arrows at the different points. At the left on each plate is shown the airflow directions in the horizontal planes in full lines for plane A above the aerofoil, and dotted lines for plane B below the aerofoil. At the right are shown the directions in the vertical planes at the four sections, 1, 2, 3 and 4.

EXAMINATION OF RESULTS

At angle of no lift.

There is little lateral flow in the horizontal plane, either above or below the aerofoil.

There is little change in direction of the airflow in vertical planes, although when the flow is over or under the aerofoil it tends to follow the profile. The changes in direction are most marked above and below the leading edge and increase from beyond the trailing edge toward the median plane of the aerofoil. There is little, if any, downwash at the rear of the aerofoil.

At angle of maximum L/D.

The lateral flow is somewhat greater than for no lift. It is most marked in the plane of the wing tip.

In vertical planes there are marked changes in direction at the leading edge, which are greater above than below. At this incidence in the plane of the tip and beyond it, there is at and in the rear of the trailing edge a slight upwash, while in planes cutting the aerofoil there is a downwash, the flow adhering to the aerofoil contour.

At angle of maximum lift.

At this incidence there is marked lateral flow, outward below and inward above the aerofoil. The lateral flow is greatest in the plane of the wing tip, and in this plane increases from the leading to the trailing edge, thereafter decreasing. The largest lateral deflection of the air stream, both above and below the aerofoil, is at the trailing edge at the wing tip.

In vertical planes there is an upwash in advance of and at the leading edge, which is more marked for thicker sections. There is also an upwash at and beyond the wing tips, which is greatest in the plane of the wing tip, and which increases more or less progressively from in front of the leading edge to the trailing edge. In planes cutting the aerofoil, the flow follows the contour and there is a definite downwash in the rear.

CONCLUSIONS

The observed deflections of the airflow due to the aerofoil are generally the same for the different aerofoil sections tested.

The flow of air is but little influenced, even at incidence of maximum lift, by the aerofoil at a distance of 1/3 chord beyond the square wing tips.

At all incidences tested the changes in direction of the airflow, both lateral and vertical, were most pronounced in plane of the wing tips and at or in the rear of the trailing edge. The lateral flow is outward below and inward above the aerofoil, is most marked in the plane of the tip, increases with the incidence of the aerofoil and appears to be not greatly influenced by the thickness of the aerofoil.

Deflection of the airflow in vertical planes increase with the incidence (lift), but for any particular conditions, as no lift or maximum lift, appear to be virtually independent of the thickness of the section. At and beyond the tip there is an upwash which is most marked in the plane of the tip and in the rear of the trailing edge. Over the aerofoil the flow follows the profile fairly closely.

The observed deflections indicate the presence of tip vortices.

APPENDIX—DESCRIPTION OF ADJUSTABLE MOUNTING OF YAWMETER

In manipulating the yawmeter in the present investigation there were two sets of adjustments necessary, one comprising two rotations about axes at right angles to align the yawmeter with the airflow direction; the other consisting of two translations, one parallel to span and the other parallel to the chord to place the yawmeter at the point at which the measurements were to be made. The latter adjustment also involved a rotation to align the motion of translation parallel to the chord for different aerofoil incidences. It was desirable, owing to the great number of observations, that as many of these adjustments as possible should be made from outside the tunnel.

The two rotations required to align the instrument with the airflow were provided as follows: The shank of the yawmeter was arranged in a bearing with axis parallel to the span and its rotation controlled by wires passing through the roof of the tunnel. The rotation was indicated by a needle on a small protractor visible from outside the tunnel, and thus determined the direction of flow in the plane normal to the span. The bearing was carried on a bell crank frame clamped to a spindle held in a chuck carried in a second bearing. The axis of the second bearing was at right angles to that of the first and its projection passed through the centre of the yawmeter head. Rotation of the chuck by means of an arm and pushrod from outside the tunnel thus rotated the yawmeter head in the plane of the aerofoil chord and span. This rotation was registered by means of a needle on a protractor visible through a window in the tunnel wall, and indicated the direction of the airflow in plane of the chord and span.

The adjustments for locating the yawmeter at the desired point were contrived in the following way:

A broad, thin, flat board was arranged close to the wall of the tunnel parallel to the aerofoil and pivotted on tubular pivots in the roof and floor of the tunnel, so that it could be set and clamped at the same incidence as the aerofoil. Short crossrails were mounted on this board so that a bridge, extending between and sliding on the rails, could, by means of a rack and pinion controlled through a shaft extending down through the upper tubular pivot, be moved parallel to the aerofoil chord. Sliding on the bridge was a small slide, carrying the chuck holding the yawmeter frame. The slide was controlled by a pushrod from outside the tunnel and enabled the yawmeter to be moved to different positions along the span of the aerofoil.

In use, the board was clamped with its plane parallel to the chord of the aerofoil. The adjustable frame of the yawmeter was then adjusted (from within the tunnel) to bring the yawmeter head ¼-inch (1/12 chord) from the aerofoil surface. With the air flowing, the bridge was traversed until the yawmeter occupied the desired position along the chord, and the slide moved to bring the yawmeter into position along the span. The meter was then rotated about the two axes until the manometers gave zero indication and the angles were read on the two protractors. Repeating the process enabled all the angles on one side of the aerofoil (upper and lower) at one incidence to be measured without stopping the airflow. The aerofoil was then reversed, by inserting the spindle in the other tip, and the process repeated for the other side of the aerofoil.

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2. Air Flow Angles for USA-27 Aerofoil.
3. Air Flow Angles for Gottingen-387 Aerofoil.

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3. Air Flow Diagrams—RAF-15—Incidence +3°. Maximum L/D.
4. Air Flow Diagrams—RAF-15—Incidence +15°. Maximum lift.
5. Air Flow Diagrams—USA-27—Incidence −5°, No lift.
6. Air Flow Diagrams—USA-27—Incidence +2°. Maximum L/D.
7. Air Flow Diagrams—USA-27—Incidence +16°. Maximum lift.
8. Air Flow Diagrams—Gottingen-387—Incidence −7.9°. No lift.
9. Air Flow Diagrams—Gottingen-387—Incidence +2°. Maximum L/D.
10. Air Flow Diagrams—Gottingen-387—Incidence +15°. Maximum lift.

LIST OF PHOTOGRAPHS

1. Front View of Apparatus.
2. Side View of Apparatus.

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AIR FLOW ABOUT WING TIPS

AEROFOIL RAF-15

	Horizontal Angle			Degrees	Vertical Angle			Degrees
	1	2	3	4	1	2	3	4
Incidence −1.3°. No lift. Plane A								
L	−0.3	−0.2	−0.2	0.0	−1.5	−1.0	−1.0	−1.5
M	−0.3	−3.5	−0.2	+1.4	−1.0	+2.0	+6.0	+5.0
N	+0.1	−2.0	+0.6	+1.2	−1.2	−1.0	+2.0	−1.5
X	+0.2	+0.3	+1.1	+1.5	−1.2	−1.0	−2.0	−1.2
Y	+0.5	+1.3	+0.8	+0.6	−1.0	−2.2	−4.0	−5.0
Z	+0.7	+0.5	+0.3	+0.2	−1.5	−1.5	+0.5	−1.2
Incidence −1.3°. No lift. Plane B								
L	−0.5	−0.2	−0.2	0.0	−1.5	−1.5	−1.7	−1.5
M	−0.4	−2.1	+0.1	+0.3	−1.5	−5.0	−6.5	−7.5
N	0.0	0.0	+0.6	+1.3	−1.5	−2.0	−1.7	−2.0
X	+0.4	−0.3	+0.4	+1.2	−1.2	−0.5	−2.0	−2.5
Y	0.0	+0.5	+0.2	−0.5	−1.0	+0.5	0.0	+1.2
Z	+0.3	+0.2	0.0	0.0	−1.2	−2.0	−1.2	−1.2
Incidence 3°. Maximum L/D. Plane A								
L	0.0	−0.4	−0.5	0.0	+3.5	+3.5	+4.2	+3.5
M	0.0	−3.5	−0.2	+0.5	+3.5	+8.5	+11.5	+13.5
N	+0.3	+2.8	+2.2	+1.5	+4.0	+5.5	+4.0	+3.5
X	+0.1	+8.0	+3.0	+1.8	+5.0	+6.0	−0.5	−1.5
Y	+0.8	+9.0	+4.2	+0.8	+5.5	+6.5	+3.5	−3.0
Z	+0.8	+0.7	+3.7	+1.5	+6.5	+12.5	−1.0	+0.7
Incidence 3°. Maximum L/D. Plane B								
L	−0.3	−0.2	−0.3	+0.3	+2.3	+4.0	+3.7	+4.0
M	−0.3	−3.1	−0.6	+0.5	+3.0	+1.5	−0.5	+1.0
N	−0.1	−4.8	−1.2	+0.5	+4.0	+3.0	0.0	−1.0
X	0.0	−6.4	−2.2	+0.3	+3.0	+3.5	−1.0	−2.0
Y	−0.5	−4.8	−3.1	−0.8	+3.0	+5.0	+1.0	0.0
Z	0.0	−4.7	−4.1	−1.0	+4.0	+5.0	0.0	+0.2
Incidence 15°. Maximum lift. Plane A								
L	+0.3	+0.2	+0.2	0.0	+15.0	+17.0	+16.0	+19.0
M	+0.5	+2.4	+1.7	+0.7	+16.0	+21.0	+19.5	+16.0
N	+2.0	+18.2	+6.4	+2.3	+17.0	+22.0	0.0	+1.0
X	+2.5	+30.5	+8.0	+2.6	+18.5	+25.0	−3.5	−1.0
Y	+2.0	+25.0	+6.3	+2.7	+20.0	+33.0	−5.5	−2.5
Z	+1.2	+0.5	+3.9	+3.0	+19.5	+40.0	−3.5	+1.0
Incidence 15°. Maximum lift. Plane B								
L	−0.6	−1.4	−1.4	−0.9	+17.0	+16.0	+17.0	+21.0
M	−2.0	−6.5	−4.8	−2.0	+17.2	+17.5	+16.0	+19.0
N	−2.2	−13.0	−7.7	−2.5	+18.5	+18.0	+4.5	+4.0
X	−3.0	−13.9	−8.8	−1.7	+19.0	+19.0	+1.0	0.0
Y	−3.1	−13.6	−8.2	−1.6	+18.0	+19.5	+3.0	+4.0
Z	−4.1	−10.8	−8.3	−1.3	+18.0	+20.0	+6.0	+8.0

Aeronautical Research Paper No. 20. TABLE NO. 2

AIR FLOW ABOUT WING TIPS

AEROFOIL USA—27

	Horizontal Angle			Degrees	Vertical Angle			Degrees
	1	2	3	4	1	2	3	4
Incidence −5°. No lift. Plane A								
L	−0.7	−1.3	−1.2	−0.5	−5.5	−3.5	−4.5	−3.5
M	−0.7	−5.4	−1.8	+0.4	−5.5	0.0	+4.0	+6.0
N	−0.7	−4.8	−1.3	0.0	−4.0	−1.0	+3.5	+4.7
X	+0.5	−1.7	−0.8	−0.3	−4.5	−3.0	−3.0	−2.5
Y	+0.5	+0.2	−0.8	−0.6	−5.0	−5.5	−5.0	−5.0
Z	+0.2	−0.6	−0.5	−0.3	−4.0	−3.5	−2.5	−3.5
Incidence −5°. No lift. Plane B								
L	−0.7	−0.9	−0.5	+0.5	−6.5	−6.5	−7.2	−8.0
M	−0.7	−2.8	−0.8	−0.5	−6.5	−10.0	−15.0	−14.5
N	−0.2	+3.0	+1.1	+0.8	−7.0	−5.5	−2.0	−2.0
X	−4.0	+2.5	+1.1	+0.7	−6.0	−4.5	−4.5	−5.5
Y	+0.6	+2.7	+1.2	−0.2	−7.5	−3.5	−2.7	−3.0
Z	+1.0	−0.8	+1.1	−0.2	−6.0	−6.7	−8.0	−5.0
Incidence +2°. Maximum L/D. Plane A.								
L	−1.0	−1.6	−1.0	0.0	+2.0	+3.2	+4.0	+5.0
M	−0.7	−4.0	−0.6	+0.5	+3.5	+7.0	+11.0	+13.0
N	0.0	+1.6	+1.2	+1.7	+4.0	+5.5	+5.0	+4.0
X	+1.0	+7.4	+3.7	+1.6	+4.0	+4.0	−3.0	−5.0
Y	+1.2	+9.1	+5.4	+1.7	+4.2	+4.0	−5.5	−6.5
Z	+0.8	+8.2	+4.2	+1.8	+5.0	+7.0	−1.5	−2.0
Incidence +2°. Maximum L/D. Plane B								
L	−0.6	−0.7	−0.5	0.0	+2.5	+2.5	+2.5	+3.5
M	−1.0	−4.3	−1.5	0.0	+3.0	0.0	−2.5	−1.7
N	−0.8	−5.0	−2.0	+0.4	+3.0	+3.0	−1.0	−1.7
X	−0.3	−7.2	−3.7	−0.4	+3.2	+4.0	−2.0	−3.0
Y	−0.2	−6.5	−4.6	−1.4	+4.0	+5.0	−1.0	−1.5
Z	0.0	−6.4	−6.0	−1.5	+5.0	+6.0	−1.5	−3.0
Incidence +16°. Maximum lift. Plane A								
L	0.0	0.0	0.0	0.0	+17.0	+21.0	+22.0	+23.5
M	+0.3	+1.0	+1.2	+1.6	+19.0	+24.5	+26.0	+28.0
N	+2.1	+18.5	+6.5	+3.2	+20.0	+15.0	+5.5	+6.0
X	+4.0	+28.2	+10.0	+3.6	+22.0	+20.0	−2.0	−2.0
Y	+3.4	+30.7	+10.0	+3.8	+23.0	+31.0	−6.0	−1.5
Z	+2.8	+15.4	+4.0	+3.2	+23.0	+51.0	−3.0	+3.0
Incidence +16°. Maximum lift. Plane B								
L	−1.1	−2.0	−1.5	−1.2	+17.5	+18.5	+20.0	+23.5
M	−2.3	−10.5	−5.2	−1.7	+16.7	+19.5	+21.0	+24.0
N	−3.2	−10.0	−10.7	−2.3	+14.7	+16.2	+1.7	+2.4
X	−2.9	−21.5	−11.2	−2.7	+20.0	+18.0	−1.5	−3.0
Y	−3.8	−19.8	−11.3	−3.3	+21.5	+17.0	+3.0	+0.2
Z	−6.5	−16.6	−12.8	3.5	+22.0	+17.0	+3.0	+4.0

Aeronautical Research Paper No. 20. TABLE NO. 3.

AIR FLOW ABOUT WING TIPS

AEROFOIL GOTTINGEN-387

	Horizontal Angle Degrees				Vertical Angle Degrees			
	1	2	3	4	1	2	3	4
Incidence −7.9°. No lift. Plane A								
L	−0.2	−1.0	−0.8	0.0	−7.0	−6.3	−7.5	−6.0
M	0.0	−6.9	−1.5	+2.7	−5.0	−1.5	+3.5	+2.0
N	+0.3	−5.5	−0.8	+1.2	−7.0	−3.0	+1.0	+2.5
X	+0.8	−0.2	0.0	+1.5	−7.0	−7.4	−6.2	−6.5
Y	+1.4	+1.3	+1.2	+0.8	−7.0	−8.0	−8.5	−10.0
Z	+1.0	+0.7	+1.2	+1.0	−6.5	−6.0	−7.0	−7.2
Incidence −7.9°. No lift. Plane B								
L	+0.9	+0.2	+0.4	+1.5	−8.5	−10.0	−10.0	−12.5
M	+0.8	−2.2	+0.5	+0.9	−9.5	−13.5	−18.5	−19.0
N	+1.8	+6.1	+2.1	+1.8	−9.0	−7.0	−2.5	−2.5
X	+2.3	+6.4	+3.1	+2.6	−9.0	−7.5	−4.5	−3.5
Y	+3.4	+5.1	+3.5	+1.5	−8.5	−7.5	−5.0	−4.3
Z	+2.0	+2.7	+3.7	+2.3	−9.0	−9.0	−8.3	−9.0
Incidence +2°. Maximum L/D. Plane A								
L	0.0	−1.0	0.0	+0.5	+2.1	+3.0	+4.8	+0.5
M	0.0	−3.0	+0.8	+2.3	+3.2	+8.0	+13.0	+14.5
N	+1.4	+3.8	+2.7	+3.3	+3.5	+4.0	+3.0	+1.0
X	+2.3	+11.0	+6.2	+3.0	+3.5	0.0	−7.0	−7.0
Y	+3.3	+12.7	+7.0	+3.5	+4.0	+2.5	−6.0	−7.2
Z	+3.2	+11.8	+6.6	+3.5	0.0	+1.0	−4.0	−3.5
Incidence +2°. Maximum L/D. Plane B								
L	−0.6	−1.2	−0.8	−0.1	+3.0	+4.0	+4.0	+6.0
M	−0.8	−4.9	−1.7	+0.2	+4.5	+2.0	−3.0	−2.0
N	−0.9	−5.2	−2.8	0.0	+4.4	+3.7	0.0	0.0
X	−0.6	−7.1	−4.2	−0.2	+5.7	+5.2	−0.5	−1.0
Y	−0.4	−8.7	−5.7	−1.6	+6.0	+7.1	−1.0	−1.3
Z	−1.0	−9.2	−6.2	−2.2	+6.3	+7.0	−3.2	−3.0
Incidence +15°. Maximum lift. Plane A								
L	0.0	0.0	+0.3	0.0	+17.5	+20.5	+20.5	+23.0
M	+1.0	+0.8	+1.5	+1.8	+19.0	+23.5	+26.5	+29.0
N	+3.4	+22.0	+6.5	+3.3	+20.0	+11.0	+3.5	+5.0
X	+5.5	+28.0	+10.0	+3.3	+22.0	+17.5	−5.0	−3.5
Y	+4.2	+30.0	+12.2	+4.0	+22.0	+26.5	−6.3	−2.7
Z	+2.9	+23.5	+7.0	+13.5	+23.5	+45.0	−6.0	+0.5
Incidence +15°. Maximum lift. Plane B								
L	0.0	−1.6	−1.5	−1.5	+17.0	+17.0	+17.0	+18.5
M	−1.3	−10.0	−4.6	−1.4	+16.5	+14.0	+12.0	+12.0
N	−2.5	−16.6	−7.0	−0.9	+19.0	+12.0	0.0	0.0
X	−2.8	−18.8	−7.1	0.0	+20.0	+14.0	−2.0	−3.0
Y	−3.2	−18.0	−7.5	0.0	+19.0	+13.0	+1.5	−3.5
Z	−4.3	−14.0	−5.8	+3.2	+19.0	+14.0	+0.5	+8.0

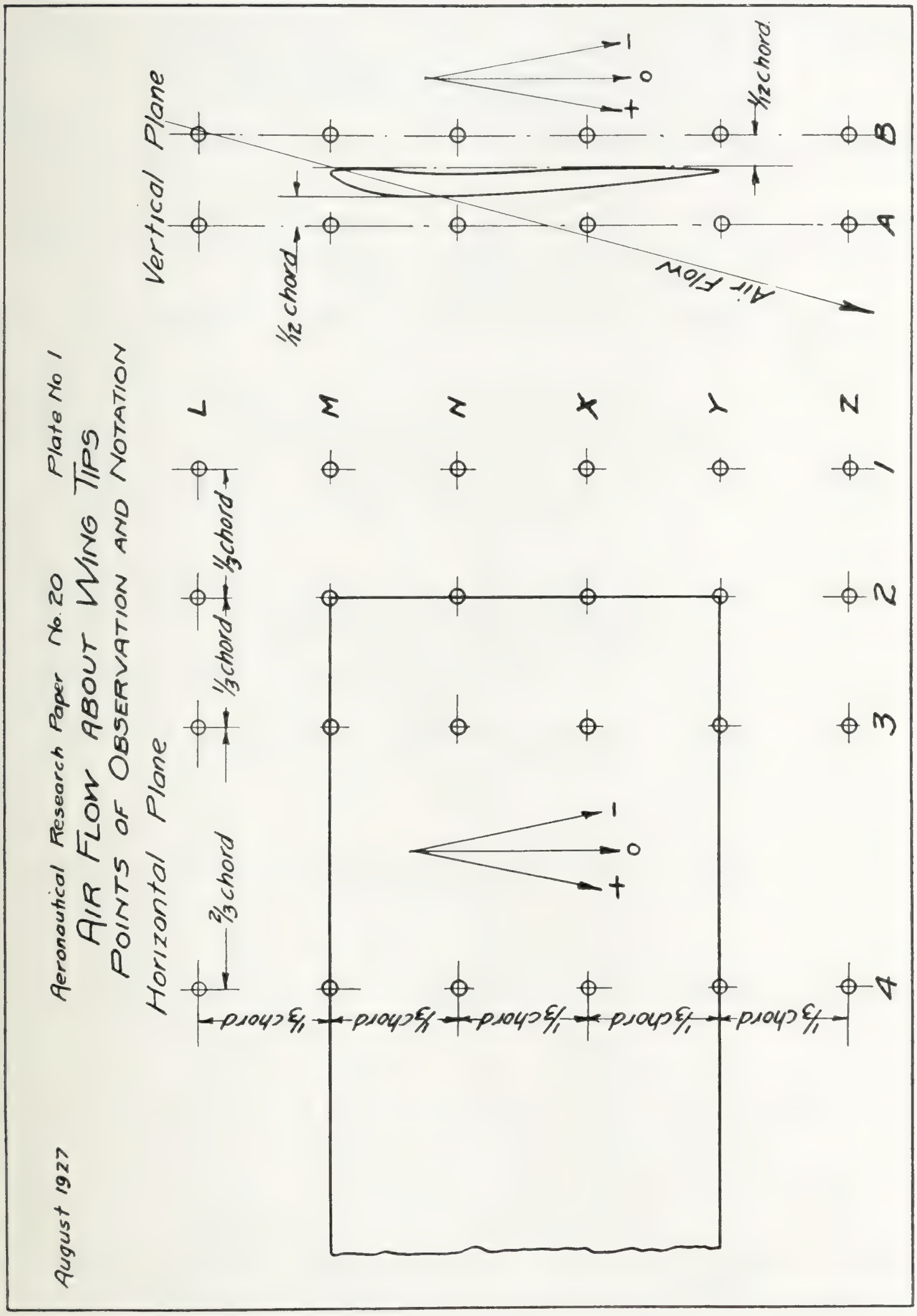

Aeronautical Research Paper No. 20
Plate No 1
AIR FLOW ABOUT WING TIPS
POINTS OF OBSERVATION AND NOTATION
Horizontal Plane
Vertical Plane
August 1927
L
M
N
X
Y
Z
1
2
3
4
A
B
2/3 chord
1/3 chord
1/3 chord
1/3 chord
1/3 chord
1/3 chord
1/3 chord
1/3 chord
1/3 chord
1/12 chord
1/12 chord
Air Flow
+
0
-
+
0
-

PLATE 1

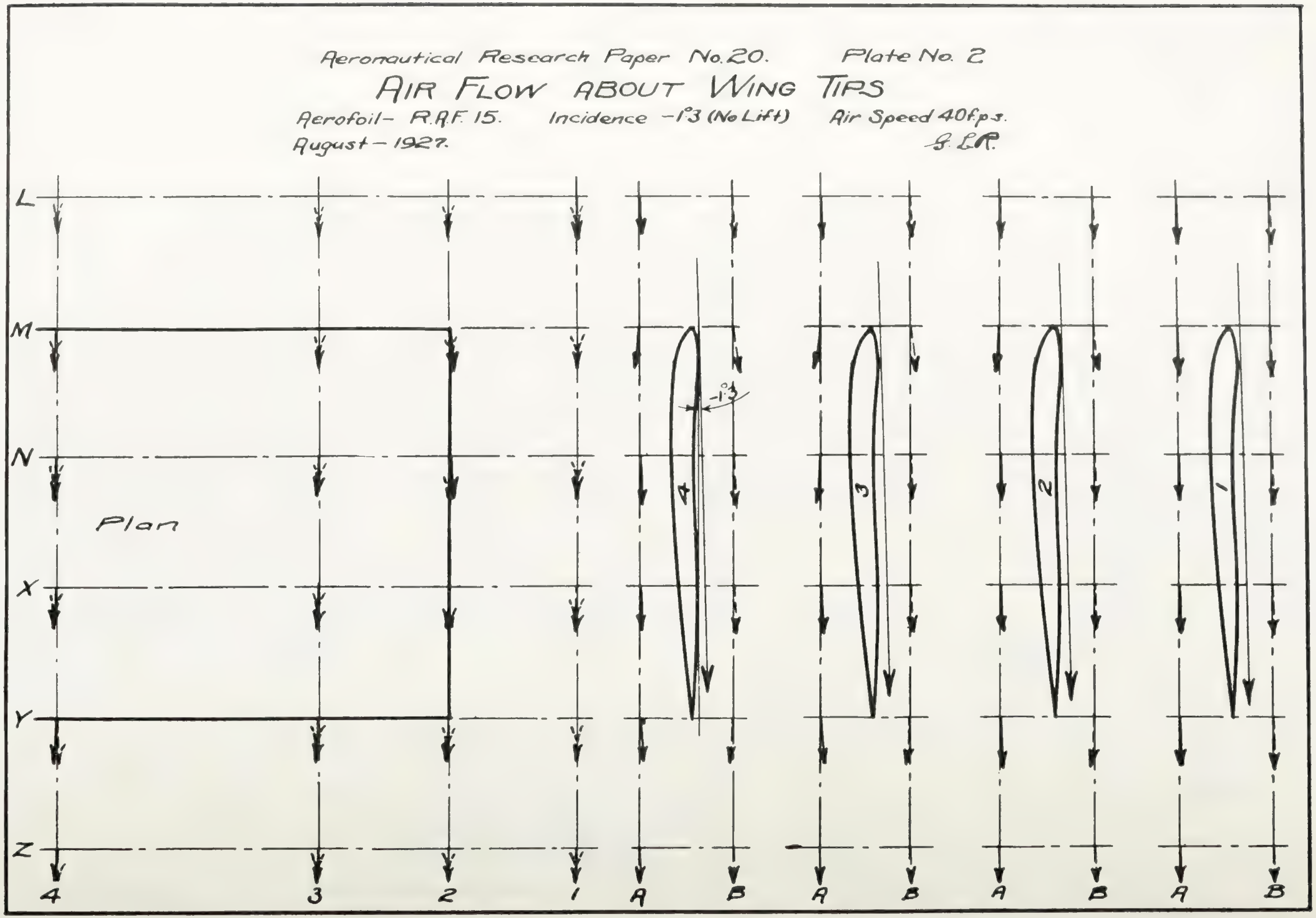

Aeronautical Research Paper No. 20.
Plate No. 2
AIR FLOW ABOUT WING TIPS
Aerofoil- R.A.F. 15.
Incidence -1°3 (No Lift)
Air Speed 40 f.p.s.
August - 1927.
G.L.R.
L
M
N
X
Y
Z
Plan
-1°3
4
3
2
1
A
B
A
B
A
B
A
B
4
3
2
1

PLATE 2

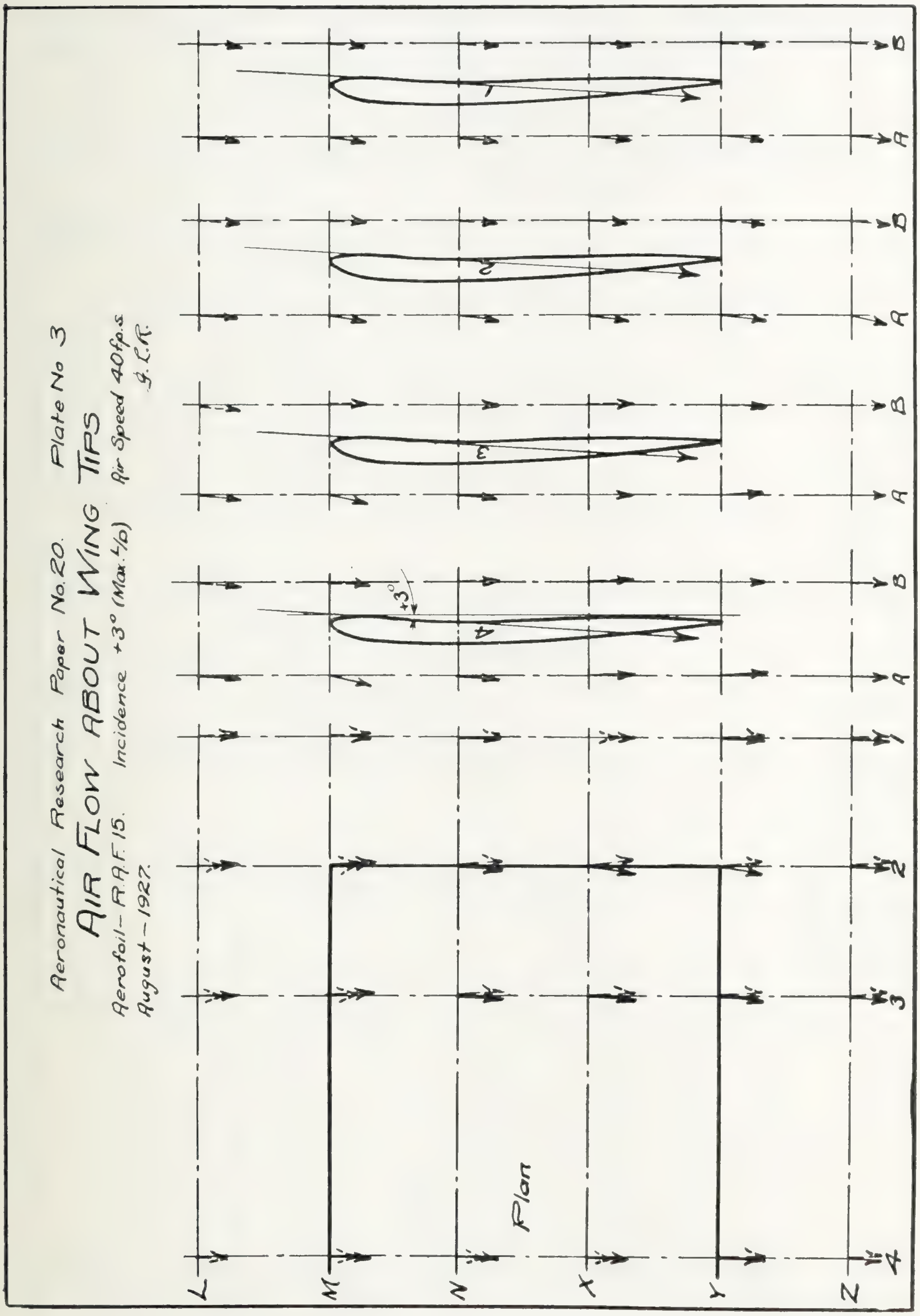

Aeronautical Research Paper No. 20.
Plate No 3
AIR FLOW ABOUT WING TIPS
Aerofoil - R.A.F. 15.
Incidence +3° (Max. L/D)
Air Speed 40 f.p.s
August - 1927.
J.E.R.
+3°
1
2
3
4
A
B
L
M
N
X
Y
Z
Plan

PLATE 3

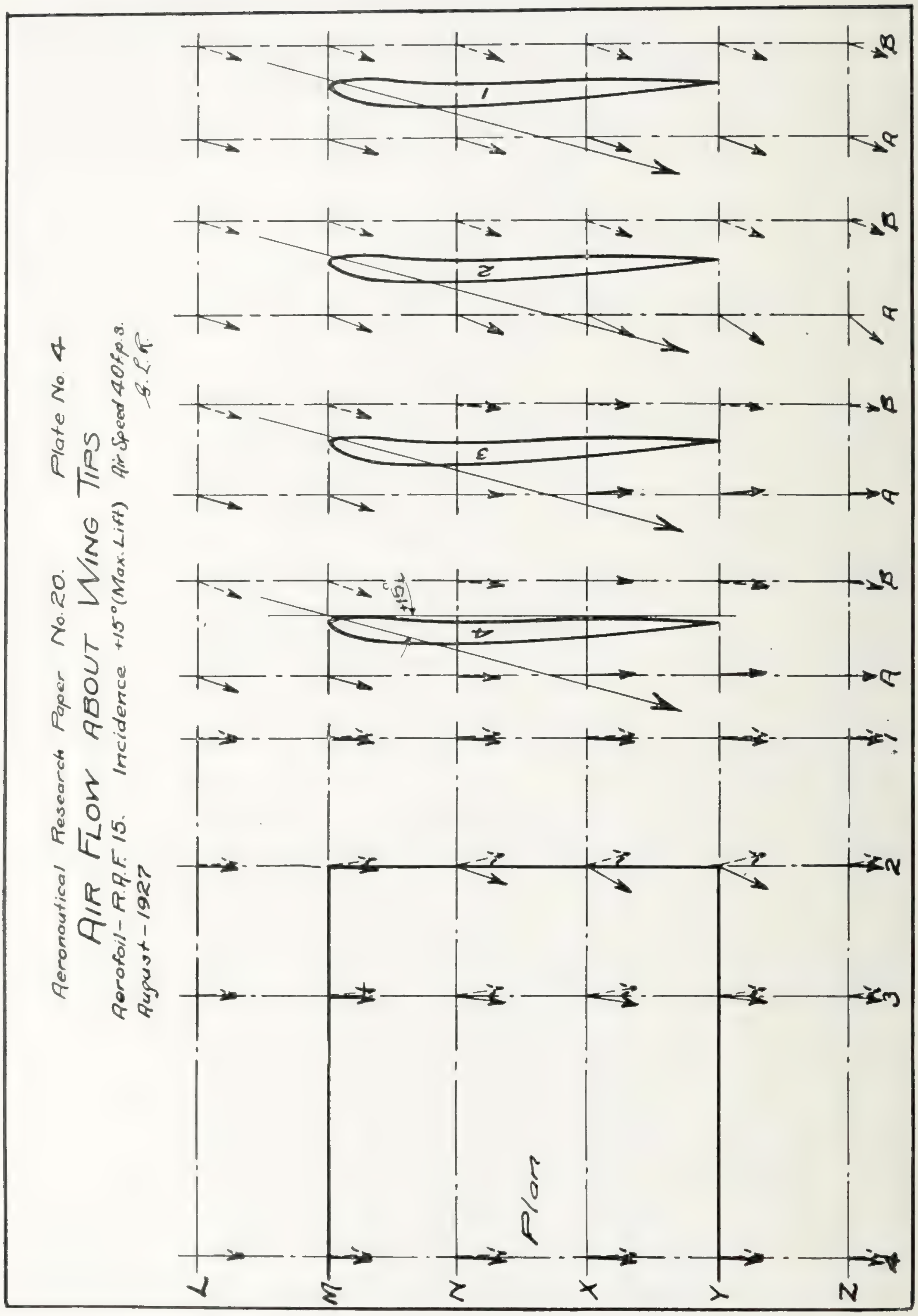

Aeronautical Research Paper No. 20.
Plate No. 4
AIR FLOW ABOUT WING TIPS
Aerofoil - R.A.F. 15.
Incidence +15° (Max. Lift)
Air Speed 40 f.p.s.
August - 1927
G.L.R.
Plan
L
M
N
X
Y
Z
1
2
3
4
A
B
+15°

PLATE 4

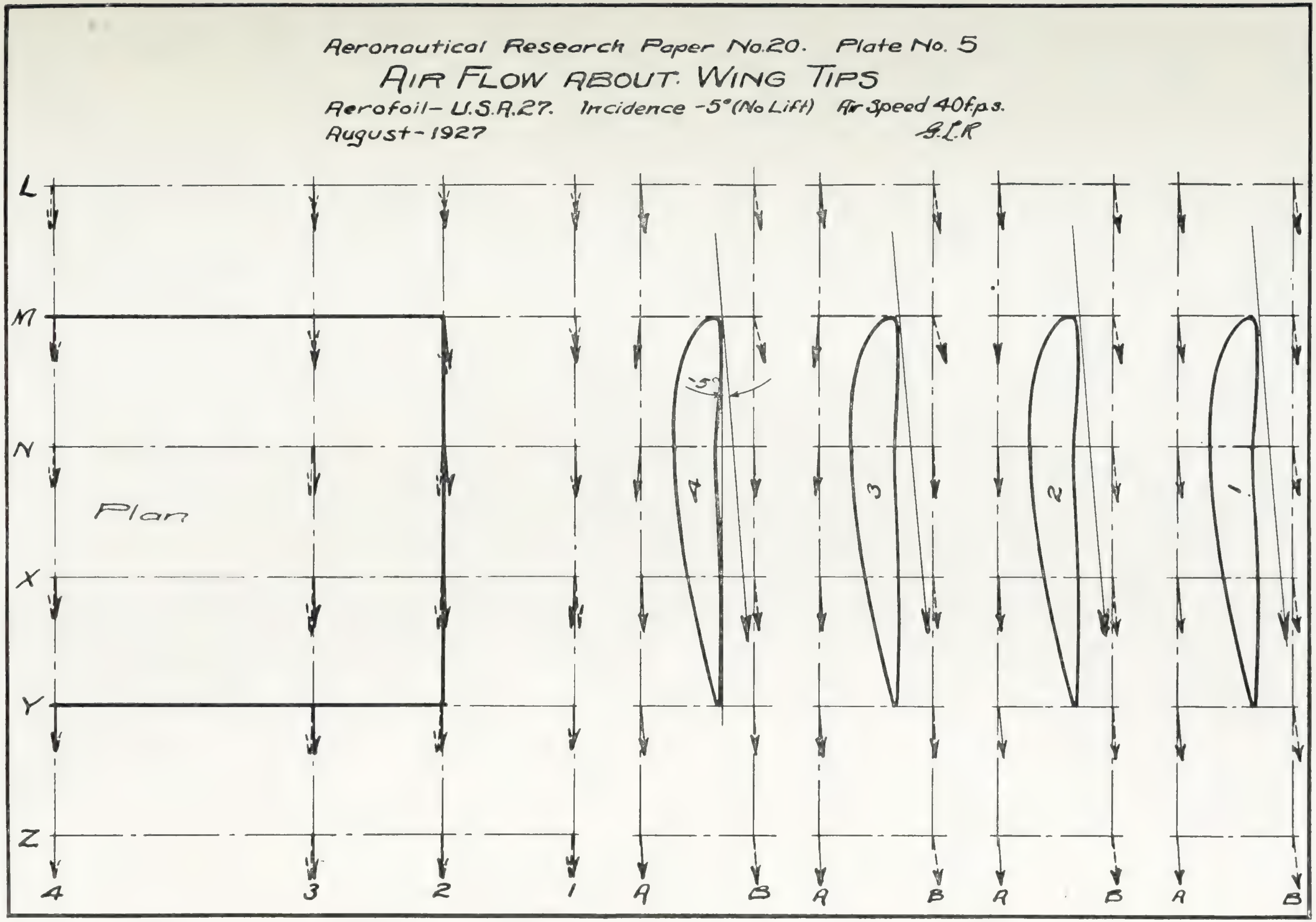

Aeronautical Research Paper No.20. Plate No. 5
AIR FLOW ABOUT WING TIPS
Aerofoil- U.S.A.27. Incidence -5°(No Lift) Air Speed 40f.p.s.
August-1927
G.L.R
L
M
N
X
Y
Z
Plan
4
3
2
1
A
B
4
3
2
1
-5°

Plate 5

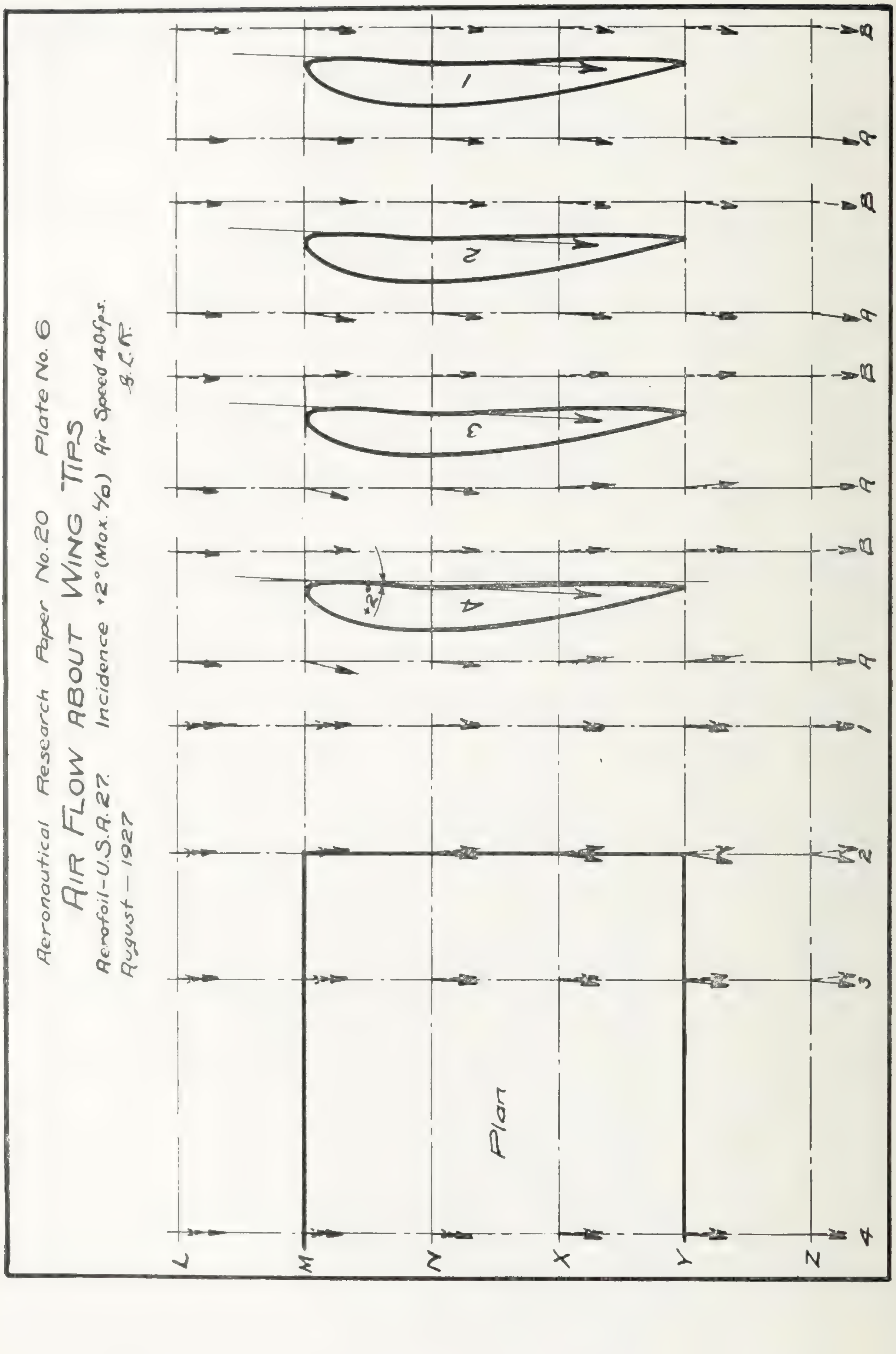

Aeronautical Research Paper No. 20 Plate No. 6
AIR FLOW ABOUT WING TIPS
Aerofoil-U.S.A. 27. Incidence +2° (Max. 4/0) Air Speed 40 f.p.s.
August — 1927 G.L.R.
1
2
3
4
+2°
A
B
Plan
L
M
N
X
Y
Z
1
2
3
4

PLATE 6

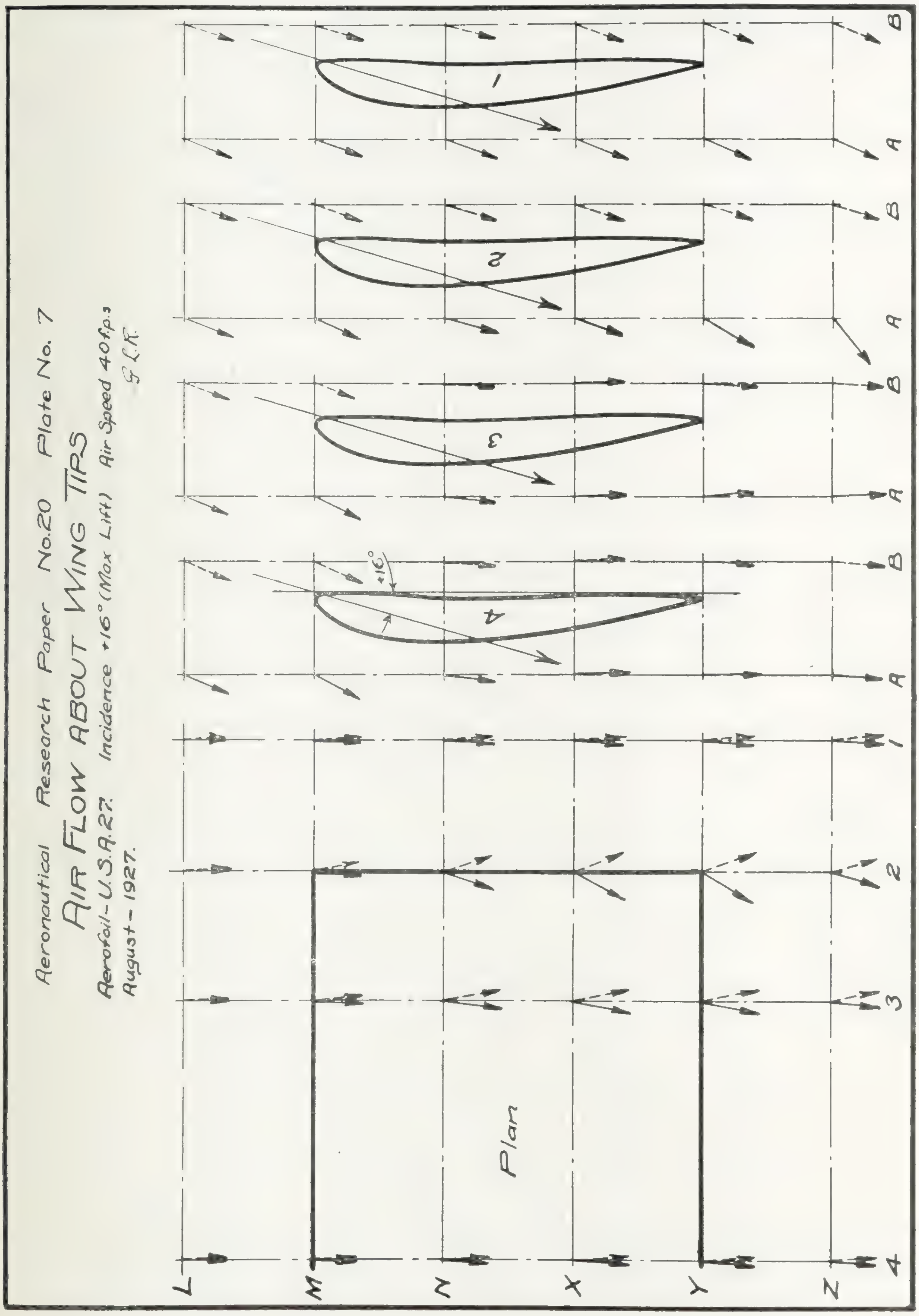

Aeronautical Research Paper No.20 Plate No. 7
AIR FLOW ABOUT WING TIPS
Aerofoil-U.S.A.27. Incidence +16° (Max Lift) Air Speed 40 f.p.s
August - 1927.
G.L.R.
+16°
1
2
3
4
A
B
A
B
A
B
A
B
1
2
3
4
Plan
L
M
N
X
Y
Z

PLATE 7

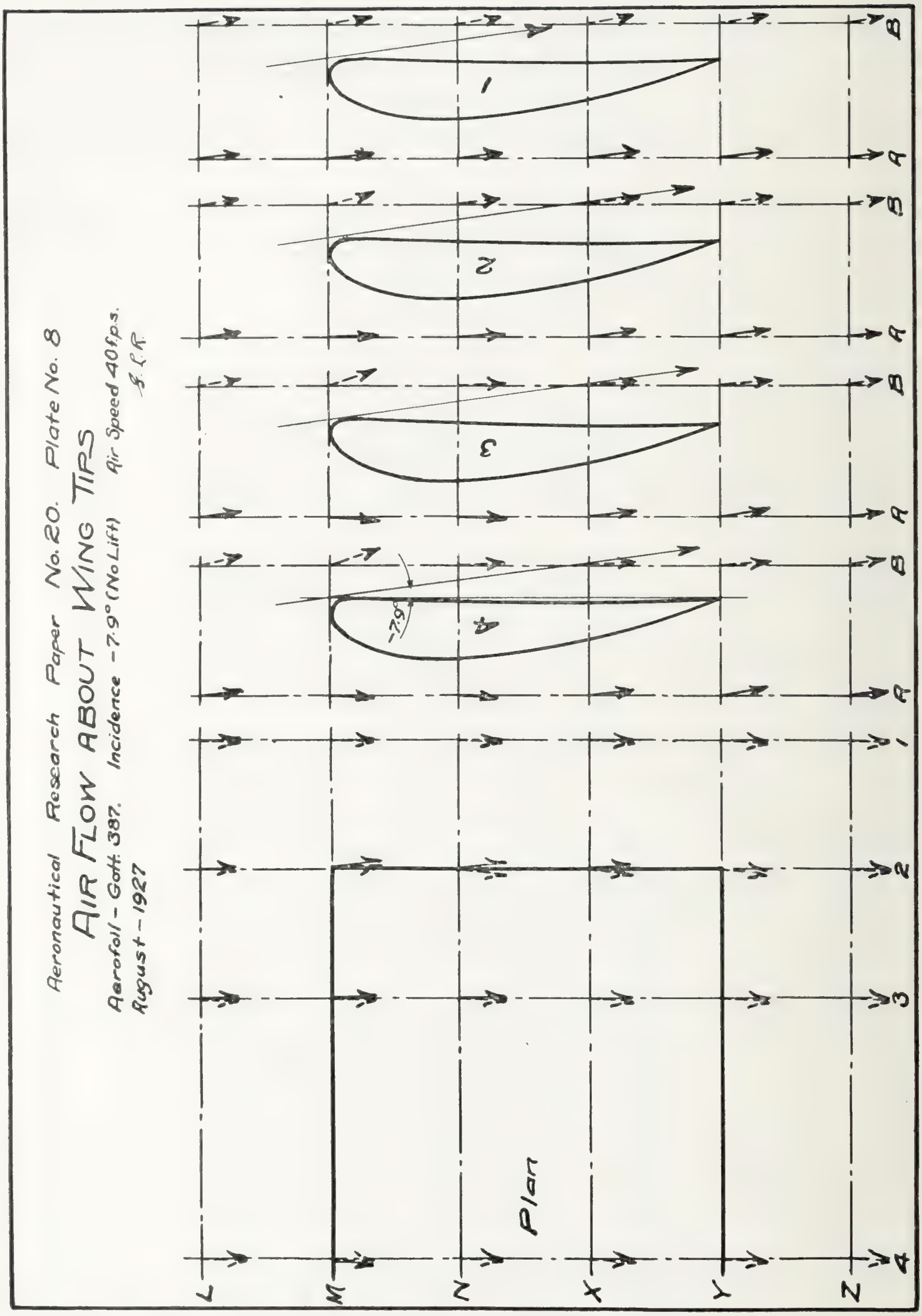

Aeronautical Research Paper No. 20. Plate No. 8
AIR FLOW ABOUT WING TIPS
Aerofoil - Gott. 387. Incidence -7.9° (No Lift) Air Speed 40 f.p.s.
August - 1927
J. L. R.
Plan
-7.9°
1
2
3
4
A
B
L
M
N
X
Y
Z

PLATE 8

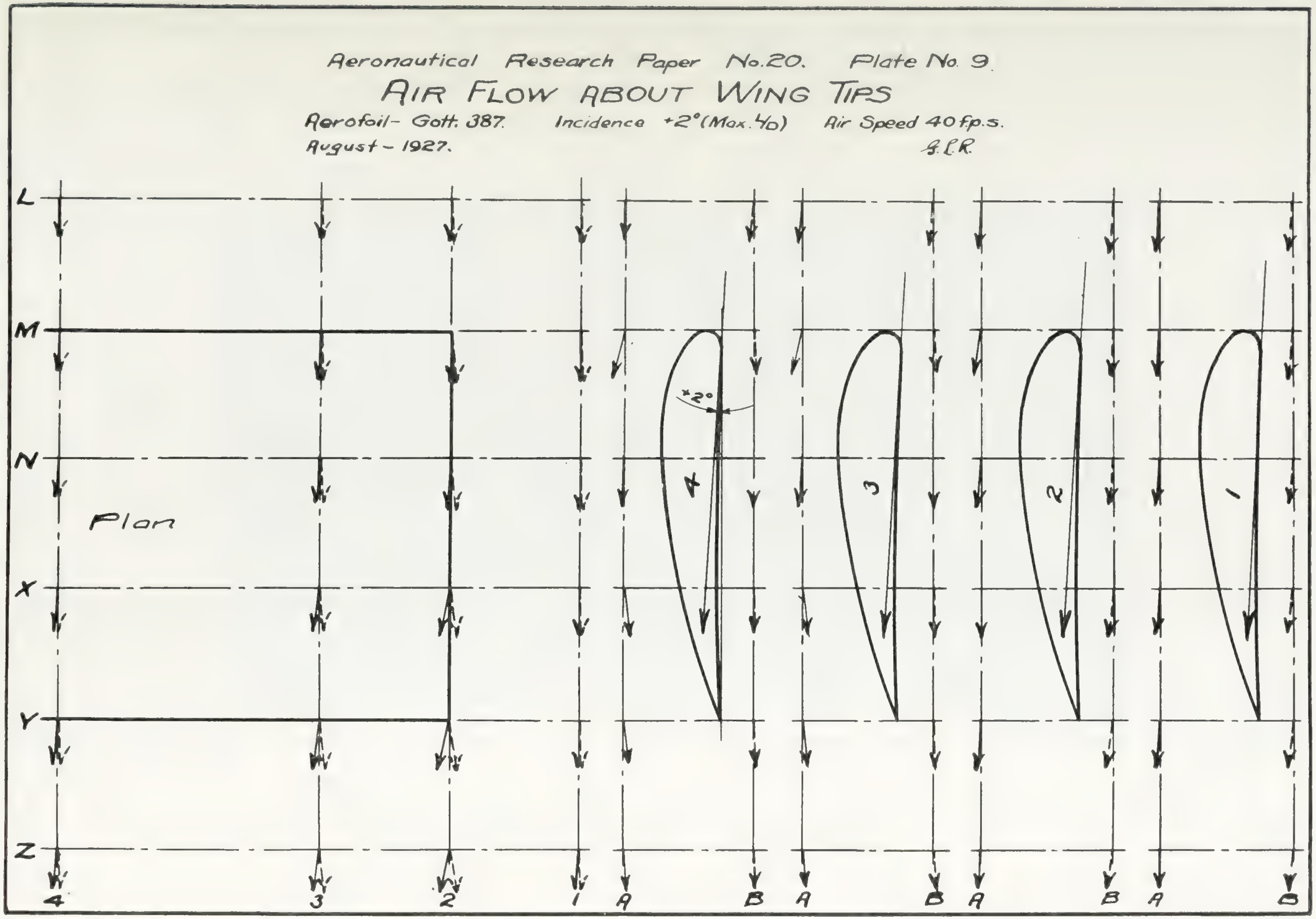

Aeronautical Research Paper No. 20. Plate No. 9.
AIR FLOW ABOUT WING TIPS
Aerofoil- Gott. 387. Incidence +2°(Max. L/D) Air Speed 40 f.p.s.
August - 1927.
G.C.R.
L
M
N
X
Y
Z
Plan
4
3
2
1
A
B
A
B
A
B
A
B
4
3
2
1
+2°

PLATE 9

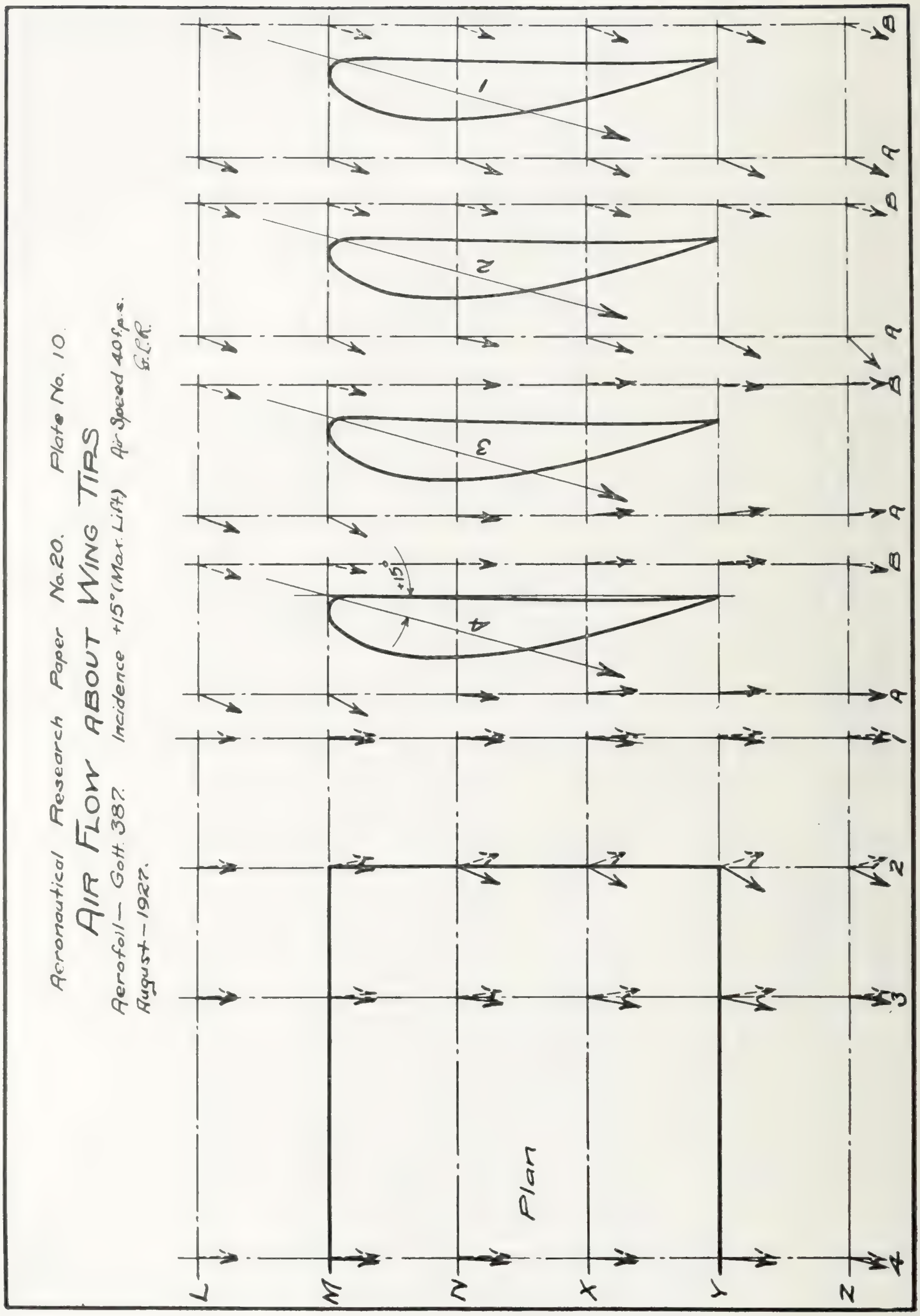
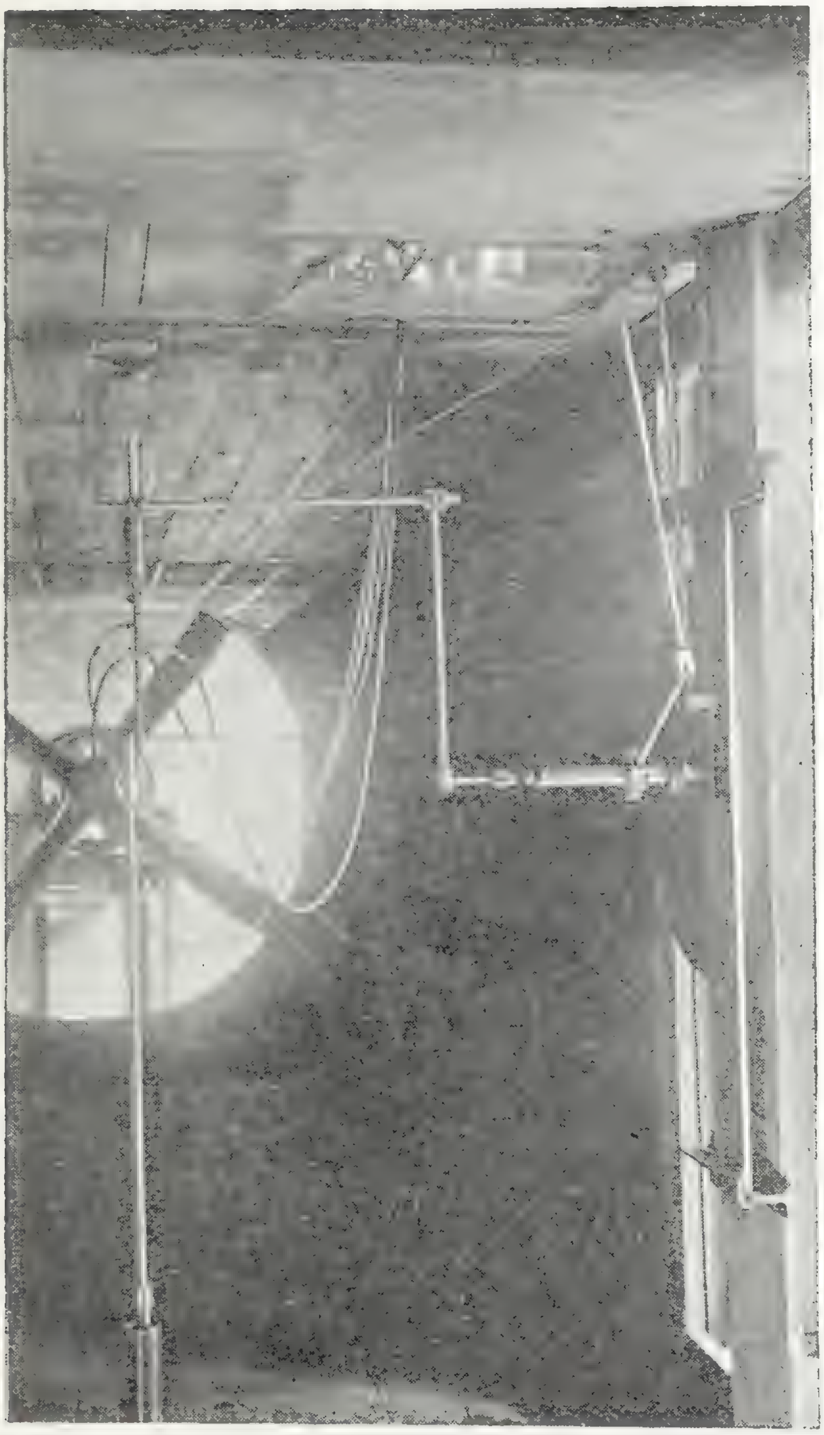
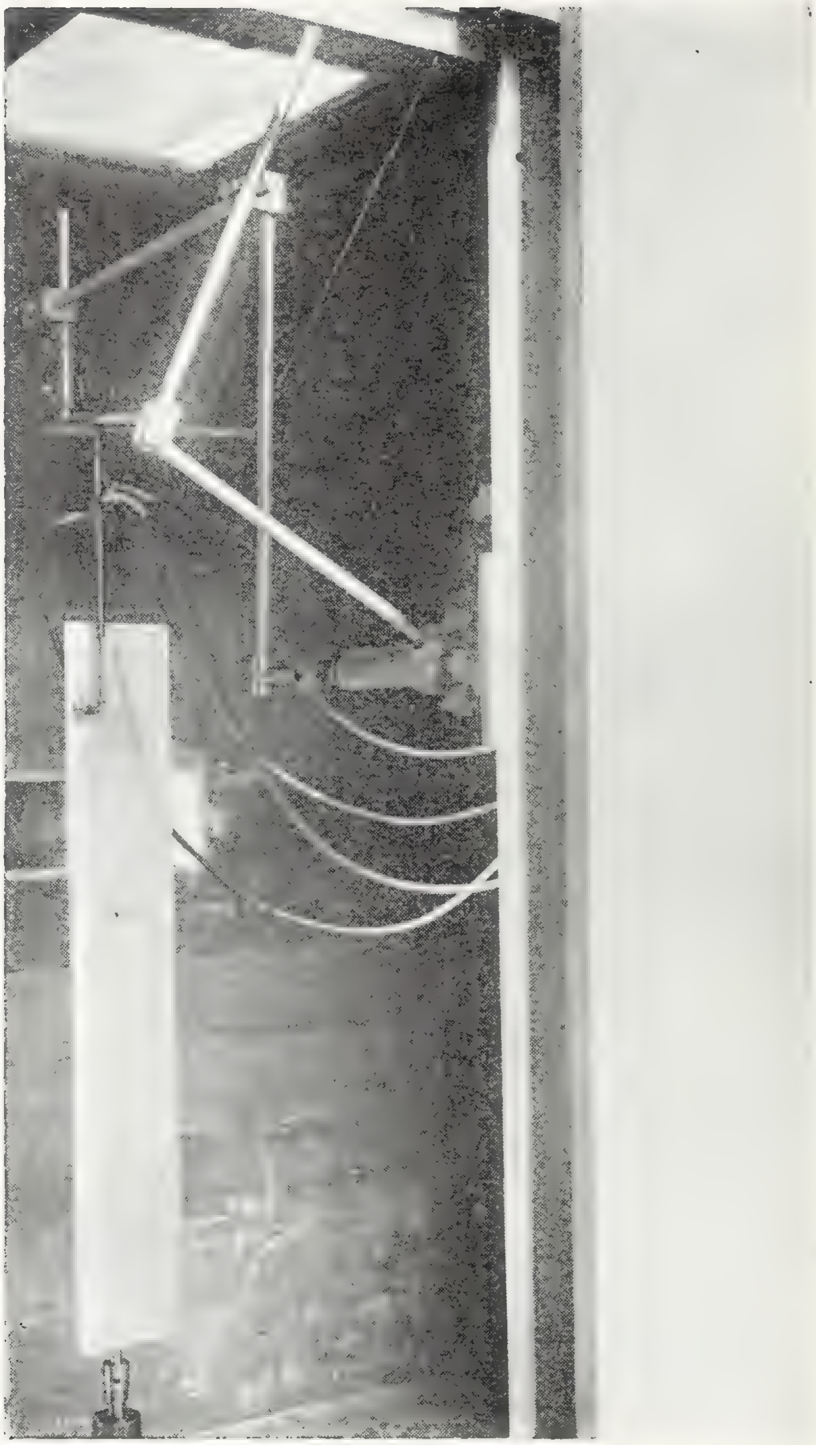

Aeronautical Research Paper No. 20. Plate No. 10.
AIR FLOW ABOUT WING TIPS
Aerofoil— Gott. 387. Incidence +15° (Max. Lift) Air Speed 40 f.p.s.
August—1927.
G.L.R.
+15°
1
2
3
4
A
B
Plan
L
M
N
X
Y
Z

PLATE 10



PHOTOGRAPH 1



PHOTOGRAPH 2

Aeronautical Research Paper No. 21
September, 1928.

INTERNATIONAL TRIALS

Tests of Two Airship Models

By J. H. Parkin, M.E., F.R.Ae.S., W. H. Coates, B.A.Sc., and
G. J. Klein, B.A.Sc.

Summary

The aerodynamic laboratory of the University of Toronto is collaborating in the international trials of wind tunnel models initiated by the British Aeronautical Research Committee for the purpose of securing information relative to the influence of test conditions on the results of tests of models in wind tunnels.

The present report deals with the tests of two airship models.

A description is given of the apparatus and technique employed in the tests.

The resistance coefficients of the two models at speeds from 20 to 80 feet per second, corresponding to a range of Reynolds' Number from about 60,000 to 250,000, are given in tabular and in graphical form.

Introduction

The results of tests made on a model in a wind tunnel are subject to certain corrections before becoming applicable to the full scale machine in flight in still air of infinite extent. These corrections are due to the conditions under which the wind tunnel tests are made and may be briefly summarized as follows:—

I. Those due to the wind tunnel:
    (*a*) Type—open or closed throat.
    (*b*) Cross-section—(1) Shape, (2) Size.
II. Those due to the air stream:
    (*a*) Speed.
    (*b*) Turbulence or flow texture.
III. Those due to the model:
    (*a*) Size.
    (*b*) Nature of surface.
IV. Those due to the support of model:
    Interference due to spindle, wires or other support employed.

As the wind tunnels of aerodynamic laboratories vary in size and design, and the apparatus and technique employed also differ, there is some uncertainty as to the extent to which the results of tests in one laboratory are comparable to those made in another.

In 1920, the Aeronautical Research Committee of Great Britain decided to institute comparative model tests in as many as possible of the aerodynamic laboratories of the world* with a view to reducing this uncertainty and securing information not otherwise available. By testing the same models in the different wind tunnels, it was expected that light would be thrown on the influence exerted by the nature of the airstream, in the tunnel, whether free or confined, its shape, size and turbulence, as well as the influence of the model supports, on the results of model tests in the tunnel.

The international trials consist of the following tests:

1. Determination of lift, drag and centre of pressure of a standard RAF-15 aerofoil, of aspect ratio 6:1 and 6″ chord, at various angles of incidence.

2. Resistance measurements at zero angle of yaw on two 1/225 scale airship models, one a model of the R-33, the other similar in form but without any cylindrical portion between the head and tail.

The trials are being participated in by the laboratories in Great Britain, France, United States, Italy and Japan. In 1921, the Associate Air Research Committee of the National Research Council of Canada offered to collaborate in the trials, and arranged to have the models tested in the laboratory at the University of Toronto. The models became available in 1923, but as the laboratory at Toronto was then dismantled for removal to a new building, it was not until September 29th, 1927, that the airship models were received.

An account of the tests and results is presented in the following report, which is issued as a research bulletin, since it forms part of the larger international research being conducted to ascertain the influence of laboratory test conditions on the results of tests of models in wind tunnels.

The report contains descriptions of equipment and technique used in the course of the general work of the laboratory, much of which ordinarily is not dealt with in the published research reports. No special refinements were introduced for these particular tests, other than careful checking of the weights and other apparatus employed. The methods and apparatus are those regularly used in the work of the laboratory.

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*R & M 954 Report for 1925-6 p.90.

DESCRIPTION OF THE MODELS

The two models (long and short) are of cast aluminum, made in sections and smoothly finished. Steel sockets and plugs are inserted at 90° intervals around the circumference at three sections 6.3 inches apart, to enable different methods of supporting the models to be employed. A plug is also inserted in the nose and a number of tailpieces, stings and spindles are supplied, as shown in figure 1.

The dimensions of each model were determined by mounting it between centres on the table of a milling machine and measuring, by means of a dial micrometer, the variation from a reference diameter. Measurements were made at four points around the circumference at each of a number of sections. The reference diameter, that at 10″ from the nose, was determined by micrometer. Sections were taken ½″ apart from the nose to a section 5″ from the nose and 1″ apart over the remainder. The measurements are tabulated in Table No. 1.

The volumes of the models were calculated from the dimensions, using Simpson's Rule, and are given in Table No. 1.

The weights and approximate positions of the centres of gravity of the models with pointed tailpieces are also given in Table No. 1.

Before each test, the models were washed in alcohol to remove all grease and dust, as it was found that the dust collected on the models during several tests affected the resistance. The plugs and other irregularities on the surface were smoothed over with wax.

APPARATUS USED

The tests at Toronto were made in the 4-ft. RAE type wind channel. The channel and its equipment have already been described in *Aeronautical Research Paper No. 16.**

The features of particular importance from the standpoint of these tests are:

1. *Honeycomb.* A honeycomb built of 20-gauge galvanized iron in the form of 3″ square cells, 30″ long, is located in the tunnel, 9 ft. ahead of the experimental region and 42″ from the faired intake. During the tests there was a distance of about 6 ft. between the exit from the honeycomb and the nose of the longer model.

2. *Cellular wall.* A cellular wall composed of wooden lattices is erected across the building 10 ft. from the exit from the regenerative cone, or the plane of rotation of the 9′ 6″ dia. four-bladed propeller. The openings through the lattices are 1¼″ square and 1¼″ long, so that there is a directive effect on the air tending to straighten out whirls. The

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*See Bulletin No. 7, 1927, School of Engineering Research, University of Toronto.

amount of open area varies in the different lattices, and the lattices are arranged so that those with the least free area are next the walls, ceiling and floor, with the object of securing a more uniform distribution of velocity in the air returning through the room.

The balance used was the drag balance of a pair of wire suspension roof balances. These balances are similar to those of the duplex wind channel at the National Physical Laboratory. There are two balances, a forward balance for measuring lift and rolling moments, and a rear balance comprising two elements or balances, one for measuring lift and the other for measuring drag. The latter is essentially a light bell crank suspended on two cone pivots (see A, fig. 6). Normally, these two parts of the rear balance are connected together and to the sting of the model by means of a completely universal joint or "wobbler" housed in a streamline guard. In these tests, since drag only was to be measured, the "wobbler" was disconnected, and the sting from the model connected through cone pivots to the vertical arm of the drag balance. (See B, fig. 6.)

The drag balance is provided with two sensitivity weights above the pivots for adjusting the centre of gravity of the moving element of the balance as near as desired to the axis of the pivots. With the airship models it was found that the restoring couple, due to the weight of the model when the suspension wires were swayed slightly from the vertical, was such as to render accurate reading of the drag impossible. Accordingly, much longer spindles for the sensitivity weights were fitted to the balance and suitably guyed (see fig. 6) to enable the weights to be placed at such a height above the pivots as to introduce a moment sufficient to neutralize the gravity moment of the model.

A useful feature of the rear balance is the mounting of the whole balance on machined rails along which it may be moved parallel to the axis of the channel by means of a screw. This enables the rear balance to be adjusted until the suspension wires are exactly vertical, as is indicated by a zero reading on the drag balance.

SUSPENSION OF THE MODELS

The model under test was supported on a wire V attached to one of the forward plugs, and a single vertical wire attached to one of the rear plugs. Steel piano wire, 0.011 inches in diameter, was used. The immediate attachment to the model was by means of small loops of wire soldered in the plugs with their planes in the wind direction, as shown at C, fig. 6. The long tailsting supplied with the model was used for transferring the drag to the balance (shown next short model in fig. 1). This sting is 0.125 inches in diameter and about 11½″ long. The vertical wire was employed to relieve the sting and balance of the weight of the model.

The upper ends of the suspension wires were attached to adjustable blocks by means of which the model could be accurately aligned in pitch and yaw.

The suspension was found quite stable. With a single V suspension attached to a plug in the middle row, the C.G. of the model was too close to the point of suspension.

It will be apparent that the suspension used interferes very little with the flow of air about the model and offers no obstruction whatever upstream from the model.

CALIBRATION OF APPARATUS

(*a*) Balance weights. The balance weights were checked against known weights.

(*b*) Balance. The leverage ratio of the balance was verified by applying known forces to the balance through a sensitive bell crank and sting, as shown in fig. 2.

(*c*) Micromanometer. The precision of the micromanometer* depends only on the accuracy of the thread of the screw. The pitch of the thread of each screw was checked by micrometer. The thread at the lower end of the screw was found to be slightly worn, but as the maximum error thereby introduced in the velocity measurement was less than ¼ of 1%, the error in pitch was neglected.

(*d*) Air Speed. The calibration of the side plate and modified Chattock micromanometer used for the measurement of air speed was based on a velocity traverse of the tunnel cross-section with a standard Pitot-static tube. The speed was held constant by manual control of the electric rheostats, using the side plate connected to a modified Chattock gauge as the speed indicator. The traverse was made with an exploring Pitot tube connected to a slanting alcohol manometer. The air velocity was measured at points three inches apart vertically and horizontally throughout the tunnel cross-section in the region occupied by the model. The average velocity through a 30″ square in the centre of the tunnel cross-section was taken as the nominal air speed corresponding to the indication on the manometer connected to the side plate.

The foregoing calibration is based on standard air, namely, air at 60° F. and 29.921 inches of mercury. Usually the conditions of the test differ from standard, and hence the true air speed differs from the "nominal" based on standard conditions. If the conditions are such that the air density is less than standard, an air speed higher than the nominal will be required to exert the same dynamic pressure. Thus, the true air speed in the tunnel is given by:

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* Described in Aero. Research Paper No. 16, Bulletin No. 7, 1927. School of Engineering Research, University of Toronto.

$$v = \left(\frac{\rho_0}{\rho}\right)^{\frac{1}{2}} v_0$$

where:

ρ_0 is density of standard air—.002373 slugs/cu. ft.

ρ is actual air density during the test

v_0 is the velocity of standard air necessary to cause the observed manometer reading.

While the presence of the model in the tunnel reduces the cross-sectional area for the flow of the air, and therefore increases the general average air velocity, in these tests about 6/10%, it is not customary to allow for this in tests of models.

Tests have shown that for models of the shape and size of the airship models, the increase in velocity due to the presence of the model is concentrated about the model and does not extend more than 4 or 5 diameters from the model in the region of the maximum diameter. In the region immediately behind the tail the influence extends about twice as far from the model, but is of smaller magnitude.

It is doubtful, therefore, whether increasing the speed in the ratio of the whole cross-sectional area of the channel to the unobstructed area with the model in place would be justifiable.

The effect of reduction of area due to the model in the velocity has been regarded, in this report, as part of the general influence of the tunnel on the measured drag, which the international trials are designed to study.

TESTS AND OBSERVATIONS

The drag in pounds of each model at zero pitch and yaw was measured at nominal air speeds of 20, 30, 40, 50, 60, 70 and 80 feet per second. The readings from these tests are tabulated in tables 3(*a*) and 4(*a*). In addition, tests at the same speeds were made to determine wire drag, sting drag and drag of sensitivity weights, as described later in this report.

The observations tabulated in this report are the averages of a number of individual readings. For air speeds of 40 f.p.s. or over, the extreme variation among the drag readings was 1¼%, and the greatest variation from the average 7/10%. The average variation among the readings was considerably less than 1% and the mean departure from the tabulated averages less than ½%.

Observations of air temperature and barometer were made at intervals throughout the tests and average figures used on the computations.

CORRECTIONS APPLIED

(*a*) *Drag of Suspension Wires.* To determine the drag of the suspension wires, a duplicate system of wires was arranged below the model (see figs. 3 and 4), and the drag measured. By comparison of the readings with and without the dummy wires, the resistance of the latter was found. The results of the tests with the dummy wires are tabulated in tables 3(*b*) and 4(*b*), and the wire drag found therefrom in column 4 of tables 3(*c*) and 4(*c*).

(*b*) *Drag of Sting.* The effect of the sting and any air flow through the guard on the resistance measurements on the model was ascertained by means of tests, using a dummy sting. A new tailpiece was made for the model, having an axial hole in it slightly larger in diameter than the sting. The model, fitted with this tailpiece, was suspended on wires as in the former tests, but steadied against longitudinal motion by an upstream inclined wire from the nose to the roof, and against lateral motion by two horizontal wires extending from side plugs in the model to the tunnel walls. A dummy sting was attached to the drag balance and inserted in the hole in the tailpiece. The forward end of the sting was supported so as to sway freely in the tailpiece on a long vertical wire extending from a knife edge at the tunnel roof through a cylindrical guard. The lower portion of the latter was reduced to fine streamline form. About one inch of wire 0.005 inches in diameter was left exposed between the end of the guard and the sting (see figs. 5 and D fig. 6). The fine wire was attached to the sting by passing it through a small vertical hole drilled through the sting and bending the end of the wire over parallel to the sting axis.

Observations were then made of the drag due to sting, balance arm and guard, and are tabulated in column 5, tables 3(*c*) and 4(*c*). The drag was found to be quite small and negative.

(*c*) *Drag due to Pressure Gradient in Tunnel.* (*Horizontal Buoyancy.*) The fall in pressure along the tunnel axis was found from the pressure differences between the static openings of a standard Pitot-static tube and the side plate of the tunnel, as indicated on a modified Chattock micromanometer. The pressure gradients are shown in *Aeronautical Research Paper No. 16*, plate 2. While the pressure variation was not uniform, it may be assumed so without appreciable error, within the region occupied by the airship models. The variations in pressure, taken from *Paper No. 16*, are tabulated in Table 2.

Where the pressure gradient in the tunnel is uniform, the drag resulting therefrom, on a body of the form here being considered, is expressed simply by the formula:

$$R = a\ V\ *$$

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* R & M 564 part 1 Report for 1918-19 vol. 1, p. 212 N.A.C.A. Technical Note No. 25—1920.

where R is the drag in pounds due to the pressure drop.

a is the pressure gradient in pounds per square foot per foot.

V is the volume of the model in cubic feet.

The computed drags due to horizontal buoyancy in the tunnel, for the two models, are tabulated in Table 2 and in column 7, Tables 3(*c*) and 4 (*c*)

(*d*) *Deflection of Drag Arm of Balance.* The deflection of the vertical arm of the drag balance** under the drag of the model allows the suspension wires to sway from the vertical position and the weight of the model then introduces an up-wind force. The magnitude of this force is given by the equation:

$$F = \frac{W d}{l}$$

where F is the upwind force in pounds.

W is the weight of the model in pounds.

d is the deflection of the drag arm at the sting attachment in inches.

l is the vertical distance between the fixed end and attachment to the model of the supporting wires, in these tests 18″.

The deflection of the drag arm was determined from tests made, using the apparatus shown in fig. 2. Known forces were applied through a sting to the drag arm, using a bell crank mounted on cone pivots, and the deflections measured with a dial micrometer. Sliding weights on the bell crank arms permitted it to be balanced, and its centre of gravity to be adjusted as near as desired to the axis of the pivots.

From the observations so made, a straight line deflection curve was plotted and the equation connecting force and deflection determined.

The deflections and corresponding upwind forces on the two models are tabulated in Tables 3*a*, 4*a*, 3*b* and 4*b*.

(*e*) *Drag of Sensitivity Weights.* The return flow of the air past the sensitivity weights, because of their size and position relative to the pivots, caused a turning moment on the drag balance such as to increase the observed reading of resistance. The increase in the reading due to this cause was determined by disconnecting the sting from the balance, leaving the model in place, and suspending from the drag arm sufficient weights to establish stability. The latter weights were suspended from the arm by wire and hung below the channel floor. The arm and wire were enclosed in a cylindrical guard for these tests, as shown in fig. 6. Readings were then taken of the drag of the weights at the different air speeds, and are tabulated in column 6, Tables 3*c* and 4*c*. The drag of

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** This arm has since been strengthened.

the weights for the long model are greater than for the short model because the former, being heavier, more weights were required, and their resistance was therefore greater.

(*f*) *Relative Movement of Tunnel and Balance.* No correction for this was necessary, since the roof balances are bolted directly to the steel framework of the tunnel.

(*g*) *Inclination of Suspension Wires to the Vertical.* The wires in each case were vertical and no correction was necessary. The drag balance is mounted on machined rails and can be adjusted fore and aft along the channel by means of a screw. Before each test the balance was adjusted until the reading was zero, the wires then being vertical.

(*h*) *Variation of Direction of Air Stream from Tunnel Axis.* The direction of the air stream is known to be very nearly parallel to the channel axis, and, as the effect of a small angle of yaw or pitch on the drag of an airship model is known to be slight, no special determination of the airstream direction was made.

NET DRAG OF MODELS

The net drag of the models, after applying all the corrections, are tabulated in column 8, Tables 3*c* and 4*c*.

COMPUTATIONS AND RESULTS

The results of the tests have been presented in the form of the usual British non-dimensional coefficient as employed in the equation for the drag.

$$D = C \rho l^2 v^2$$

and

$$C = \frac{D}{\rho l^2 v^2}$$

where R—net drag in pounds after applying all corrections.

C—non-dimensional resistance coefficient expressing resistance per unit volume.

ρ—mass density of air under standard conditions, namely, .002373 slugs per cubic foot.

l—(volume)$^{1/3}$ in feet.

v—nominal air speed in feet per second.

It is well known* that with the method of velocity measurement employed, the data in the drag equation are for standard air, hence the standard air density and nominal air speed are used

The calculated resistance coefficients are tabulated in Tables 3*d* and 4*d*.

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*R. & M. 68. Report for 1912-13, p. 60.

To facilitate comparison with the results of tests in other laboratories, values of *vl* and of Reynolds' Number have been tabulated in Tables 3*d* and 4*d*. In both cases the value of *l* used is the cube root of the volume of the model. The air speed and kinematic viscosity used in the calculation of Reynolds' Number:

$$\frac{vl}{\nu} = \frac{\rho vl}{\mu}$$

are the values corresponding to the actual condition of the air during the tests.

Under standard conditions, 60°F. and 29.921 inches, the density of air is .002373 slugs per cu. ft., and the viscosity 3.7324×10^{-7} slugs/(ft.×secs.) from which the kinematic viscosity is 1.5728×10^{-4} sq. ft./secs. From graphs based on these figures the density and viscosity corresponding to the observed temperature and pressure during the test were determined, and are given in Tables 3 and 4. These values were in turn used in the calculation of Reynolds' Number.

PRESENTATION OF RESULTS

The results of the tests are presented in graphical form in figs. 7-12.

The resistances of the models under different conditions of support are plotted in figs. 7, 8 and 9, together with the corrections applied for drag-arm deflection, wire and sensitivity weight drag and drag due to pressure gradient.

In figs. 7 and 8, the graphs are plotted on nominal air speed and show clearly the relative magnitudes of the corrections (that due to sting was too small to plot).

The graphs in fig. 9 are plotted, using logarithmic scales, and show the manner in which the resistances vary with speed.

The resistance coefficients for the two models are plotted in figs. 10 and 11, in the former on *vl*, and in the latter on Reynolds' Number. These diagrams permit the results to be compared with those from other laboratories.

In fig. 12 the resistance coefficients are compared with the published coefficients for similar models under like conditions.

DISCUSSION OF RESULTS

The drawing of general conclusions must await the collection and analysis of the results of the tests in all the participating laboratories, and discussion of the present results at this time is therefore unnecessary. There are, however, certain features of the results here presented to which attention should be directed.

1. The accuracy of the observations at speeds below 40 f.p.s. is low, due to the small absolute magnitude of the forces measured.

2. Even with the simple wire suspension employed, the drag of the suspension is more than 50% of the drag of the model. In fact, for the short model, at low speeds, the drag of the wires almost equals that of the model. In other words, the drag of 4.5 ft. of drawn steel wire .011 inch in diameter, presenting a total area of 0.594 sq. in. to the wind, has more than half the resistance of the stream-line model with a cross-sectional area of about 13.86 sq. ins.

3. The effects of drag-arm deflection, pressure gradient and drag of sensitivity weights are seen to be quite small. The sting effect is negligible.

4. In general, all the resistances for speeds of 40 f.p.s. or over, may be expressed by an equation of the form $D=aV^n$, as is apparent from the logarithmic plottings of fig. 9.

The index n varies for the different elements. For the bare short model the index is about 2.4, and for the long model, roughly, 2.1. The index for the wires averages 1.75, for pressure gradient 1.98, and for sensitivity weights 2.20. For the models with the wires the index lies between the values for the model and those for the wires, being just under 2 for the models with dummy wires, and just over 2 for the models with a single set of wires.

5. The range of *vl* or Reynolds' Number available in this tunnel and covered in these tests apparently lies in a region of critical flow for these models, particularly for the short model. This is clear from the graphs of figs. 10 and 11, and also from those for the model alone in fig. 9.

The prediction of full scale drag of an airship from model tests under such conditions would be very hazardous.

6. The influence of the addition of the cylindrical mid-portion in the long model on the air flow and resistance is very marked.

7. The graph of resistance coefficient for the long model, compared with those for the L-33 and R-33 models in fig. 12, indicates large effects on the resistance due to flow texture. However, the graphs all appear to be tending to the same resistance at the higher values of *vl*.

A.R.P. No. 21. TABLE NO. 1

INTERNATIONAL TRIALS

AIRSHIP MODELS

DIMENSIONS OF MODELS

(Dimensions in Inches)

Long Model				Short Model			
Distance from Nose	Diameter	Distance from Nose	Diameter	Distance from Nose	Diameter	Distance from Nose	Diameter
0.0	0.000	18.0	4.194	0.0	0.000	11.0	4.192
0.5	1.318	19.0	4.170	0.5	1.290	12.0	4.183
1.0	1.862	20.0	4.131	1.0	1.852	13.0	4.149
1.5	2.268	21.0	4.045	1.5	2.262	14.0	4.101
2.0	2.574	22.0	3.934	2.0	2.580	15.0	4.014
2.5	2.852	23.0	3.784	2.5	2.854	16.0	3.892
3.0	3.074	24.0	3.602	3.0	3.076	17.0	3.722
3.5	3.271	25.0	3.402	3.5	3.270	18.0	3.532
4.0	3.437	26.0	3.168	4.0	3.438	19.0	3.324
4.5	3.583	27.0	2.936	4.5	3.584	20.0	3.098
5.0	3.712	28.0	2.664	5.0	3.710	21.0	2.843
6.0	3.916	29.0	2.352	6.0	3.916	22.0	2.561
7.0	4.063	30.0	2.012	7.0	4.058	23.0	2.240
8.0	4.147	31.0	1.632	8.0	4.148	24.0	1.885
9.0	4.186	32.0	1.208	9.0	4.186	25.0	1.501
10.0	4.200	33.0	0.743	10.0	4.196	26.0	1.068
11.0	4.200	34.0	0.195	....	.....	27.0	0.592
12.0	4.200	34.28	0.000	....	.....	27.97	0.000
13.0	4.200	....	.....	....	.....	....	.....
14.0	4.201	....	.....	....	.....	....	.....
15.0	4.200	....	.....	....	.....	....	.....
16.0	4.200	....	.....	....	.....	....	.....
17.0	4.200	....	.....	....	.....	....	.....

Length........34.280 inches
Volume....... 0.18906 cu. ft.
$(\text{Volume})^{2/3}$.... 0.32939 sq. ft.
$(\text{Volume})^{1/3}$.... 0.57393 ft.
Weight........ 8.992 lbs.
C.G...........15.15 inches from nose.

Length........27.970 inches
Volume....... 0.13812 cu. ft.
$(\text{Volume})^{2/3}$.... 0.26720 sq. ft.
$(\text{Volume})^{1/3}$.... 0.51691 ft.
Weight........ 6.25 lbs.
C.G...........12.08 inches from nose.

A.R.P. No. 21. TABLE NO. 2

INTERNATIONAL TRIALS

AIRSHIP MODELS

DRAG DUE TO PRESSURE GRADIENT IN TUNNEL

Air Speed Nominal f.p.s.	Pressure Drop		Drag on Model	
	Ins. water/foot	Pds./sq.ft./ft.*	Long Model vol. =0.18906	Short Model vol. =0.13812
20	.0005	.0026	.00049	.00036
30	.0010	.0052	.00098	.00072
40	.0020	.0104	.00197	.00144
50	.0033	.0172	.00324	.00237
60	.0045	.0234	.00442	.00323
70	.0061	.0318	.00602	.00440
80	.0080	.0416	.00786	.00574

*Twelve inches of water column = 62.36757 pds./sq. ft.

A.R.P. No. 21. TABLE NO. 3

INTERNATIONAL MODELS

AIRSHIP MODELS

SHORT MODEL

Air temperature—82°F.
Air density—.002251 slugs/cu. ft.
Barometer—29.58 inches.
Air viscosity—3.859×10^{-7} slugs/(ft. secs.)
Kinematic viscosity of air—1.714×10^{-4} sq. ft./sec.

(*a*) DRAG OF MODEL AND SUPPORTS

Air Speed		Observed Drag (av.)	Drag Arm Deflection	Deflection Correction	Net Drag
Nominal	True				
20	20.52	.00493	.0005	.00018	.00511
30	30.78	.00978	.0010	.00036	.01014
40	41.04	.01723	.0018	.00063	.01786
50	51.30	.02725	.0029	.00101	.02826
60	61.56	.03987	.0042	.00147	.04134
70	71.82	.05520	.0059	.00204	.05724
80	82.08	.07250	.0077	.00267	.07517
(*b*) DRAG OF MODEL WITH DUMMY WIRES					
20	20.52	.00662	.0007	.00024	.00686
30	30.78	.01395	.0015	.00051	.01446
40	41.04	.02450	.0026	.00090	.02540
50	51.30	.03837	.0041	.00141	.03978
60	61.56	.05479	.0058	.00202	.05681
70	71.82	.07450	.0079	.00274	.07724
80	82.08	.09690	.0103	.00357	.10047

(*c*) Drag of Model

Air Speed		Drag					Net Drag of Model
Nominal	True	Observed	Wire	Sting	Weights	Press. Grad.	
20	20.52	.00511	.00175	−.00000	.00010	.00036	.00290
30	30.78	.01014	.00432	−.00006	.00020	.00072	.00496
40	41.04	.01786	.00754	−.00007	.00050	.00144	.00845
50	51.30	.02826	.01152	−.00008	.00080	.00237	.01365
60	61.56	.04134	.01547	−.00017	.00130	.00323	.02151
70	71.82	.05724	.02000	−.00030	.00160	.00440	.03154
80	82.08	.07517	.02530	−.00032	.00220	.00574	.04225

(*d*) Coefficients

Air Speed		vl * $l=0.51691$	Reynolds' Number	Resistance Coefficient
Nominal	True			
20	20.52	10.34	61,874	.01139
30	30.78	15.51	92,811	.00867
40	41.04	20.68	123,748	.00833
50	51.30	25.85	154,685	.00860
60	61.56	31.01	185,622	.00942
70	71.82	36.18	216,559	.01015
80	82.08	41.36	247,496	.01041

*Based on nominal speed and $l=(\text{volume})^{1/3}$.

A.R.P. No. 21. TABLE No. 4

INTERNATIONAL TRIALS

AIRSHIP MODELS

LONG MODEL

Air temperature—80°F.
Air density—.002260 slugs/cu. ft.
Barometer—29.58 inches.
Air viscosity—3.847×10^{-7} slugs/(ft. secs.)
Kinematic viscosity of air—1.702×10^{-4} sq. ft./sec.

(*a*) DRAG OF MODEL AND SUPPORTS

Air Speed		Observed Drag (av.)	Drag Arm Deflection	Deflection Correction	Net Drag
Nominal	True				
20	20.48	.00553	.0006	.00029	.00582
30	30.72	.01218	.0013	.00065	.01283
40	40.96	.02258	.0024	.00120	.02378
50	51.20	.03620	.0038	.00192	.03812
60	61.44	.05180	.0055	.00275	.05455
70	71.68	.07012	.0074	.00372	.07384
80	81.92	.09125	.0097	.00484	.09609
(*b*) DRAG OF MODEL WITH DUMMY WIRES					
20	20.48	.00762	.0008	.00040	.00802
30	30.72	.01630	.0017	.00086	.01716
40	40.96	.02958	.0031	.00157	.03115
50	51.20	.04718	.0050	.00251	.04969
60	61.44	.06650	.0071	.00353	.07003
70	71.68	.08918	.0095	.00473	.09391
80	81.92	.11582	.0123	.00614	.12196

(*c*) DRAG OF MODEL

Air Speed		Drag					Net Drag of Model
Nominal	True	Observed	Wire	Sting	Weights	Press. Grad.	
20	20.48	.00582	.00220	−.00005	.00020	.00049	.00298
30	30.72	.01283	.00433	−.00013	.00030	.00098	.00735
40	40.96	.02378	.00737	−.00020	.00060	.00197	.01404
50	51.20	.03812	.01157	−.00028	.00090	.00324	.02269
60	61.44	.05455	.01548	−.00035	.00140	.00442	.03360
70	71.68	.07384	.02007	−.00043	.00180	.00602	.04638
80	81.92	.09609	.02587	−.00045	.00250	.00786	.06031

(*d*) COEFFICIENTS

Air Speed		vl * $l=0.57393$	Reynolds' Number	Resistance Coefficient
Nominal	True			
20	20.48	11.48	69,052	.00953
30	30.72	17.22	103,578	.01043
40	40.96	22.96	138,104	.01122
50	51.20	28.70	172,630	.01161
60	61.44	34.44	207,156	.01194
70	71.68	40.18	241,682	.01211
80	81.92	45.92	276,208	.01206

*Based on nominal speed and $l=(\text{volume})^{1/3}$.

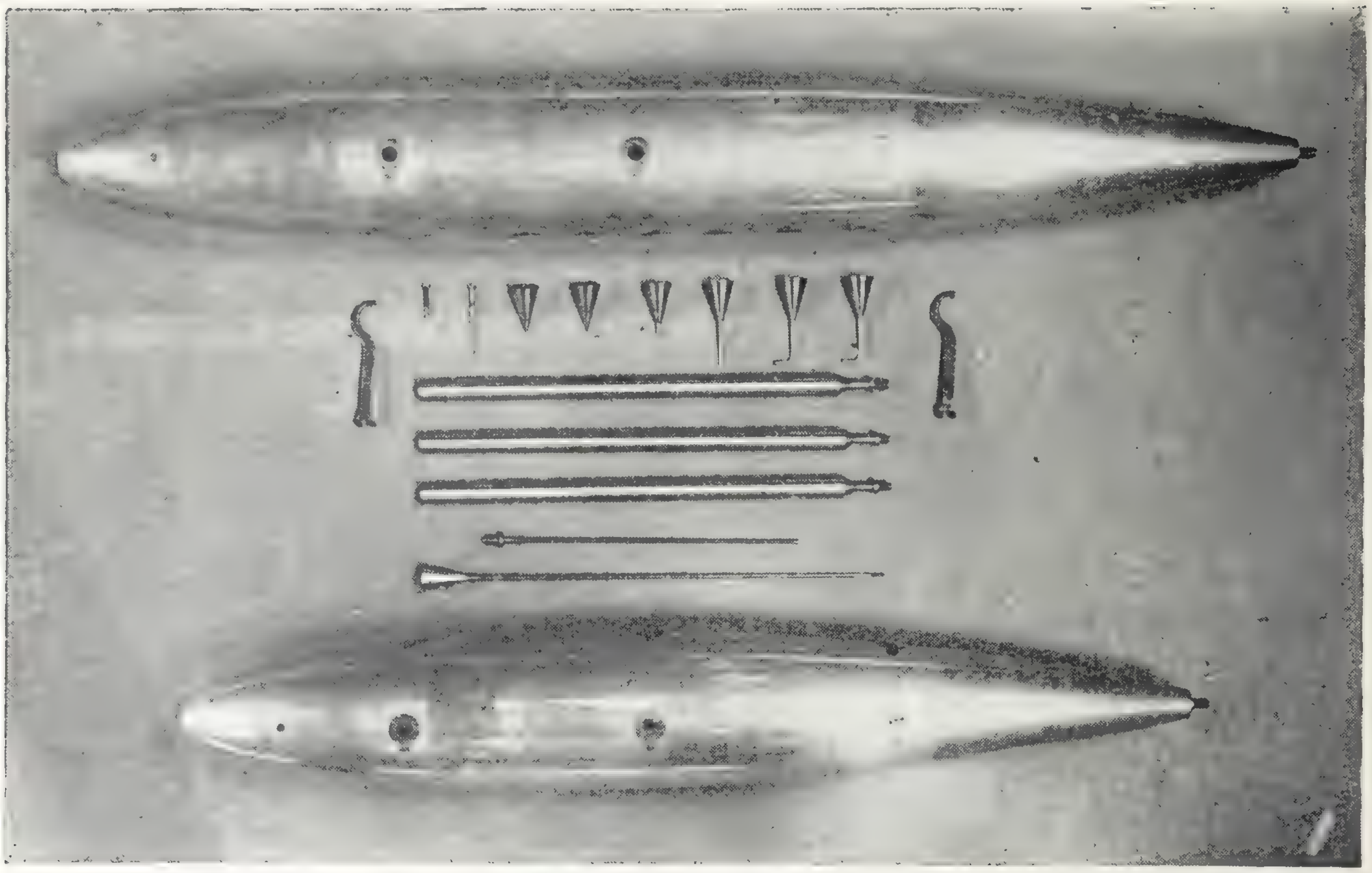

FIG. 1

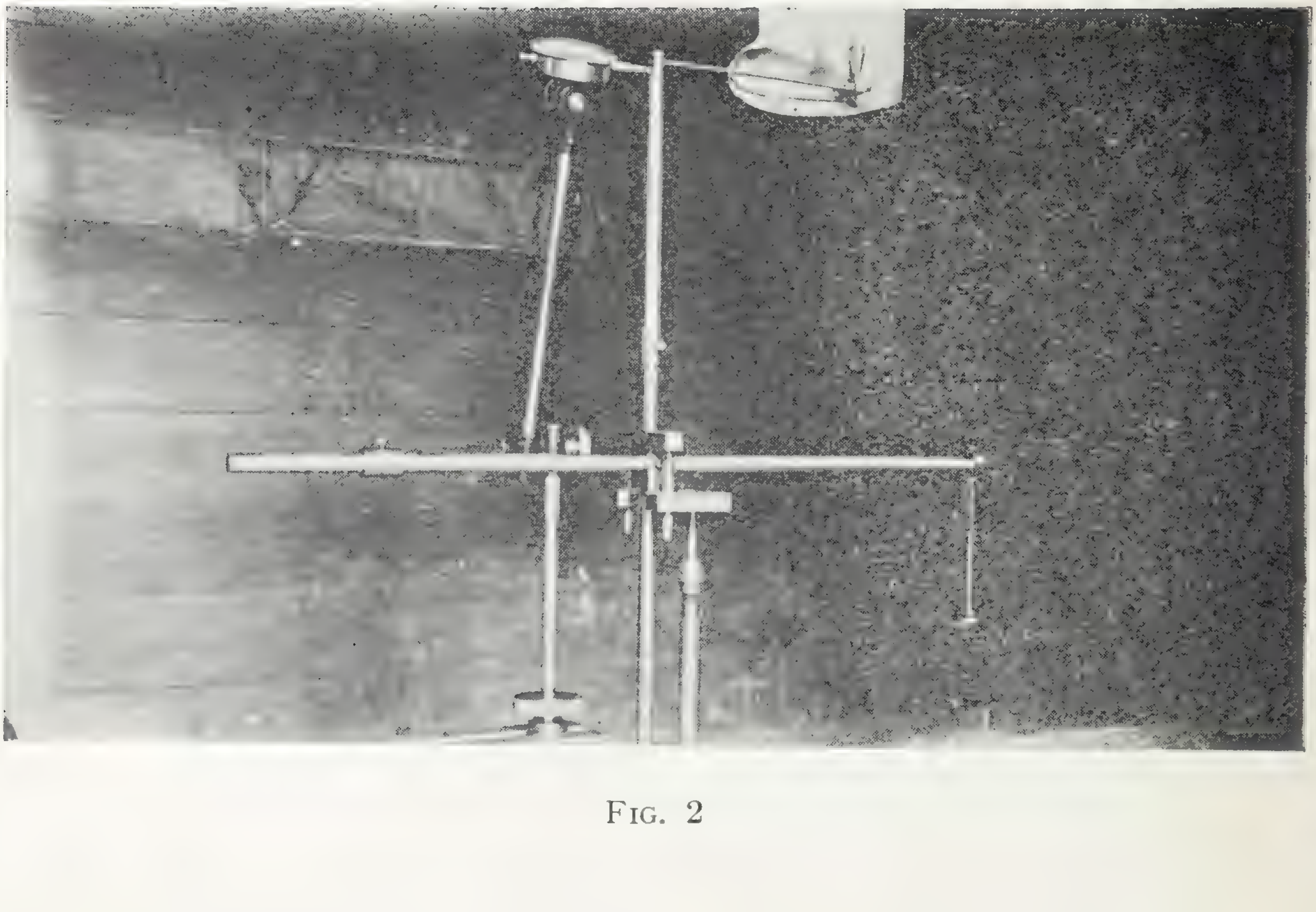

FIG. 2

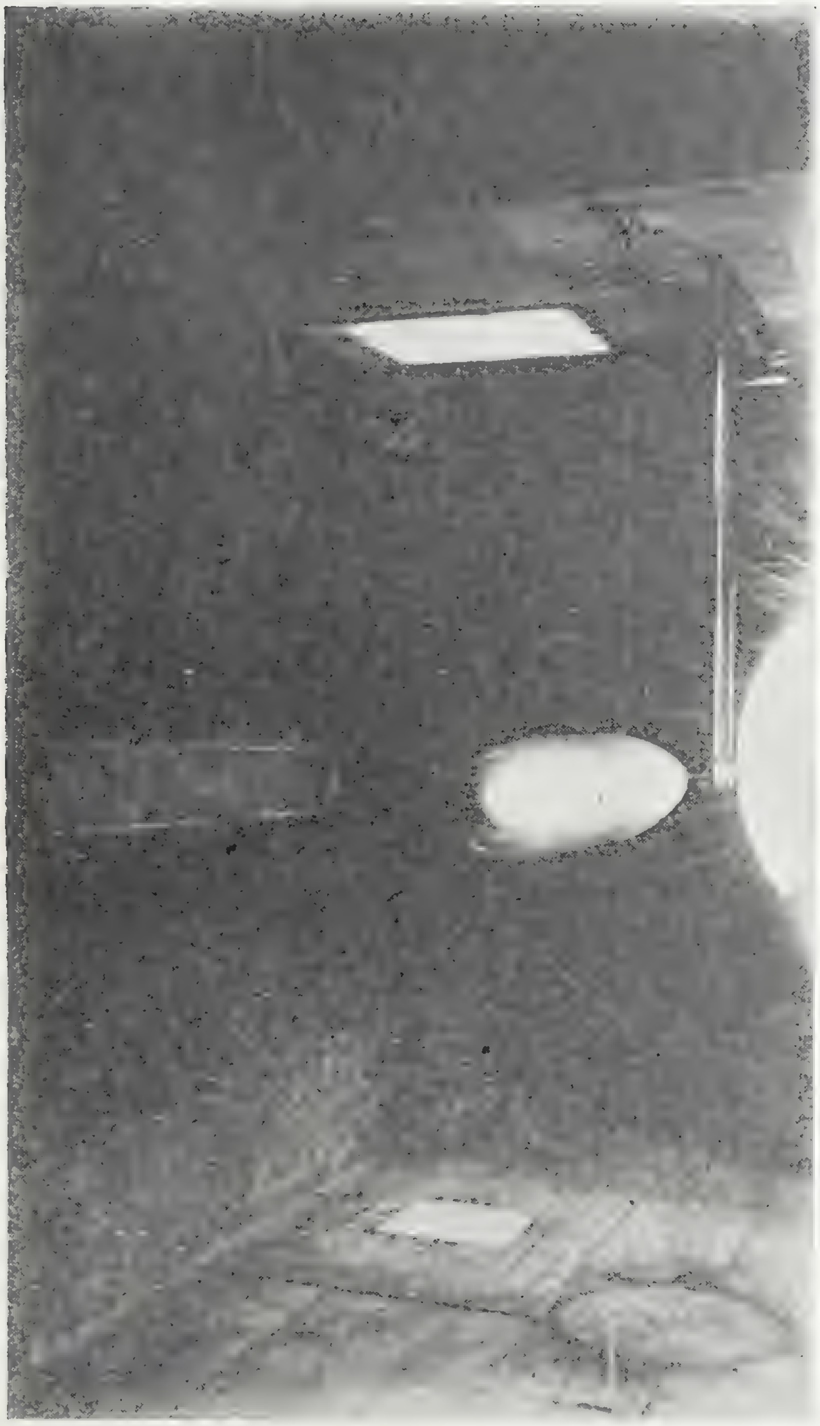

FIG. 3

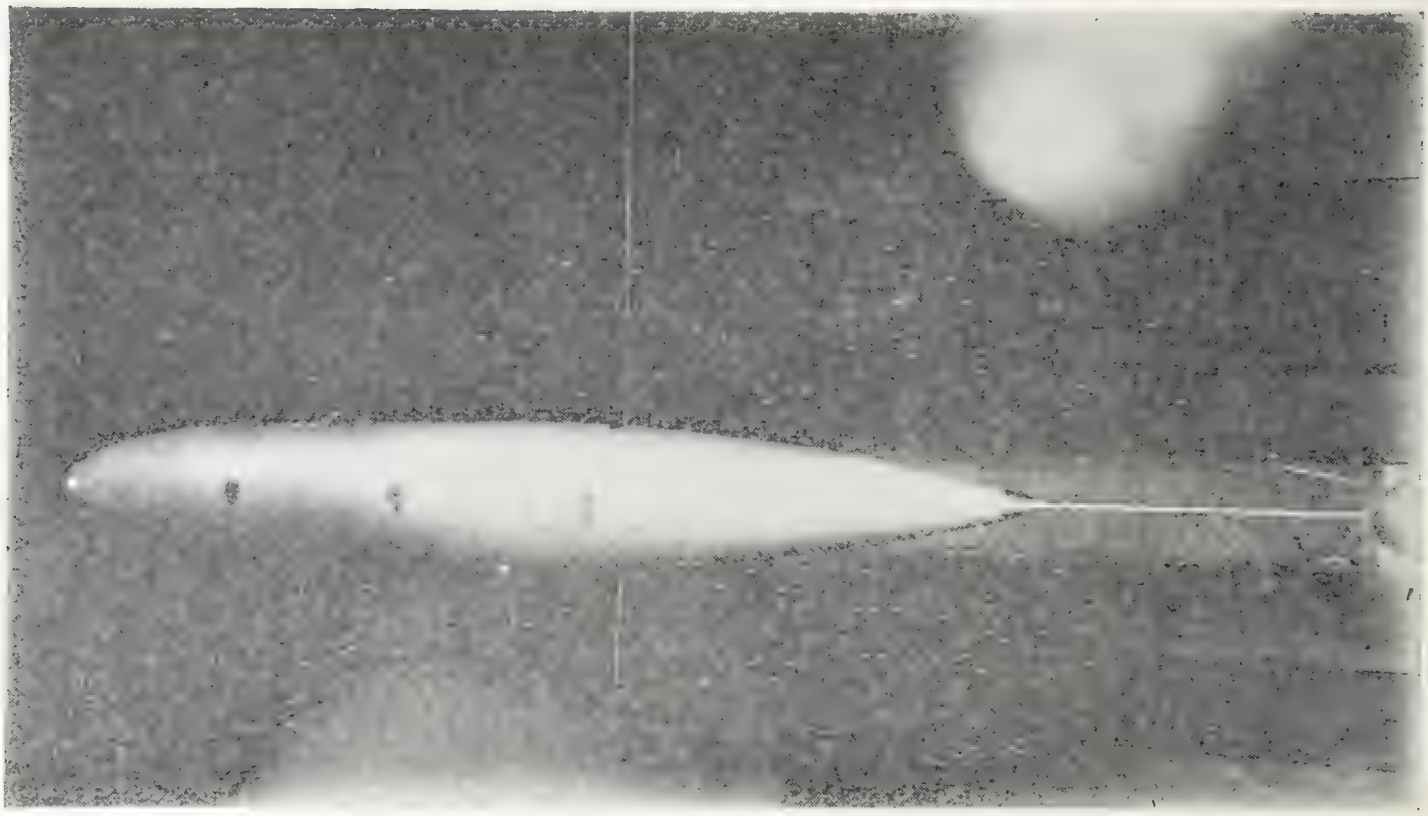

FIG. 4

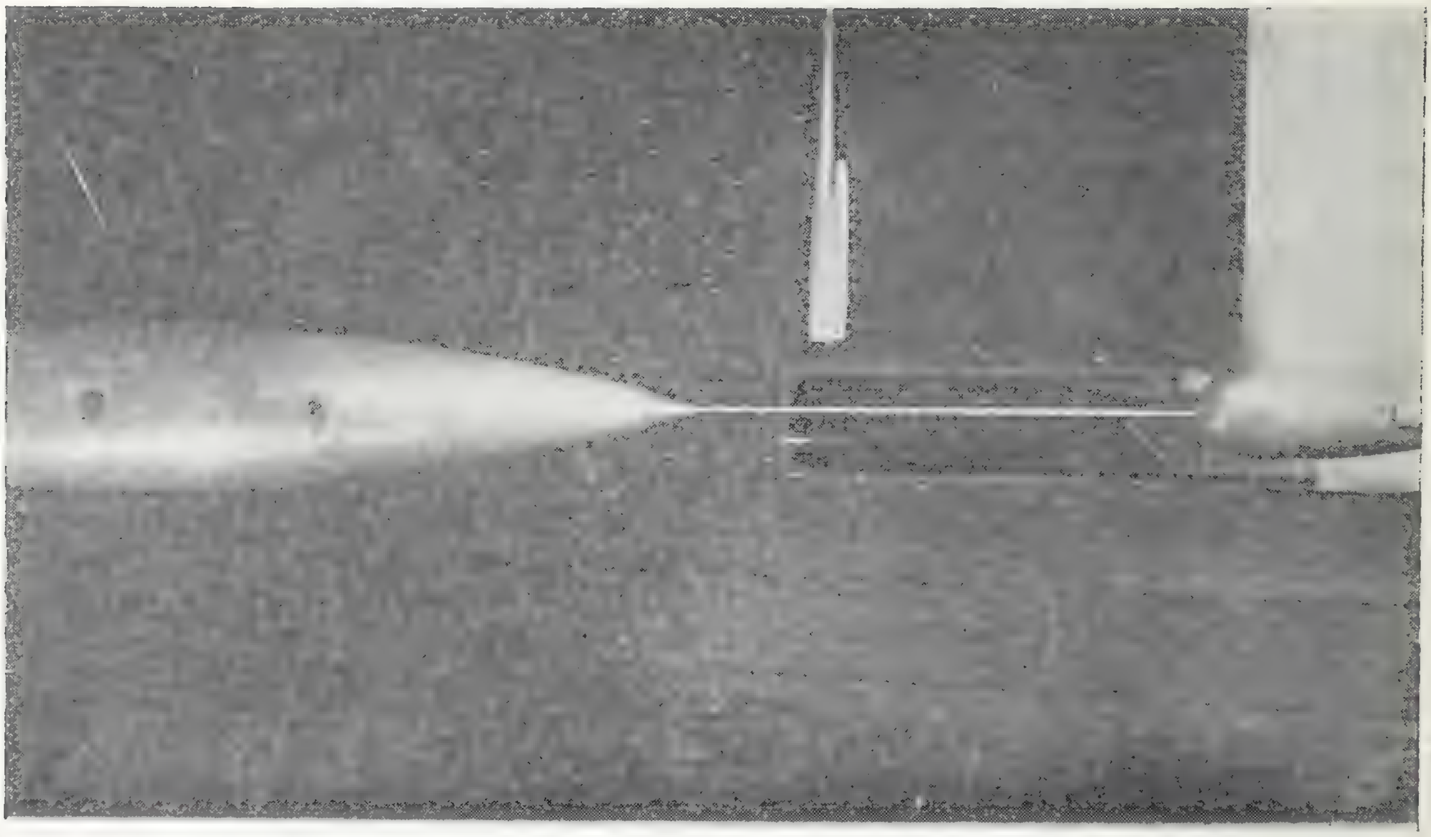

FIG. 5

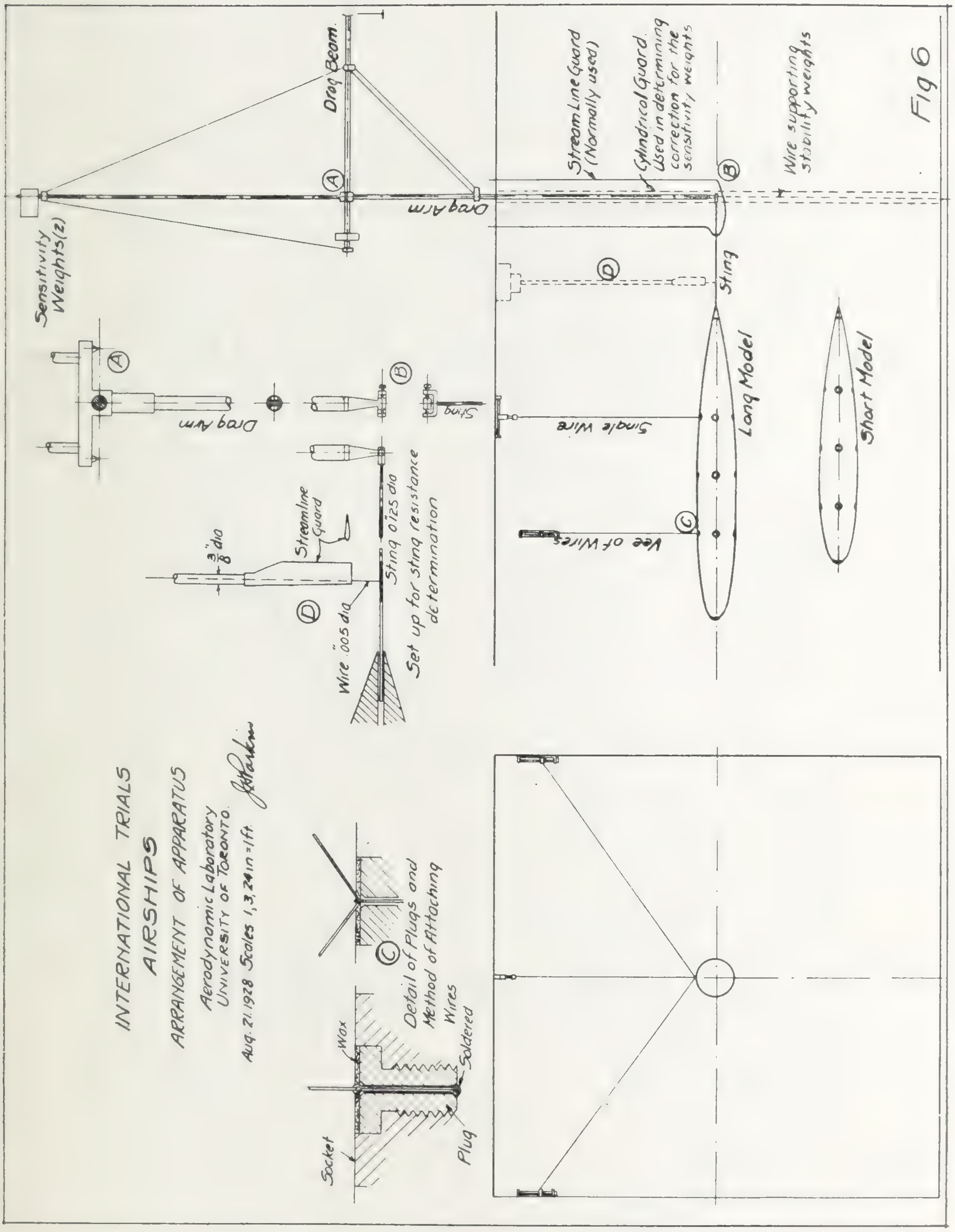
INTERNATIONAL TRIALS
AIRSHIPS
ARRANGEMENT OF APPARATUS
Aerodynamic Laboratory
UNIVERSITY OF TORONTO.
Aug. 21. 1928 Scales 1, 3, 24 in = 1 ft.
Sensitivity Weights (2)
Drag Beam
A
Drag Arm
Stream Line Guard (Normally used)
Cylindrical Guard Used in determining correction for the sensitivity weights
B
Wire supporting stability weights
Fig 6
D
Sting
Long Model
Short Model
Single Wire
C
Vee of Wires
A
Drag Arm
B
Sting
3/8" dia
Streamline Guard
D
Wire .005" dia
Sting 0.125 dia
Set up for sting resistance determination
Socket
Wax
Plug
Soldered
C
Detail of Plugs and Method of Attaching Wires

FIG. 6

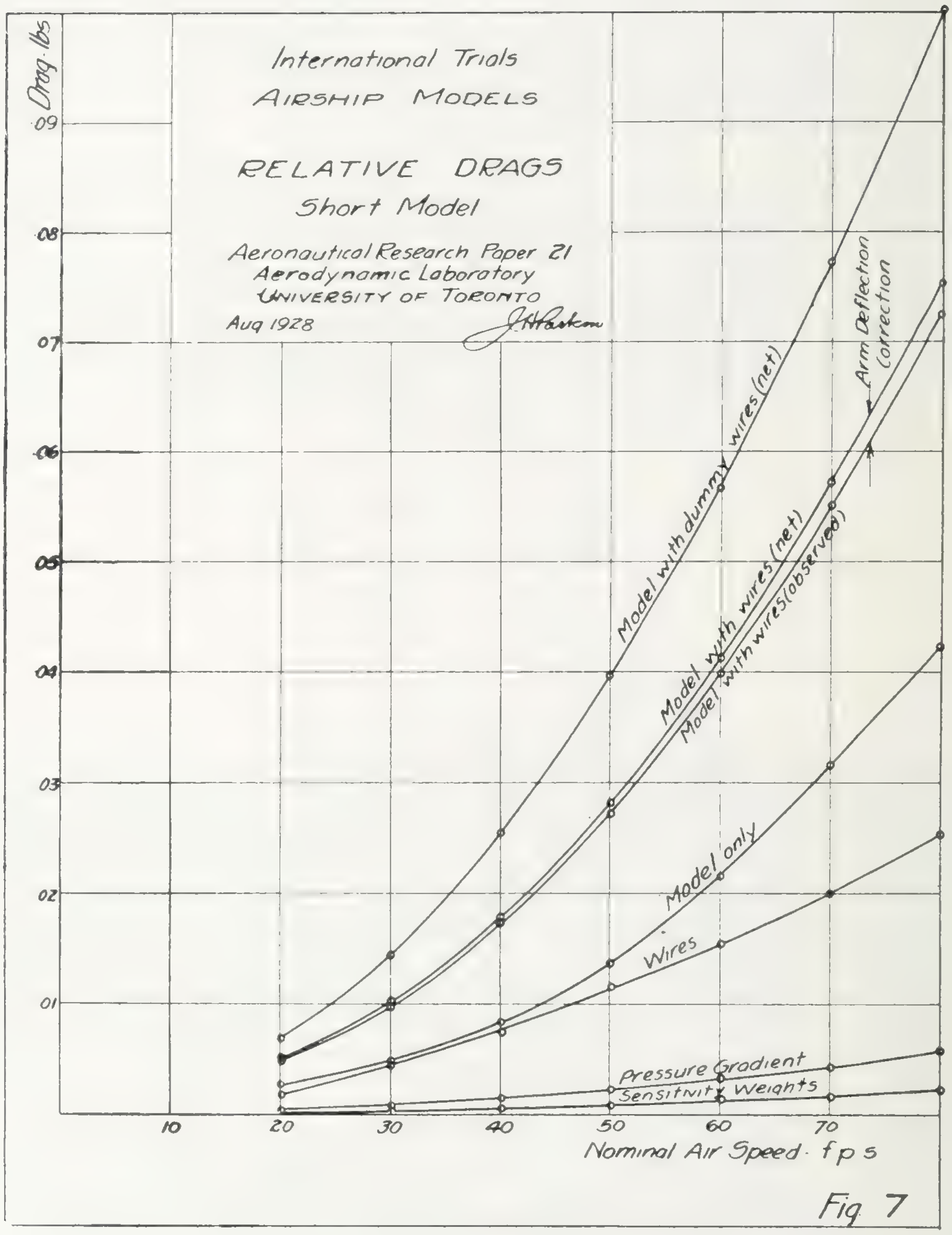
International Trials
AIRSHIP MODELS
RELATIVE DRAGS
Short Model
Aeronautical Research Paper 21
Aerodynamic Laboratory
UNIVERSITY OF TORONTO
Aug 1928
J.H. Parkin
Drag. lbs
.09
.08
.07
.06
.05
.04
.03
.02
.01
10
20
30
40
50
60
70
Nominal Air Speed. f p s
Model with dummy wires (net)
Model with wires (net)
Model with wires (observed)
Arm Deflection Correction
Model only
Wires
Pressure Gradient
Sensitivity Weights
Fig 7

FIG. 7

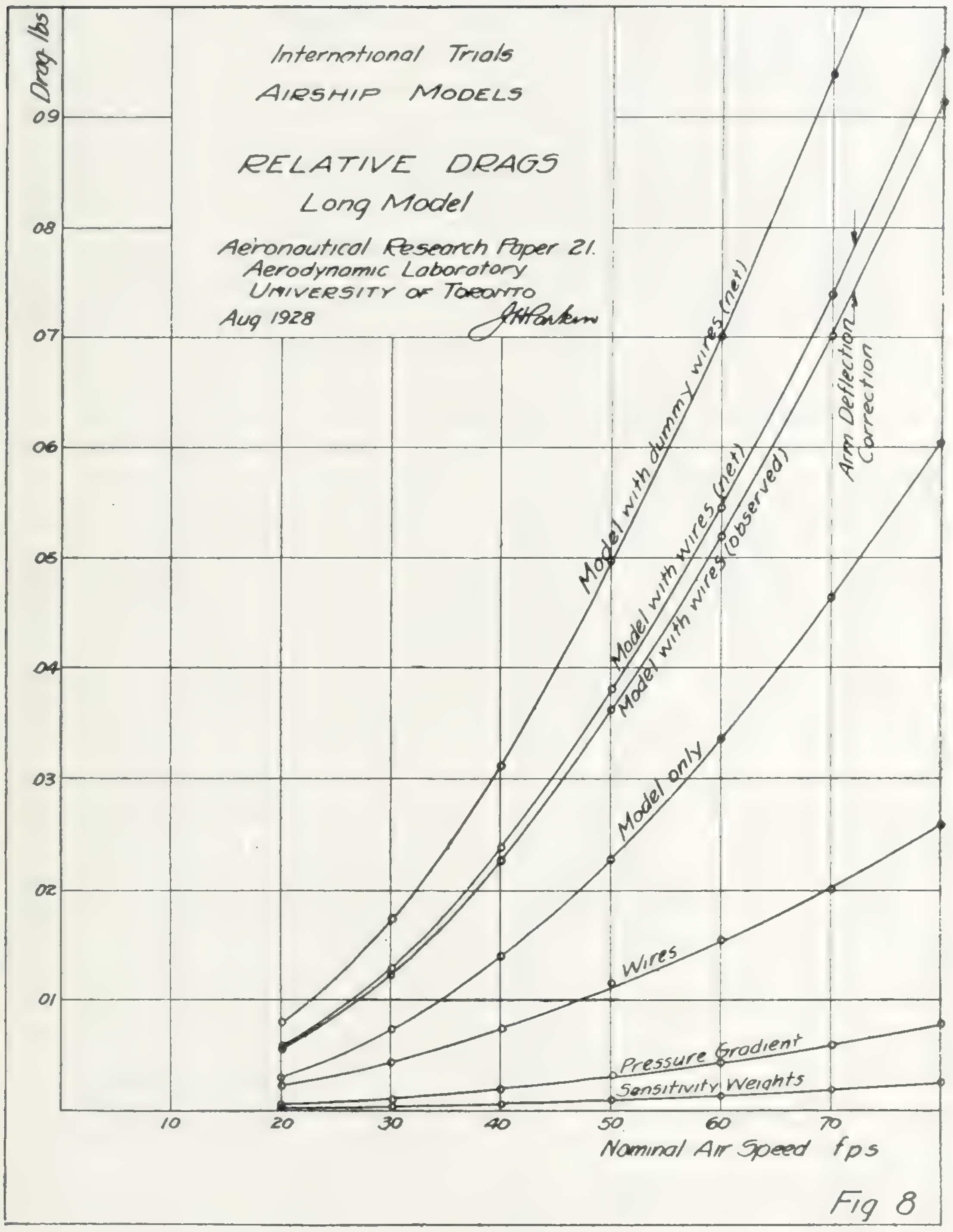
International Trials
AIRSHIP MODELS
RELATIVE DRAGS
Long Model
Aëronautical Research Paper 21.
Aerodynamic Laboratory
UNIVERSITY OF TORONTO
Aug 1928
JHParkin
Drag lbs
.09
.08
.07
.06
.05
.04
.03
.02
.01
10
20
30
40
50
60
70
Nominal Air Speed fps
Model with dummy wires (net)
Model with wires (net)
Model with wires (observed)
Arm Deflection Correction
Model only
Wires
Pressure Gradient
Sensitivity Weights
Fig 8

FIG. 8

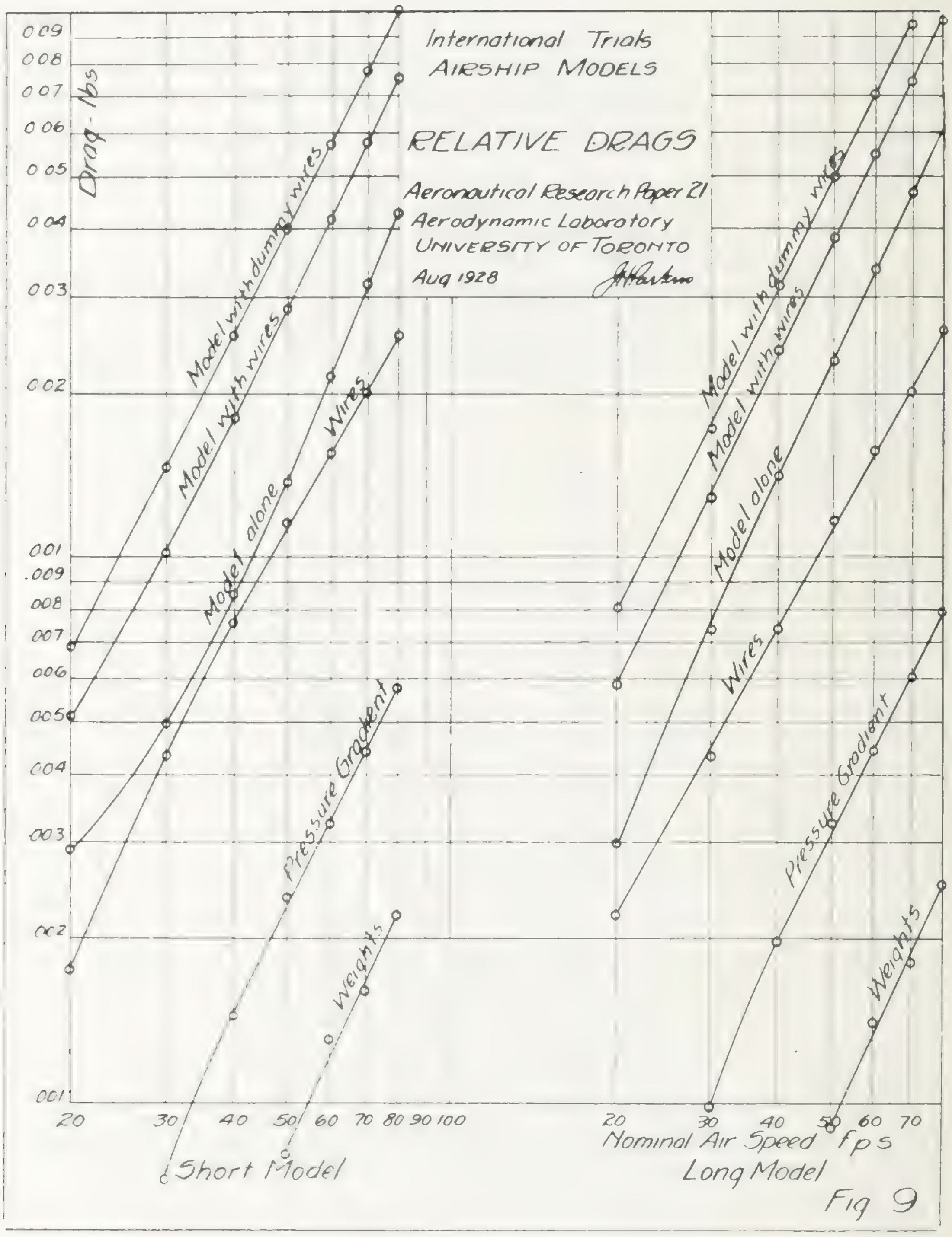
International Trials
AIRSHIP MODELS
RELATIVE DRAGS
Aeronautical Research Paper 21
Aerodynamic Laboratory
UNIVERSITY OF TORONTO
Aug 1928
Drag - lbs
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
.009
.008
.007
.006
.005
.004
.003
.002
.001
Model with dummy wires
Model with wires
Model alone
Wires
Pressure Gradient
Weights
20
30
40
50
60
70
80
90
100
Short Model
Nominal Air Speed f p s
Long Model
Fig 9

FIG. 9

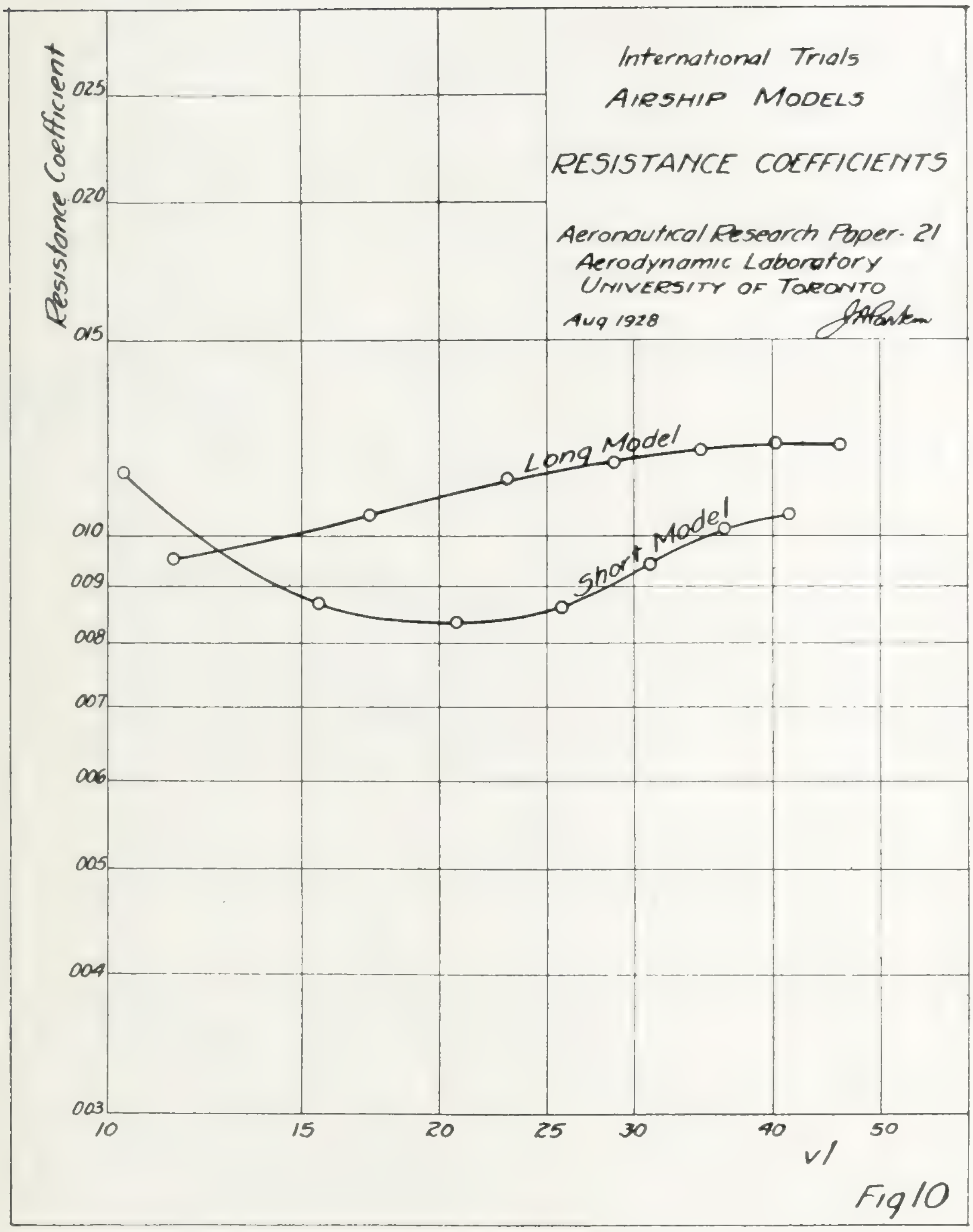
International Trials
AIRSHIP MODELS
RESISTANCE COEFFICIENTS
Aeronautical Research Paper-21
Aerodynamic Laboratory
UNIVERSITY OF TORONTO
Aug 1928
Resistance Coefficient
025
020
015
010
009
008
007
006
005
004
003
10
15
20
25
30
40
50
vl
Long Model
Short Model
Fig 10

FIG. 10

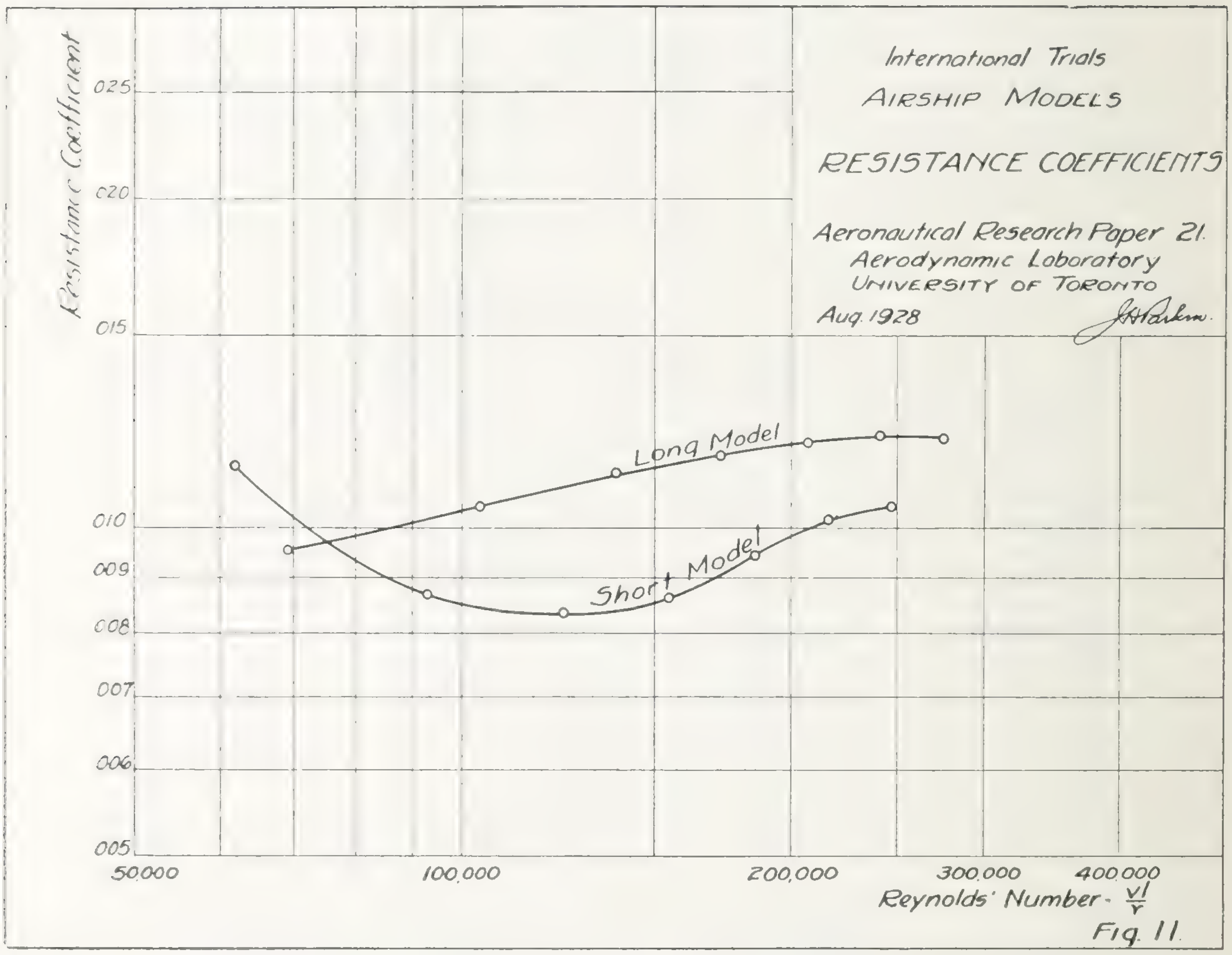
International Trials
AIRSHIP MODELS
RESISTANCE COEFFICIENTS
Aeronautical Research Paper 21.
Aerodynamic Laboratory
UNIVERSITY OF TORONTO
Aug. 1928
J.H. Parkin.
Resistance Coefficient
.025
.020
.015
.010
.009
.008
.007
.006
.005
Long Model
Short Model
50,000
100,000
200,000
300,000
400,000
Reynolds' Number - $\frac{Vl}{\nu}$
Fig. 11.

FIG. 11

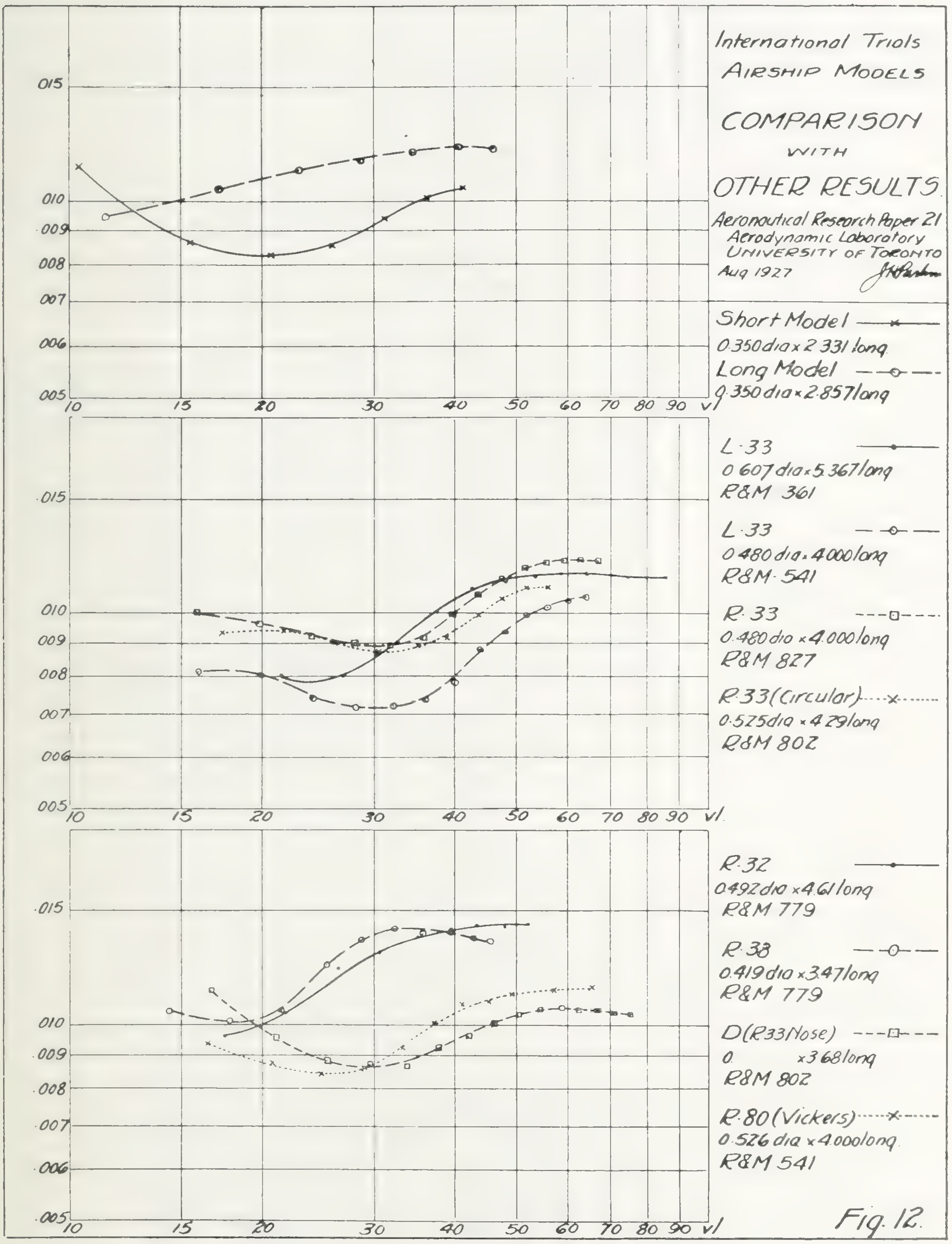
International Trials
Airship Models
COMPARISON
WITH
OTHER RESULTS
Aeronautical Research Paper 21
Aerodynamic Laboratory
University of Toronto
Aug 1927
Short Model
0.350 dia × 2.331 long
Long Model
0.350 dia × 2.857 long
L-33
0.607 dia × 5.367 long
R&M 361
L-33
0.480 dia × 4.000 long
R&M 541
R-33
0.480 dia × 4.000 long
R&M 827
R-33 (circular)
0.575 dia × 4.29 long
R&M 802
R-32
0.492 dia × 4.61 long
R&M 779
R-38
0.419 dia × 3.47 long
R&M 779
D (R33 Nose)
0. × 3.68 long
R&M 802
R-80 (Vickers)
0.526 dia × 4.000 long
R&M 541
.015
.010
.009
.008
.007
.006
.005
10
15
20
30
40
50
60
70
80
90
vl
Fig. 12.

FIG. 12

Aeronautical Research Paper No. 22.
June, 1928.

THE INTERFERENCE BETWEEN THE BODY AND WINGS OF AIRCRAFT

By
J. H. Parkin, M.E., F.R.Ae.S., and G. J. Klein, B.A.Sc., Jun. A.S.M.E,*

Summary

Tests on certain aircraft having disclosed discrepancies between the characteristics estimated from the figures for the individual components and those observed for the complete model, the present investigation was undertaken to determine the nature and magnitude of the effects of interference between the body and wings of aircraft and to ascertain the factors influencing the interference.

The models were made up of standard 3″ × 18″ duralumin aerofoils and fuselages of wood.

The aerofoils employed in the investigation were:

(*a*) RAF-15—a thin section.
(*b*) USA-27—a section of medium thickness.
(*c*) Gottingen-387—a thick section.
The fuselages used were:
(*a*) Fuselage A—of streamline form.
(*b*) Fuselage B—of cabin form.
(*c*) Fuselage C—of usual open cockpit form.

A number of typical monoplane and biplane combinations of wings and fuselages were tested, and in some cases the effect of fairing between wings and fuselage was studied.

The results of the investigation indicate that the interference effects are dependent on the shape of the fuselage, the aerofoil section and the relative position of the fuselage and aerofoil. The better the aerodynamic form of the fuselage, and the thicker the aerofoil section, the greater are the interference effects and the more marked the influence of the relative vertical position of wings and body on the interference.

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*Part of this paper constituted the thesis submitted by Mr. Klein for the degree of B.A.Sc. Mr. Klein was assisted in making the observations at different times by W. H. Coates, B.A.Sc., P. G. Stanley, B.A.Sc., and G. L. Roberts, B.A.Sc.

Interference between wings and body tends, in general, to lower the critical angle and increase the drag of the combination as compared with the individual components. It may increase or decrease the lift.

From an aerodynamic standpoint, the best position for the wing is at the top of the fuselage, and the worst that at the bottom.

Fillets and fairing may improve a combination having poor characteristics, but have little effect if the arrangement already possesses good characteristics.

An explanation of the cause of the observed interference effects is advanced.

INTRODUCTION

There is now available for the aircraft designer a large mass of data secured from the many tests that have been made on models of aircraft components in wind tunnels, and the design of aircraft is based largely on this information.

The experimental data pertaining to aerofoils is very extensive and has been derived practically wholly from tests of models in wind tunnels. The usual wind tunnel test of an aerofoil is made on a model of aspect ratio six and as a monoplane. In certain cases, particularly for aerofoils in common use or employed as standards of comparison, the aerofoil is also tested as a biplane, usually with gap equal to chord, zero stagger and zero decalage. In addition to the characteristics of the biplane as a whole, the characteristics of the individual wings are frequently determined.

Usually the drag coefficient for an aerofoil is found to be very nearly the same when tested as a monoplane, or as the upper or the lower wing of a biplane. The lift coefficient, on the other hand, for the aerofoil as the upper wing of a biplane is about 95% of that of the aerofoil as a monoplane, while the lift coefficient as the lower wing of a biplane is only about 85% of that as a monoplane. These differences are due to interference between the upper and lower wings of the biplane combination.

In aerodynamics, interference is the mutual influence of two or more flow patterns when superimposed.

There is also available much information concerning the aerodynamic properties of fuselages, undercarriages, floats, tail units, struts, wires, etc., derived in a similar way from wind tunnel tests.

With this information it is possible to estimate fairly accurately the aerodynamic forces on each full scale component, when the latter is free from interference due to the presence of other components. In the complete aircraft, however, each component interferes more or less with the flow about the other elements, and this interference is commonly allowed for by the designer by increasing the sum of the drags of the components by from 10-30%, depending on the form and arrangement of the parts. Well streamlined parts, smooth curves and generous fillets are considered to reduce interference. The lift of the complete aircraft is taken to be the sum of the lifts of the components.

The figures estimated in this way are obviously liable to be in error unless the machine under consideration is similar to a previous design on which definite information is available regarding the extent of the interference effects. It is therefore usual to check the estimated figures by means of a wind tunnel test on a complete model of the aircraft.

It is a well known fact that the forces on the complete model frequently differ considerably from those calculated because of the difficulty encountered in estimating correctly the allowances to be made for the effects of interference.

During the summer of 1926, tests were conducted at Toronto on a complete model of a high-speed seaplane. The drag of the model was found to be much greater than anticipated, and upon further experimental investigation it was found that the drag of the wings was greatly increased by the presence of the fuselage.

Within the last few years, the interference effect of various forms of bodies on the characteristics of airscrews has been investigated more or less thoroughly, but data on the interference between bodies, wings and undercarriage is very limited (refer to bibliography). The present investigation was, therefore, undertaken to determine the magnitude of the interference effects between aircraft fuselages and wings, and the factors influencing the interference.

MODELS

The models used were made up of duralumin aerofoils and fuselages of mahogany.

In order that the results might be as generally useful as possible, aerofoil sections of three principal types were employed in the investigation:

(*a*) A thin section—RAF-15.
(*b*) A section of medium thickness—USA-27.
(*c*) A thick section—Gottingen-387.

The aerofoils were of 3″ chord, 18″ span (aspect ratio 6) with square tips, cut on the Nichols generating machine. The aerofoils were tapped at each end 1″ back from the leading edge to receive end spindles. The ordinates of the aerofoil sections are given in Table 3.

The fuselages (see Diagram 1 and Photos 1 and 2) were designed to represent, as far as possible, existing types and present-day practice in fuselage design.

Fuselage A represents the well streamlined form as used in racing aircraft. Its shape is that of RAF 30 aerofoil, with its fineness ratio altered to suit the desired overall dimensions. Its tail ends in a vertical knife edge and all cross-sections are elliptical.

Fuselage B typifies the usual commercial cabin form with a V windshield set out on the nose. The cross-section is circular at the airscrew spinner and changes gradually to rectangular at the leading edge of the wing. All other sections are rectangular.

Fuselage C is of the ordinary open cockpit form with flat nose, straight lines and arched back. The cross-sections are rectangular with semi-circular or semi-elliptical tops.

The number of variables in the different models was limited by making the fuselage models to the same general dimensions, thereby enabling the comparison of models having different fuselages to be more readily made. In all models the distance from the plane of rotation of the airscrew to the leading edge of the aerofoil was made equal to the chord of the aerofoil, and from the airscrew to the tail knife edge 4½ chords. Fuselages A and B were made with rounded noses, representing spinners on the propeller. Fuselage C had a flat nose, as in fuselages fitted with a nose radiator. The depth of fuselages A and B was made equal to the aerofoil chord, and that of fuselage C, 3/4 of the aerofoil chord. Thus, in some biplane models, the fuselage completely filled the gap between the wings, while in others the gap was only partially filled. The width of the fuselage in each case was made equal to 2/3 of the depth.

The fuselages were made with transverse slots fitted with plugs. The aerofoils were passed through the slots and fastened in proper position with wedges or screws, the original fuselage form being restored by filling the spaces with plasticene. For the parasol arrangements, the aerofoil was fastened to the fuselage by screws passing through distance pieces of brass tubing to maintain the proper spacing.

A number of monoplane and biplane combinations of the aerofoils and fuselages, covering as far as possible the usual aircraft arrangements, were tested for each aerofoil section, as follows:—

Monoplanes:

(*a*) The aerofoil at the bottom of fuselages A, B and C.

(*b*) The aerofoil midway between top and bottom of fuselages A, B and C.

(*c*) The aerofoil at the top of fuselages A, B and C.

(*d*) The aerofoil at the top of fuselage C and the fuselage faired to a rectangular cross-section under the wing.

(*e*) A parasol arrangement with fuselage C. The gap between the top of the fuselage and the aerofoil was made ½" or 1/6 of the aerofoil chord.

(*f*) With fuselage A some tests were made with fillets between the aerofoil and fuselage.

Biplanes:

In all biplane arrangements the gap equalled the chord and the stagger and decalage were zero.

With fuselages A and B only one arrangement was tested, that with the fuselage filling the gap between the wings.

The arrangements tested with fuselage C were:

(*a*) Fuselage on upper surface of lower wing.

(*b*) Fuselage midway between the wings.

(*c*) Fuselage on lower surface of upper wing.

(*d*) Fuselage on lower surface of upper wing and faired as in monoplane combination (*d*).

The angle between the geometric chord of the aerofoil and the centre line of the fuselage (thrust line) was made equal to the angle of attack, giving maximum ratio of lift to drag for the aerofoil alone.

APPARATUS AND METHOD OF TEST

The investigation was made in the 4-foot closed RAE type wind tunnel of the University of Toronto. The tunnel is equipped with a spindle balance of NPL type and wire suspension roof balances. The latter are similar in general design to those used on the British 7 × 14 wind tunnel. A detailed description of the tunnel and equipment, with the exception of the wire balances, is given in *Aeronautical Research Paper No. 16*, Bulletin No. 7 of the School of Engineering Research, University of Toronto.

The majority of the tests were made during the latter part of the summer of 1927. The tests with the RAF-15 aerofoil were made early in the summer of 1928.

Two methods of supporting the models were used:

1. For the thick aerofoils, USA-27 and Gottingen-387, end spindles and the NPL balance were used. For the biplanes the form of spindle support was that used in a previous research and described in *Aeronautical Research Paper No. 19.* The balance and spindle were shielded by means of a cylindrical guard. The spindle effects were determined in the usual way by making a second set of measurements, using a dummy spindle and guard.

The fuselages, when tested alone, were supported in a similar way, using long spindles screwed into brass sockets inserted in the model. These different points will be made clear by an examination of the photographs.

2. The RAF-15 aerofoil was so thin that there was danger of an end spindle tearing out of the model; consequently, the tests with this aerofoil were delayed until the completion of the wire suspension type roof balances. The measurements on the models with the RAF-15 aerofoil were then made on these balances. With the roof balances the models were supported in an inverted position by means of two wires attached to the leading edge of the lower wing, and a sting fitted into the rear of the fuselage. The wires extended up to a lift balance, and the sting connected through a "wobbler" to a combination lift and drag balance.

The optical alignment apparatus described in *Aeronautical Research Paper No. 16* was employed in aligning the models supported on spindles. With the wire suspension arrangement the level described on page 10 and shown in Fig. 9 of *Aeronautical Research Paper No. 16*, was used.

Air speed was measured by means of side plate and a micromanometer of the Chattock type, described in *Aeronautical Research Paper No. 16*. The air speed was 40 f.p.s. throughout.

The results have been corrected for spindle or wire interference, but no corrections for channel wall interference have been applied.

CALCULATIONS

The effects of interference are shown by the differences between the forces on the complete model and the sum of the forces on the individual components.

A difficulty was encountered in determining the area of the aerofoil to be used in calculating its lift and drag when attached to the fuselage. With a fuselage attached to an aerofoil, part of one surface of the aerofoil is covered by the fuselage. While it would be possible, using pressure distributions and estimating surface friction, to calculate the lift and drag of an aerofoil under the conditions described, where the areas of the upper and lower surfaces are not equal, it was considered such a tedious procedure was unnecessary for the purposes of the present report.

In this paper, the smaller of the exposed areas of the upper and lower surfaces of the aerofoil when attached to a fuselage is assumed to be the aerofoil area. That is, the assumption is made that when a fuselage is attached to one surface of an aerofoil, the other surface of the aerofoil, opposite that occupied by the fuselage, is fuselage surface and not aerofoil surface.

The lift and drag of each aerofoil was calculated from the British non-dimensional coefficients determined for the same models and given in *Aeronautical Research Paper No. 19.*

The equations employed are:

$$L = K_L \rho S\ V^2$$
$$D = K_L \rho S\ V^2$$

L—lift in pounds.
D—drag in pounds.
S—area in square feet.
V—air speed in feet per second.
ρ—mass density of air (.002373 slugs per cubic foot).
K_L—non-dimensional lift coefficient.
K_D—non-dimensional drag coefficient

The RAF-15 aerofoil is not dealt with in the latter report. Monoplane and biplane measurements were made on this aerofoil, using the wire suspension roof balances to determine the coefficients for the purposes of this report.

The sum of the lifts of the separate components of the complete model is the "calculated" lift of the model, that is, the lift *without* interference; the measured lift on the complete model is the lift *with* interference, and a comparison of these two lifts shows the interference effect in each case. A similar procedure was followed with the drags.

PRESENTATION OF THE RESULTS

The characteristics of the complete models, with and without interference, are expressed in pounds. The area to be used in calculating coefficients presented some difficulty, and it was considered that the results would be quite as useful expressed in pounds as expressed in coefficient form based on some arbitrary area.

Values of lift, drag and lift/drag, as measured on the complete model (with interference) are tabulated in Table 1. The corresponding values calculated from the characteristics of the components (without interference) are listed in Table 2.

The results are presented graphically in two ways, each possessing advantages over the other for certain purposes:

1. The characteristics, lift, drag and lift/drag are plotted on incidence of fuselage (thrust line) in diagrams 2-56.
2. Polar curves of lift plotted against drag are plotted in diagrams 57-65 inclusive.

In the polar curves the characteristics of all the models having the same fuselage and aerofoil may be compared directly with one another, and with the characteristics of the aerofoil alone.

Scales of K_L and K_D, based on nominal aerofoil area, are provided on the polar curves.

The effect of interference is shown by the difference between values as measured on the complete model and as calculated from the components.

The system used to designate the various combinations is as follows:

M—designates a monoplane.

B—designates a biplane.

1, 2, 3, etc., designates the position of the aerofoil or aerofoils, accord-to Diagram 1.

F—designates a fairing, as shown in Diagram 1 or Photo 4.

Thus, for fuselage C and aerofoil RAF-15, B-15-F is a biplane with one aerofoil on top of the fuselage, and the second, one chord below the first and the top of the fuselage faired, as shown in Diagram 1.

DISCUSSION OF RESULTS

The results of the investigation show that interference depends on:

(*a*) The fuselage.

(*b*) The aerofoil.

(*c*) The relative vertical position of fuselage and aerofoil.

The results indicate that the more nearly the fuselage approaches a streamline form and the thicker the aerofoil, the more marked will be the effects of interference, and the more important the relative vertical position of fuselage and aerofoil.

Briefly, the effects of interference are:

(*a*) The angle at which *stalled air-flow begins* is equal to, or less than the critical angle for the aerofoil alone.

(*b*) The drag is equal to, or greater than the sum of the drags of the component parts. Some models with fuselage C were exceptions to this rule, but whenever the drag was reduced the lift was reduced at the same time, indicating a shielding effect. Apparently, the flat nose produced a strong upwash at positive angles of incidence, and a downwash at negative angles, resulting in a reduction of the air velocity about the aerofoil near the fuselage.

(*c*) The lift may be greater or less than the sum of the lifts of the component parts.

It is well known that an aerofoil stalls earlier at the medial section than at the tips and that a thick aerofoil has a more sudden stall than a thin aerofoil. It is, therefore, to be expected that any interference at or near the medial section would have considerable influence on the angle at which stalled air-flow begins, and that this influence would be greater for thick aerofoils than for thin ones.

In monoplanes using the RAF-15 aerofoil the critical angle was practically the same for the complete models and for the aerofoil only. The lift curves all indicate the same effects of interference with the RAF-15 aerofoil as with the USA-27. The RAF-15, being a much thinner section, the lift curves of models with this aerofoil are more faired than the curves for the same monoplane with the USA-27 section.

In monoplanes using the USA-27 aerofoil, the critical angles were equal to, or less than, the critical angle of the aerofoil, except in the case of monoplane 1, fuselage A. In this model, maximum lift occurred at 19° fuselage incidence when the calculated maximum lift occurred at 14°. The curves for this model show that it was affected considerably by interference, the lift curve deviated decidedly from the straight line lift curve which this aerofoil possesses, and the drag was greatly increased, indicating that the aerofoil close to the fuselage was likely stalled. To investigate this point further, pressure distribution tests were made and it was found that even at $-4°$ incidence of the fuselage, stalled air-flow existed over part of the aerofoil close to the fuselage, and that as the incidence was increased the stalled area of the aerofoil increased uniformly until at 19° incidence practically all of the aerofoil was stalled.

A similar case was referred to by Mr. Cowley during the discussion of Mr. Pierson's paper, "The Use of the Wind Channel for Performance Prediction," in the Journal of the Royal Aeronautical Society, February, 1928. This particular case was a seaplane model of a proposed Schneider Trophy machine, whose maximum lift occurred at an angle of attack of 35°. It was found that the wings stalled at a normal angle of attack and that the extra lift was due to the floats. This model was probably a low-wing monoplane with a medium thick aerofoil and a well streamlined fuselage.

Monoplane 1, fuselage B with USA-27 aerofoil, began to stall at 11° fuselage incidence. Increasing the incidence increased the stalled area to such an extent that the lift decreased. At 18° fuselage incidence, the lift suddenly increased to approximately the calculated value. This was due to a sudden decrease in the stalled area of the aerofoil by the downwash from the nose of the fuselage. The lift curves for monoplane 1, fuselage B, RAF-15 aerofoil, and monoplane 2, fuselage A, RAF-15 aerofoil, show similar tendencies.

In monoplanes using the Gottingen-387 aerofoil the critical angles were all less than for the aerofoil, except for monoplane 6, fuselage C. All the monoplanes with the Gottingen-387 on top or through the middle of the fuselage exhibited double stalls. This was due to an initial partial stall and a final complete stall of the aerofoil. Biplane 13, fuselage A, also exhibited a double stall.

For each biplane, the interference effects on the critical angle, and the characteristics in general, are similar to those for the two corresponding monoplanes, but are less marked, due to the interference between the two aerofoils.

The effect of varying the relative vertical position of fuselage and aerofoil can be seen from the polar curves. For a given aerofoil and fuselage the following general conclusions may be drawn:

(*a*) Placing the aerofoil at the bottom of the fuselage, results in the poorest aerodynamic characteristics. The maximum lift and the critical angle are low, and if the fuselage is well streamlined, such as fuselage A, part of the aerofoil is stalled even at −4° fuselage incidence.

(*b*) Placing the aerofoil midway between the top and bottom of the fuselage results in good characteristics. The minimum drag is the lowest, and up to about 6° to 8° fuselage incidence the drag is low. Except for monoplane 2, fuselage B, aerofoil RAF-15 or USA-27, the maximum lift and the critical angle were low.

(*c*) Placing the aerofoil at the top of the fuselage secures very good characteristics. The minimum drag is slightly higher than in the case of the aerofoil passing through the middle of the fuselage. The maximum lift and critical angle are high.

(*d*) The parasol arrangement, in general, possesses the highest maximum lift and greatest critical angle. The minimum drag is the highest of the monoplanes tested.

(*e*) With fuselage A, the biplane characteristics are midway between the characteristics for the two corresponding monoplane arrangements, but with fuselages B and C the biplane drag for a given lift is definitely lower than for any of the monoplane arrangements. Where the fuselage does not fill the gap between the wings it appears best to place the lower wing at the bottom of the fuselage and the upper wing parasol.

(*f*) Fillets improve the characteristics of models that would otherwise have poor characteristics, especially when partial stall exists at low angles of incidence. If, however, the model already has good characteristics, fillets will have practically no effect on the characteristics.

The foregoing interference effects may be explained in the following way, using as an example a low-wing monoplane with a well streamlined fuselage.

The fuselage is surrounded by a system of curved streamlines, and, if a vertical plane be passed through this system, near the fuselage, the flow in this plane, at the bottom of the fuselage (the position of the wing) is curvilinear and the curvature concave upward.

This curvilinear flow past the cambered aerofoil in effect increases the camber of the aerofoil. This can be visualized by picturing the streamlines and aerofoil camber as readjusted until the streamlines are straight. The aerofoil then becomes deeply cambered.

The effective camber of the aerofoil being thus increased, the critical angle is made smaller. Had the aerofoil been at the top of the fuselage, the reverse would have been the case; the streamlines would have been concave downward and the effective camber of the aerofoil near the fuselage decreased. All the results show strong evidence that the effective camber of the aerofoil close to the fuselage is changed. Partial stall and doublestalls are two of the principal resulting effects.

The effects with the different fuselages may be explained in the same way.

Fuselage A had good streamline form and the interference effects and the changes in effective camber were more marked with it than with any other fuselage.

Fuselage B showed the same general tendencies, but to a less extent. It should be noted that in monoplane 3, fuselage A, the decrease in effective camber resulted in a lower lift/drag ratio than for monoplane 2, fuselage A. However, in monoplane 3, fuselage B, the decreased effective camber seems to be counteracted by the windshield acting as part of the wing, resulting in a higher lift/drag ratio for this model than was calculated.

Fuselage C, as has already been stated, showed indications of shielding the aerofoil close to the fuselage causing the polar curves for each aerofoil to be very close together. Above 8° the results are very much the same as for fuselages A or B.

BIBLIOGRAPHY.

Applied Aerodynamics—Bairstow.

Messungen bei verschiedener gegenseitiger Anordnung von Flügel und Rumpf—Prandtl. Ergebnisse der Aerodynamischen Versuchsanstadt zu Gottingen—I. Leiferung, p. 118, translated in NACA Technical Note 75, December, 1921.

Beeinflussung von Tragflügeln durch Motorgondeln—Prandtl.

Ergebnisse der Aerodynamischen Versuchsanstalt zu Gottingen—III Lieferung, page 115.

Effects of Protruding Gasoline Tanks upon the Characteristics of an Aerofoil—Jacobs. NACA Technical Note No. 249, October, 1926.

Experiments on Models of Complete Aeroplanes—Effect of Body of F E 2 B on the Characteristics of the Main Planes—Relf. R & M No. 198, part I, Report for 1915-16, p. 144.

Measurements of the Drag of the Bodies of a Model Fighter and Bomber Machine —Fage & Collins. R & M 348 Report for 1917-18 I, p. 248.

Tests on a Model of the DH-10 Machine—Relf & Lavender. R & M 559 Report for 1918-19 I, p. 489.

Forces, Moments and Interferences on a 1/12 Scale Model of B E 2 E Fitted with RAF-18 Wing Section—Gates & Jones. R & M 658 Report for 1919-20 I, p. 279.

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LIST OF DIAGRAMS

Characteristics plotted on incidence

No.	Model Arrangement			
1.	Relative positions of aerofoil and fuselage.			
2.	Fuselage A,	USA-27,	Monoplane	1
3.	"	"	"	2
4.	"	"	"	3
5.	"	"	"	3F
6.	"	"	Biplane	13
7.	"	Gottingen-387,	Monoplane	1
8.	"	"	"	2
9.	"	"	"	3
10.	"	"	Biplane	13
11.	Fuselage B,	USA-27	Monoplane	1
12.	"	"	"	2
13.	"	"	"	3
14.	"	"	Biplane	13
15.	"	Gottingen-387,	Monoplane	1
16.	"	"	"	2
17.	"	"	"	3
18.	"	"	Biplane	13
19.	Fuselage C,	USA-27,	Monoplane	3
20.	"	"	"	4
21.	"	"	"	5
22.	"	"	"	5F
23.	"	"	"	6
24.	"	"	Biplane	15
25.	"	"	"	15F
26.	"	"	"	26
27.	"	"	"	37

28. Fuselage C, Gottingen-387, Monoplane 3
29. " " " 4
30. " " " 5
31. " " " 5F
32. " " " 6
33. " " Biplane 15
34. " " " 15F
35. " " " 26
36. " " " 37
37. Fuselage A, RAF-15, Monoplane 1
38. " " " 1F
39. " " " 2
40. " " " 3
41. " " " 3F
42. " " Biplane 13
43. " " " 13F
44. Fuselage B, " Monoplane 1
45. " " " 2
46. " " " 3
47. " " Biplane 13
48. Fuselage C, RAF-15 Monoplane 3
49. " " " 4
50. " " " 5
51. " " " 5F
52. " " " 6
53. " " Biplane 15
54. " " " 15F
55. " " " 26
56. " " " 37

POLAR CURVES

57. Fuselage A, RAF-15
58. " USA-27
59. " Gottingen-387
60. Fuselage B, RAF-15
61. " USA-27
62. " Gottingen-387
63. Fuselage C, RAF-15
64. " USA-27
65. " Gottingen-387.

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4. Front view of set-up with fuselage A, showing fairing.
5. Monoplane set-up with fuselage B.
6. Biplane set-up with fuselage B.
7. Biplane set-up with fuselage B, showing dummy.
8 and 9. Views of monoplane with fuselage C, showing fairing.
10. Biplane set-up with fuselage C, showing dummy spindle and guard.

THE INTERFERENCE BETWEEN THE BODY AND WINGS OF AIRCRAFT

TABLE 1

EXPERIMENTAL VALUES FOR COMPLETE MODELS

Fuselage Aerofoil Position	Incidence (degrees)	A USA-27 M-1	A USA-27 M-2	A USA-27 M-3	A USA-27 M-3-F	A USA-27 B-13
Lift..... (pounds)	−4	.1389	.1347	.1692	.1625	.2935
	0	.3225	.3536	.3694	.3864	.6535
	+4	.5060	.5441	.5784	.5832	.9719
	8	.6421	.7391	.7702	.7887	1.2702
	12	.7875	.7765	.9507	.9465	1.5826
	16	.9013	.6393	.9890	1.0031	1.7728
Drag.... (pounds)	−4	.0334	.0246	.0293	.0273	.0608
	0	.0401	.0280	.0320	.0305	.0689
	+4	.0622	.0445	.0486	.0467	.1030
	8	.0904	.0680	.0730	.0728	.1527
	12	.1303	.1264	.1024	.0990	.2207
	16	.1727	.2002	.1404	.1454	.2857
L/D....	−4	4.15	5.97	5.77	5.95	4.82
	0	8.04	12.65	11.55	12.68	9.50
	+4	8.14	12.18	11.90	12.50	9.42
	8	7.10	10.88	10.55	10.82	8.32
	12	6.04	6.14	9.30	9.55	7.20
	16	5.22	3.19	7.08	6.89	6.20

TABLE 2

CALCULATED VALUES FOR COMPLETE MODELS

Lift..... (pounds)	−4	.1537	.1457	.1584	.1537	.2525
	0	.3607	.3421	.3711	.3607	.6207
	+4	.5477	.5197	.5637	.5477	.9368
	8	.7314	.6950	.7531	.7314	1.2531
	12	.8840	.8395	.9100	.8840	1.5488
	16	.9160	.8707	.9425	.9160	1.7602
Drag.... (pounds)	−4	.0246	.0245	.0252	.0246	.0483
	0	.0259	.0247	.0266	.0259	.0501
	+4	.0414	.0395	.0426	.0414	.0803
	8	.0653	.0622	.0670	.0653	.1273
	12	.0950	.0805	.0976	.0950	.1857
	16	.1281	.1222	.1315	.1281	.2484
L/D....	−4	6.25	5.82	6.30	6.25	5.23
	0	13.90	13.92	13.95	13.90	12.38
	+4	13.21	13.15	13.20	13.21	11.66
	8	11.20	11.18	11.22	11.20	9.85
	12	9.30	9.27	9.33	9.30	8.34
	16	7.15	7.36	7.16	7.15	7.08

THE INTERFERENCE BETWEEN THE BODY AND WINGS OF AIRCRAFT

TABLE 1 (*a*)

EXPERIMENTAL VALUES FOR COMPLETE MODELS

Fuselage Aerofoil Position	Incidence (degrees)	B USA-27 M-1	B USA-27 M-2	B* USA-27 M-3	B USA-27 B-13	C USA-27 M-3	C USA-27 M-4	C USA-27 M-5	C USA-27 M-5-F
Lift........	−4	.1378	.1565	.0395	.2796	.1475	.1502	.1521	.1630
(pounds)	0	.3604	.3742	.2791	.6616	.3394	.3704	.3600	.3779
	+4	.5583	.5803	.4941	.9844	.5430	.5905	.5560	.5820
	8	.7519	.7762	.7049	1.3149	.7268	.7766	.7348	.7786
	12	.7983	.9422	.8748	1.5617	.7356	.8532	.8990	.9376
	16	.6396	.9708	.9909	1.7398	.5868	.9510	.9738	.9598
Drag.......	−4	.0337	.0352	.0377	.0564	.0339	.0317	.0340	.0327
(pounds)	0	.0354	.0384	.0312	.0609	.0327	.0341	.0343	.0354
	+4	.0518	.0559	.0412	.0924	.0512	.0523	.0510	.0521
	8	.0810	.0826	.0629	.1399	.0757	.0776	.0726	.0783
	12	.1303	.1177	.0939	.2058	.1217	.1431	.1009	.1086
	16	.2083	.1569	.1362	.2903	.2000	.1900	.1438	.1660
L/D.......	−4	4.09	4.45	1.05	4.95	4.35	4.74	4.47	4.99
	0	9.90	9.75	8.95	10.86	10.38	10.84	10.50	10.68
	+4	10.80	10.38	12.00	10.64	10.83	11.30	10.90	11.16
	8	9.28	9.40	11.20	9.40	9.60	10.00	10.10	9.94
	12	6.12	8.00	9.31	7.60	6.05	5.95	8.90	8.63
	16	3.06	6.20	7.26	6.00	2.93	5.00	6.77	5.78

TABLE 2 (*a*)

CALCULATED VALUES FOR COMPLETE MODELS

Lift........	−4	.1454	.1454	.1454	.2386	.1513	.1513	.1605	.1513
(pounds)	0	.3421	.3421	.3421	.5801	.3518	.3518	.3730	.3518
	+4	.5203	.5203	.5303	.8774	.5261	.5261	.5681	.5261
	8	.6999	.6999	.6999	1.1743	.7175	.7175	.7608	.7175
	12	.8470	.8470	.8470	1.4555	.8701	.8701	.9221	.8701
	16	.8856	.8856	.8856	1.6586	.9033	.9033	.9573	.9033
Drag.......	−4	.0337	.0337	.0337	.0546	.0337	.0337	.0351	.0337
(pounds)	0	.0338	.0338	.0448	.0561	.0350	.0350	.3364	.0350
	+4	.0481	.0481	.0481	.0839	.0505	.0505	.0528	.0505
	8	.0714	.0714	.1714	.1288	.0746	.0746	.0782	.0746
	12	.1007	.1007	.1007	.1840	.1030	.1029	.1083	.1030
	16	.1319	.1319	.1319	.2422	.1359	.1357	.1428	.1359
L/D.......	−4	.431	.431	.431	.437	4.50	4.50	4.57	4.50
	0	10.01	10.01	10.01	10.30	10.00	10.00	10.50	10.00
	+4	10.80	10.80	10.80	10.45	10.40	10.40	10.75	10.40
	8	9.80	9.80	9.80	9.13	9.62	9.62	9.74	9.62
	12	8.40	8.40	8.40	7.90	8.45	8.45	8.50	8.45
	16	6.71	6.71	6.71	6.85	6.74	6.74	6.80	6.74

*By mistake these readings were taken with respect to the incidence of the aerofoil chord instead of that of the fuselage axis.

THE INTERFERENCE BETWEEN THE BODY AND WINGS OF AIRCRAFT

TABLE 1 (*b*)

EXPERIMENTAL VALUES FOR COMPLETE MODELS

Fuselage Aerofoil Position	Incidence (degrees)	C USA-27 M-6	C USA-27 B-15	C USA-27 B-15-F	C USA-27 B-26	C USA-27 B-37
Lift.....	−4	.1797	.3154	.2922	.2928	.3149
(pounds)	0	.3818	.6755	.6651	.6547	.6796
	+4	.5648	.9959	.9727	.9677	.9773
	8	.7655	1.2883	1.3051	1.2290	1.2724
	12	.9050	1.6042	1.5423	1.5580	1.6222
	16	.9619	1.8755	1.8708	1.7969	1.7908
Drag....	−4	.0339	.0620	.0557	.0621	.0590
(pounds)	0	.0369	.0665	.0623	.0666	.0646
	+4	.0535	.0986	.0946	.1006	.0935
	8	.0802	.1384	.1448	.1474	.1412
	12	.1086	.1952	.2010	.2076	.2110
	16	.1424	.2860	.2748	.2804	.2996
L/D....	−4	5.30	5.10	5.24	4.71	5.34
	0	10.32	10.15	10.67	9.82	10.50
	+4	10.53	10.10	10.30	9.62	10.45
	8	9.54	9.30	9.00	8.33	9.00
	12	8.33	8.22	7.67	7.50	7.69
	16	6.75	6.56	6.80	6.40	5.98

TABLE 2 (*b*)

CALCULATED VALUES FOR COMPLETE MODELS

Lift.....	−4	.1653	.2661	.2580	.2701	.2595
(pounds)	0	.3838	.6425	.6236	.6517	.6252
	+4	.5846	.9731	.9441	.9871	.9471
	8	.7823	1.3003	1.2613	1.3193	1.2668
	12	.9486	1.6096	1.5616	1.6336	1.5696
	16	.9840	1.8305	1.7765	1.8585	1.7863
Drag....	−4	.0356	.0597	.0584	.0603	.0582
(pounds)	0	.0371	.0615	.0602	.0622	.0590
	+4	.0539	.0931	.0909	.0942	.0907
	8	.0800	.1426	.1389	.1440	.1390
	12	.1108	.2015	.1959	.2045	.1973
	16	.1463	.2666	.2587	.2705	.2613
L/D....	−4	4.65	4.46	4.42	4.48	4.46
	0	10.36	10.45	10.33	10.48	10.60
	+4	10.81	10.45	10.40	10.47	10.43
	8	9.77	9.12	9.10	9.15	9.10
	12	8.55	8.00	7.99	8.17	7.94
	16	6.72	6.85	6.86	6.86	6.84

THE INTERFERENCE BETWEEN THE BODY AND WINGS OF AIRCRAFT

TABLE 1 (*c*)

EXPERIMENTAL VALUES FOR COMPLETE MODELS

Fuselage Aerofoil Position	Incidence (degrees)	A Gott-387 M-1	A Gott-387 M-2	A Gott-387 M-3	A Gott-387 B-13
Lift...... (pounds)	−4	.2276	.2365	.2710	.4540
	0	.3890	.4565	.4896	.7969
	+4	.5684	.6592	.6809	1.1012
	8	.7105	.8513	.8720	1.4055
	12	.8364	.8496	1.0426	1.6598
	16	.6147	.5986	.7673	1.5256
Drag..... (pounds)	−4	.0396	.0260	.0324	.0646
	0	.0542	.0350	.0411	.0890
	+4	.0816	.0567	.0629	.1298
	8	.1134	.0841	.0880	.1858
	12	.1557	.1508	.1228	.2504
	16	.2343	.2211	.2157	.3793
L/D.....	−4	5.75	9.10	8.35	7.03
	0	7.20	13.05	11.90	8.96
	+4	6.97	11.60	10.80	8.50
	8	6.27	10.10	9.90	7.57
	12	5.83	5.63	8.50	6.63
	16	2.62	2.70	3.56	4.02

TABLE 2 (*c*)

CALCULATED VALUES FOR COMPLETE MODELS

Lift...... (pounds)	−4	.2480	.2352	.2554	.4453
	0	.4475	.4250	.4510	.7665
	+4	.6447	.6121	.6637	1.1043
	8	.8314	.7894	.8554	1.4102
	12	.9820	.9310	1.0090	1.6977
	16	.5711	.5437	.5887	1.9235
Drag..... (pounds)	−4	.0267	.0255	.0274	.0544
	0	.0353	.0336	.0363	.0724
	+4	.0544	.0518	.0557	.1092
	8	.0821	.0781	.0843	.1579
	12	.1141	.1059	.1173	.2205
	16	.2199	.1996	.2153	.2945
L/D.....	−4	9.30	9.23	9.33	8.20
	0	12.68	12.65	12.41	10.60
	+4	11.48	11.80	11.86	10.11
	8	10.13	10.70	10.15	8.95
	12	8.60	9.12	8.60	7.70
	16	2.73	2.72	2.73	6.54

THE INTERFERENCE BETWEEN THE BODY AND WINGS OF AIRCRAFT

TABLE 1 (*d*)

EXPERIMENTAL VALUES FOR COMPLETE MODELS

Fuselage Aerofoil Position	Incidence (degrees)	B Gott-387 M-1	B Gott-387 M-2	B* Gott-387 M-3	B Gott-387 B-13	C Gott-387 M-3	C Gott-387 M-4	C Gott-387 M-5
Lift....... (pounds)	−4	.2478	.2589	.1603	.4702	.2572	.2772	.2676
	0	.4638	.4752	.3688	.8224	.4486	.5027	.4635
	+4	.6548	.6890	.5847	1.1622	.6527	.7268	.6569
	8	.8207	.8870	.8051	1.4768	.8561	.9278	.8493
	12	.6840	.8017	.9809	1.6746	.6515	.8922	1.0254
	16	.6007	.5874	.8680	1.3436	.5952	.6504	.6018
Drag...... (pounds)	−4	.0359	.0359	.0389	.0610	.0349	.0338	.0348
	0	.0470	.0446	.0389	.0809	.0454	.0443	.0446
	+4	.0688	.0692	.0522	.1209	.0635	.0688	.0608
	8	.1026	.0995	.0793	.1733	.0945	.0979	.0899
	12	.1773	.1641	.1146	.2472	.1725	.1841	.1248
	16	.2334	.2417	.2004	.3884	.2322	.2614	.2342
L/D......	−4	6.90	7.20	4.12	7.70	7.36	8.2	7.69
	0	9.86	10.66	9.50	10.15	9.86	11.34	10.40
	+4	9.52	9.95	11.20	9.54	10.28	10.55	10.80
	8	8.01	8.92	10.15	8.50	9.06	9.48	9.44
	12	3.85	4.88	8.48	6.78	3.77	5.06	8.22
	16	2.57	2.43	4.32	3.46	2.56	2.52	2.57

TABLE 2 (*d*)

CALCULATED VALUES FOR COMPLETE MODELS

Lift....... (pounds)	−4	.2349	.2349	.2349	.4159	.2433	.2433	.2583
	0	.4250	.4250	.4250	.7161	.4365	.4365	.4630
	+4	.6127	.6127	.6127	1.0235	.6307	.6307	.6691
	8	.7943	.7943	.7943	1.3225	.8148	.8148	.8533
	12	.9385	.9385	.9385	1.5934	.9666	.9666	1.0236
	16	.5586	.5586	.5586	1.8131	.5677	.5677	.6003
Drag...... (pounds)	−4	.0347	.0347	.0347	.0611	.0358	.0358	.0372
	0	.0427	.0427	.0427	.0769	.0440	.0440	.0461
	+6	.0604	.0604	.0604	.1110	.0631	.0631	.0667
	8	.0873	.0873	.0873	.1562	.0910	.0910	.0956
	12	.1187	.1187	.1187	.2164	.1224	.1224	.1291
	16	.2093	.2093	.2093	.2854	.2055	.2055	.2275
L/D......	−4	6.77	6.77	6.77	6.80	6.80	6.80	6.80
	0	9.95	9.95	9.95	9.32	9.92	9.92	10.02
	+4	10.14	10.14	10.14	9.23	9.99	9.99	10.01
	8	9.10	9.10	9.10	8.48	8.95	8.85	8.93
	12	7.80	7.80	7.80	7.36	7.89	7.89	7.93
	16	2.62	2.62	2.62	6.35	2.76	2.75	2.64

*By mistake these readings were taken with respect to the incidence of the aerofoil chord instead of that of the fuselage axis.

THE INTERFERENCE BETWEEN THE BODY AND WINGS OF AIRCRAFT

Table 1 (*e*)

EXPERIMENTAL VALUES FOR COMPLETE MODELS

Fuselage Aerofoil Position	Incidence (degrees)	C Gott-387 M-5-F	C Gott-387 M-6	C Gott-387 B-15	C Gott-38 B-15-F	C Gott-387 B-26	C Gott-387 B-37
Lift........	−4	.2608	.2693	.4783	.4712	.4743	.4754
(pounds)	0	.4651	.4639	.8164	.8012	.7848	.8129
	+4	.6666	.6508	1.1286	1.1634	1.0777	1.1232
	8	.8631	.8553	1.4390	1.4815	1.3893	1.4536
	12	1.0194	1.0306	1.7375	1.7789	1.7006	1.6551
	16	.5963	.7757	1.6542	1.7446	1.6134	1.4511
Drag.......	−4	.0363	.0366	.0661	.0686	.0687	.0610
(pounds)	0	.0437	.0457	.0891	.0897	.0871	.0808
	+4	.0649	.0661	.1265	.1346	.1209	.1210
	8	.0920	.0977	.1797	.1893	.1796	.1839
	12	.1263	.1328	.2455	.2566	.2512	.2445
	16	.2341	.2172	.3682	.3748	.3784	.4037
L/D.......	−4	7.16	7.36	7.23	6.87	6.90	7.80
	0	10.62	10.14	9.15	8.95	9.00	10.05
	+4	10.28	9.85	8.91	8.66	8.90	9.20
	8	9.38	8.75	8.01	7.83	7.74	8.36
	12	8.06	7.76	7.08	6.84	6.76	6.26
	16	2.55	3.57	4.50	4.65	4.48	3.60

Table 2 (*e*)

CALCULATED VALUES FOR COMPLETE MODELS

Lift........	−4	.2433	.2659	.4633	.4508	.4696	.4496
(pounds)	8	.4365	.4760	.7938	.6712	.8053	.7725
	+4	.6307	.6881	1.1256	1.0931	1.1416	1.0956
	8	.8148	.8883	1.4624	1.4201	1.4846	1.4250
	12	.9666	.0531	1.7621	1.7106	1.7886	1.7196
	16	.5677	.6178	1.9999	1.9353	2.0298	1.9533
Drag.......	−4	.0358	.0379	.0659	.0643	.0666	.0654
(pounds)	0	.0440	.0471	.0846	.0827	.0856	.0824
	+4	.0631	.0677	.1231	.1200	.1246	.1200
	8	.0910	.0979	.1731	.1685	.1755	.1691
	12	.1224	.1312	.2373	.2307	.2410	.2324
	16	.2055	.2335	.3151	.3051	.3191	.3082
L/D.......	−4	6.89	7.01	7.03	7.01	7.05	6.89
	0	9.92	10.10	9.38	8.12	9.40	9.37
	+4	9.99	10.16	9.14	9.11	9.16	9.14
	8	8.95	9.07	8.45	8.43	8.45	8.42
	12	7.89	8.04	7.43	7.40	7.40	7.40
	16	2.76	2.64	6.35	6.35	6.36	6.34

THE INTERFERENCE BETWEEN THE BODY AND WINGS OF AIRCRAFT

TABLE 1 (*f*)

EXPERIMENTAL VALUES FOR COMPLETE MODELS

Fuselage Aerofoil Position	Incidence (degrees)	A RAF-15 M-1	A RAF-15 M-1-F	A RAF-15 M-2	A RAF-15 M-3	A RAF-15 M-3-F	A RAF-15 B-13	A RAF-15 B-13-F
Lift.......	−4	.035	.025	.019	.033	.027	.059	.029
(pounds)	0	.270	.265	.263	.260	.267	.440	.439
	+4	.446	.462	.469	.463	.4665	.755	.781
	8	.600	.605	.629	.651	.6585	1.084	1.082
	12	.693	.695	.691	.705	.7195	1.310	1.297
	16	.646	.666	.652	.641	.6645	1.284	1.276
Drag......	−4	.0285	.0178	.0155	.0164	.0162	.0320	.0336
(pounds)	0	.0263	.0216	.0202	.02145	.02155	.04055	.04015
	+4	.0439	.0374	.0368	.0379	.03705	.06875	.06875
	8	.0754	.0756	.0703	.0645	.0655	.12413	.11383
	12	.1294	.1265	.1462	.1451	.1526	.2054	.2055
	16	.2256	.2283	.2304	.2360	.2412	.3980	.3994
L/D......	−4	1.227	1.404	1.225	2.0	1.665	1.845	.863
	0	10.25	12.27	13.0	12.1	12.5	10.85	10.94
	+4	10.15	12.35	12.72	12.2	12.6	11.02	11.35
	8	7.90	8.0	8.95	10.1	10.05	8.73	9.50
	12	5.35	5.49	4.72	4.85	4.71	6.37	6.31
	16	2.86	2.915	2.82	2.71	2.75	3.22	3.19

TABLE 2 (*f*)

CALCULATED VALUES FOR COMPLETE MODELS

Lift.......	−4	.0062	.0059	.0058	.0064	.0051	.0389	.0376
(pounds)	0	.2465	.2380	.2330	.2610	.2420	.4394	.4245
	+4	.4277	.4137	.4047	.4355	.4202	.7633	.7374
	8	.6154	.5954	.5824	.6274	.6052	1.0743	1.0353
	12	.6830	.6610	.6460	.6960	.6714	1.2900	1.2456
	16	.5957	.5762	.5737	.6171	.5957	1.2157	1.1752
Drag......	−4	.0137	.0134	.0131	.0139	.0136	.0250	.0243
(pounds)	0	.0168	.0163	.0160	.0171	.0166	.0343	.0332
	+4	.0325	.0316	.0309	.0331	.0320	.0630	.0610
	8	.0583	.0565	.0553	.0572	.0572	.1130	.1094
	12	.1284	.1244	.1213	.1306	.1262	.2026	.1995
	16	.2099	.2032	.1990	.2135	.2063	.3997	.3862
L/D......	−4	.450	.44	.44	.460	.375	1.56	1.55
	0	14.66	14.5	14.56	14.66	14.56	12.80	12.77
	+4	13.16	13.05	13.10	13.15	13.15	12.10	12.10
	8	10.55	10.52	10.50	10.95	10.56	9.5	9.47
	12	5.31	5.3	5.31	5.33	5.31	6.35	6.35
	16	2.84	2.84	2.88	2.89	2.93	3.04	3.04

THE INTERFERENCE BETWEEN THE BODY AND WINGS OF AIRCRAFT

Table 1 (*g*)

EXPERIMENTAL VALUES FOR COMPLETE MODEL

Fuselage Aerofoil Position	Incidence (degrees)	B RAF-15 M-1	B RAF-15 M-2	B RAF-15 M-3	B RAF-15 B-13	C RAF-15 M-3	C RAF-15 M-4	C RAF-15 M-5
Lift.	−4	.016	.016	.034	.0505	.022	.011	.022
(pounds)	0	.246	.254	.267	.4515	.262	.253	.258
	+4	.448	.458	.473	.804	.459	.452	.460
	8	.637	.659	.668	1.1075	.620	.637	.653
	12	.691	.691	.727	1.3175	.654	.703	.715
	16	.644	.662	.686	1.2855	.623	.669	.675
Drag.	−4	.0217	.0212	.0213	.0334	.0218	.0215	.0218
(pounds)	0	.0249	.0263	.0244	.0422	.0267	.0255	.0259
	+4	.0420	.0427	.0413	.0737	.0434	.0409	.0425
	8	.0726	.0748	.0693	.1224	.0795	.0801	.0702
	12	.1478	.1556	.1645	.2239	.1501	.1552	.1500
	16	.2298	.2400	.2561	.4209	.2290	.2364	.2424
L/D.	−4	.737	.755	1.6	1.512	1.01	.512	1.01
	0	9.88	9.66	11.0	10.70	9.81	9.92	9.96
	+4	10.67	10.75	11.45	10.9	10.60	11.04	10.83
	8	8.77	8.80	9.65	9.05	7.80	7.95	9.3
	12	4.67	4.44	4.41	5.89	4.36	4.53	4.77
	16	2.80	2.76	2.675	3.055	2.72	2.83	2.79

Table 2 (*g*)

CALCULATED VALUES FOR COMPLETE MODEL

Lift.	−4	.0055	.0055	.0055	.0352	.0072	.0072	.0075
(pounds)	0	.2330	.2330	.2330	.4113	.2390	.2390	.2528
	+4	.4053	.4053	.4053	.7157	.4176	.4176	.4393
	8	.5873	.5873	.5873	1.0103	.6023	.6023	.6333
	12	.6495	.6495	.6495	1.2149	.6716	.6716	.7066
	16	.5886	.5886	.5886	1.1546	.5983	.5983	.6293
Drag.	−4	.0223	.0223	.0223	.0328	.0231	.0231	.0236
(pounds)	0	.0249	.0249	.0249	.0414	.0261	.0261	.0268
	+4	.0395	.0395	.0395	.0678	.0417	.0417	.0433
	8	.0645	.0645	.0645	.1154	.0675	.0675	.0704
	12	.1315	.1315	.1315	.2007	.1352	.1352	.1413
	16	.2087	.2087	.2087	.3852	.2152	.2152	.2255
L/D.	−4	.246	.246	.246	1.07	.312	.312	.317
	0	9.35	9.35	9.35	9.94	9.15	9.15	9.44
	+4	10.26	10.26	10.26	10.55	10.01	10.01	10.12
	8	9.1	9.1	9.1	8.76	8.93	9.93	9.00
	12	4.94	4.94	4.94	6.07	4.96	4.96	5.00
	16	2.82	2.82	2.82	3.00	2.78	2.78	2.79

THE INTERFERENCE BETWEEN THE BODY AND WINGS OF AIRCRAFT

TABLE 1 (*h*)

EXPERIMENTAL VALUES FOR COMPLETE MODELS

Fuselage		C	C	C	C	C	C
Aerofoil	Incidence	RAF-15	RAF-15	RAF-15	RAF-15	RAF-15	RAF-15
Position	(degrees)	M-5-F	M-6	B-15	B-15-F	B-26	B-37
Lift........	−4	.021	.025	.042	.047	.034	.063
(pounds)	0	.252	.258	.434	.431	.408	.448
	+4	.457	.446	.757	.744	.724	.754
	8	.643	.635	1.067	1.075	1.023	1.051
	12	.726	.721	1.306	1.284	1.301	1.313
	16	.674	.658	1.300	1.286	1.285	1.285
Drag.......	−4	.0219	.0227	.0376	.0376	.0369	.0351
(pounds)	0	.0262	.0264	.0457	.0460	.0453	.0442
	+4	.0411	.0424	.0756	.0743	.0736	.0735
	8	.0693	.0702	.1250	.1253	.1220	.1223
	12	.1528	.1421	.2025	.2141	.1906	.2177
	16	.2457	.2359	.4066	.4077	.4101	.4171
L/D.......	−4	.96	1.10	1.116	1.25	.921	1.79
	0	9.62	9.77	9.50	9.36	9.00	10.15
	+4	11.12	10.50	10.02	10.02	9.85	10.25
	8	9.28	9.05	8.54	8.59	8.39	8.59
	12	4.75	5.07	6.54	5.99	6.83	6.025
	16	2.74	2.79	3.20	3.15	3.13	3.08

TABLE 2 (*h*)

CALCULATED VALUES FOR COMPLETE MODELS

Lift........	−4	.0072	.0078	.0417	.0410	.0423	.0400
(pounds)	0	.2390	.2610	.4540	.4430	.4620	.4426
	+4	.4176	.4551	.7861	.7701	.8041	.7715
	8	.6023	.6563	1.1123	1.0733	1.1323	1.0873
	12	.6716	.7316	1.3376	1.3026	1.3626	1.3116
	16	.5983	.6513	1.2678	1.2388	1.2883	1.2323
Drag.......	−4	.0231	.0240	.0354	.0348	.0358	.0348
(pounds)	0	.0261	.0273	.0451	.0443	.0456	.0441
	+4	.0417	.0443	.0750	.0735	.0761	.0734
	8	.0675	.0723	.1275	.1246	.1295	.1249
	12	.1352	.1459	.2197	.2150	.2231	.2142
	16	.2152	.2327	.4246	.4153	.4311	.4128
L/D.......	−4	.312	.325	1.18	1.18	1.18	1.15
	0	9.15	9.56	10.05	10.00	10.10	10.02
	+4	10.01	10.3	10.47	10.5	10.55	10.50
	8	8.93	9.07	8.73	8.61	8.75	8.71
	12	4.96	5.01	6.1	6.06	6.1	6.11
	16	2.78	2.80	2.99	2.80	2.99	2.99

THE INTERFERENCE BETWEEN THE BODY AND WINGS OF AIRCRAFT

Table 3

ORDINATES OF AEROFOIL SECTIONS

Distance from Leading Edge	—RAF-15—		—USA-27—		—Gottingen-387—	
	Lower Ordinate	Upper Ordinate	Lower Ordinate	Upper Ordinate	Lower Ordinate	Upper Ordinate
0	+0.20	0.20	1.77	1.77	2.61	3.61
1.25	− .45	1.90	0.50	3.80	1.35	6.74
2.5	− .73	2.80	0.33	5.07	0.81	7.98
5	− .90	3.90	0.17	6.93	0.36	9.87
7.5	−1.00	4.60	0.10	8.22	0.18	11.32
10	−1.00	5.05	0.00	9.17	0.13	12.40
15	−0.80	5.58	0.10	10.50	0.00	13.83
20	−0.50	5.76	0.35	11.33	0.08	14.77
30	−0.10	5.80	0.95	11.90	0.22	15.36
40	−0.05	5.58	1.17	11.57	0.38	14.88
50	−0.28	5.17	0.90	10.77	0.54	13.48
60	−0.47	4.68	0.25	9.52	0.54	11.59
70	−0.62	4.07	0.10	8.00	0.54	9.16
80	−0.67	3.28	0.05	6.03	0.50	6.58
90	−0.35	2.24	0.15	3.65	0.2	3.61
100	+0.30	0.30	0.67	0.67	0.00	0.37

Table 4

LIFT AND DRAG FOR FUSELAGES A, B AND C

Fuselage	Incidence	A*	B	C
Lift.............	−4°	−.0027	−.0030	−.0017
	0	.0000	.0000	−.0010
	+4	+.0027	+.0033	+.0031
	8	.0054	.0103	.0073
	12	.0080	.0155	.0136
	16	.0107	.0256	.0183
Drag............	−4°	.0032	.0124	.0128
	0	.0027	.0118	.0123
	+4	.0032	.0118	.0131
	8	.0048	.0140	.0155
	12	.0074	.0176	.0173
	16	.0112	.0209	.0215

*Due to the small value of lift and drag on Fuselage A, and to the relatively large values of spindle correction, it was difficult to get accurate results. The above results for this fuselage are taken from the probable curve through the points found by experiment.

THE INTERFERENCE BETWEEN THE BODY AND WINGS OF AIRCRAFT

TABLE 5

LIFT AND DRAG COEFFICIENTS FOR THE AEROFOILS

	Fuselage Incidence*	—RAF-15—		—USA-27—		—Gott-387—	
		K_L	K_D	K_L	K_D	K_L	K_D
Monoplane:	−4°	.0067	.0079	.1173	.0160	.1878	.0176
	0	.1840	.0105	.2701	.0174	.3354	.0244
	+4	.3173	.0219	.4082	.0287	.4806	.0383
	8	.4555	.0398	.5446	.0453	.6183	.0576
	12	.5050	.0903	.6563	.0657	.7284	.0799
	16	.4450	.1482	.6784	.0876	.4206	.1489
Upper wing of Biplane:	−4°	.0112	.0081	.1015	.0161	.1602	.0196
	0	.1621	.0110	.2355	.0168	.2876	.0251
	+4	.2866	.0218	.3573	.0277	.4179	.0393
	8	.4120	.0410	.4834	.0460	.5420	.0593
	12	.5115	.0680	.6044	.0716	.6651	.0855
	16	.4200	.1330	.6859	.0975	.7657	.1180
Lower wing of Biplane:	−4°	.0197	.0081	.0895	.0173	.1708	.0184
	0	.1630	.0123	.2229	.0183	.2785	.0264
	+4	.2760	.0226	.3333	.0293	.3880	.0390
	8	.3780	.0391	.4374	.0445	.4949	.0530
	12	.4420	.0764	.5340	.0599	.5814	.0716
	16	.4730	.1545	.6052	.0775	.6466	.0911

*The angle between the aerofoil chord and the fuselage axis was 2°−0′ for USA-27 and Gott-387, and 3°−0′ for RAF-15.

THE INTERFERENCE BETWEEN THE BODY AND WINGS OF AIRCRAFT

THE RELATIVE POSITIONS OF AEROFOIL AND FUSELAGE.

Geo. J. Klein.

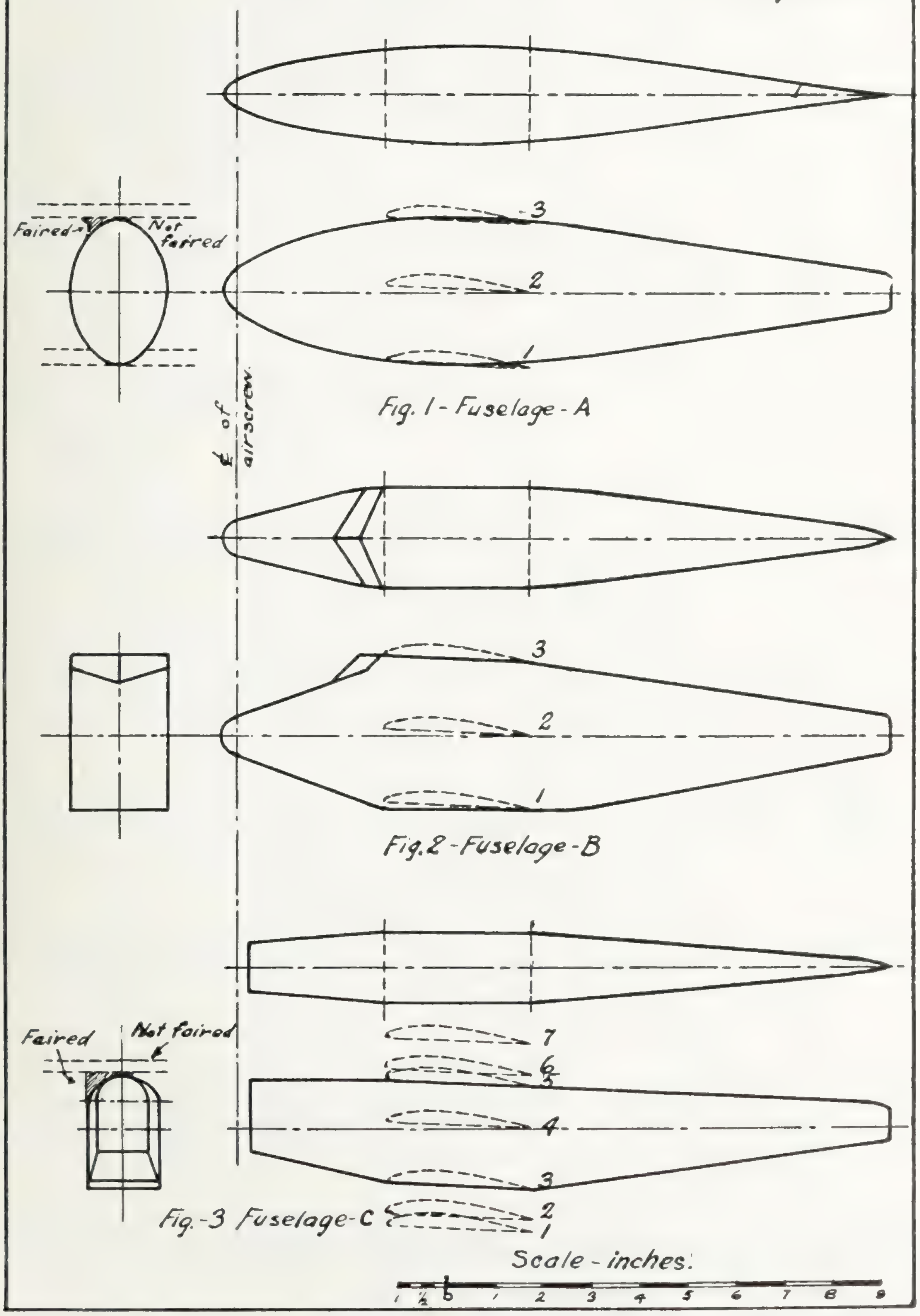


Fig. 1 - Fuselage - A

Fig. 2 - Fuselage - B

Fig.-3 Fuselage-C

PLATE - 1.

Monoplane I, Fuselage A, Aerofoil - U.S.A. - 27.
Angle between Aerofoil chord and Axis of Fuselage = 2°-0'.
Air speed - 40 f.p.s.

Geo. J. Klein

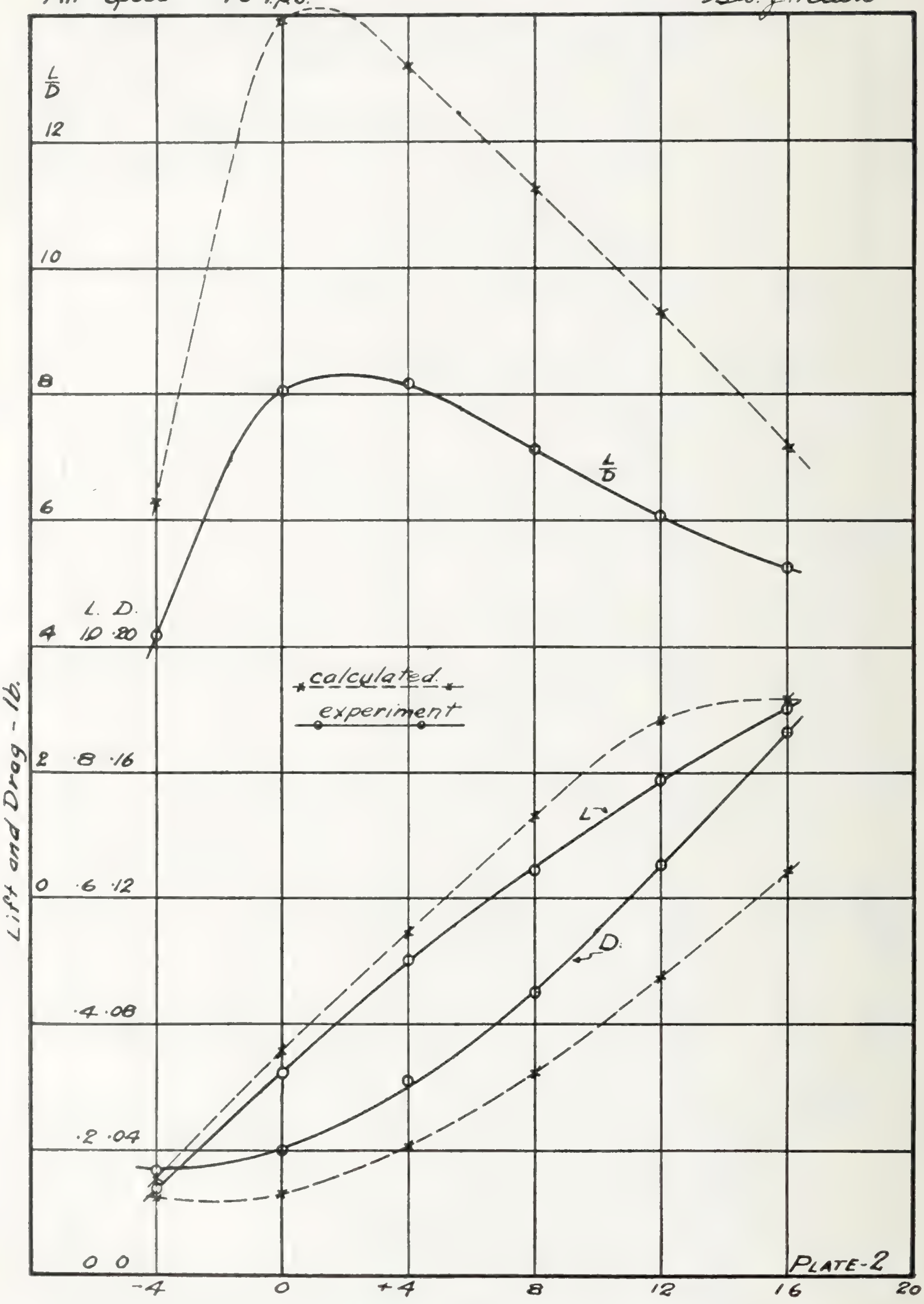
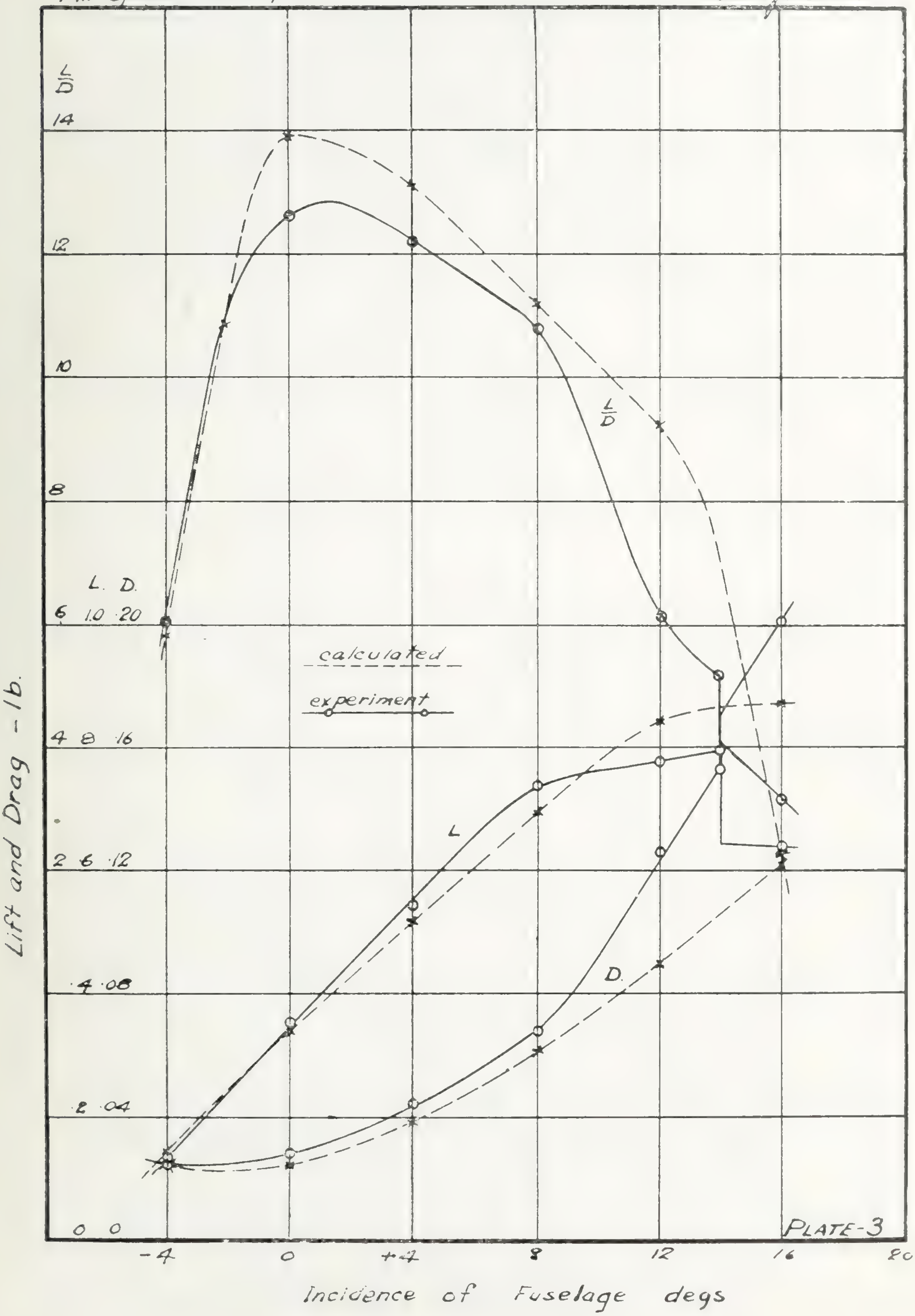
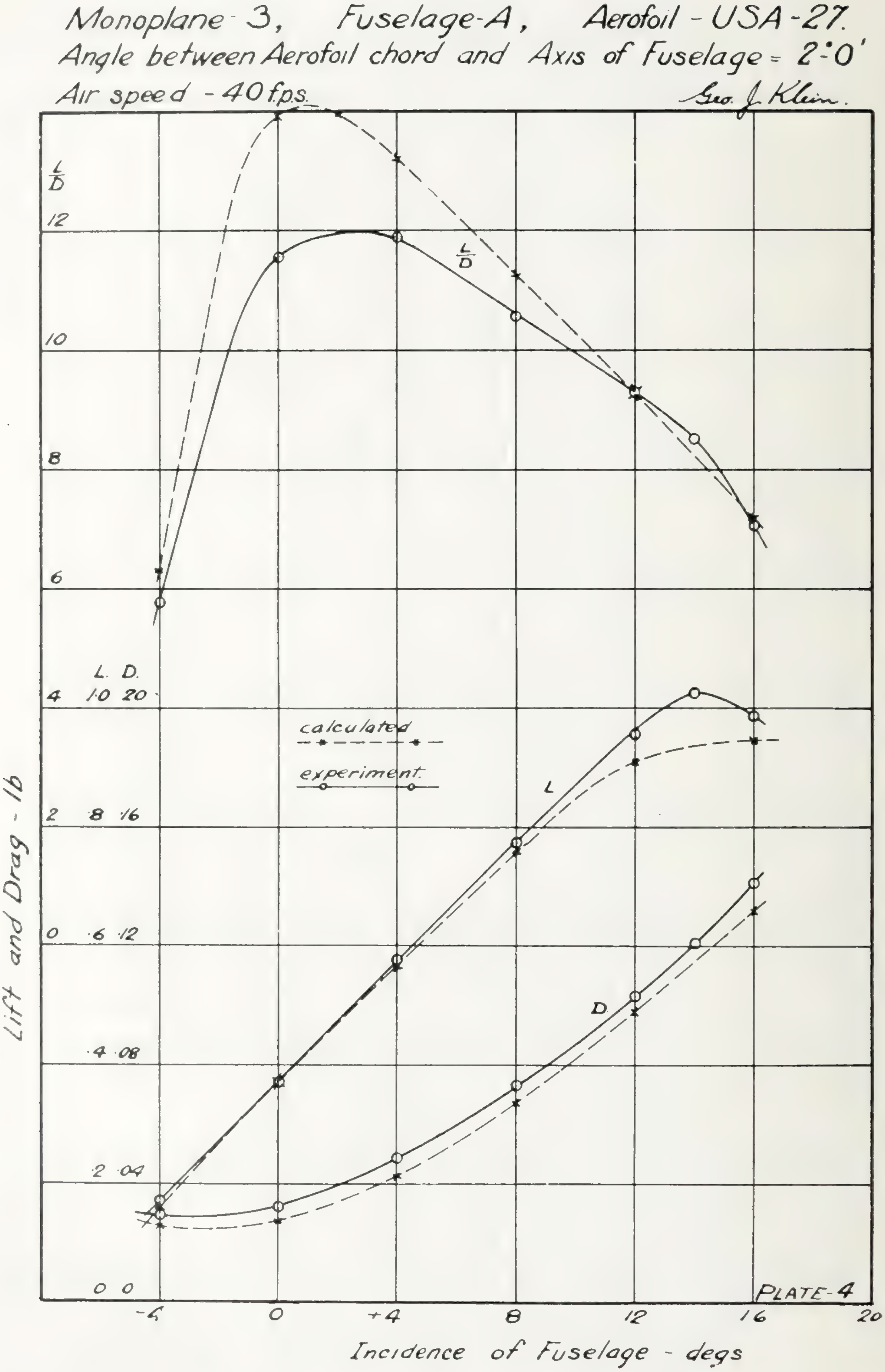


PLATE-2

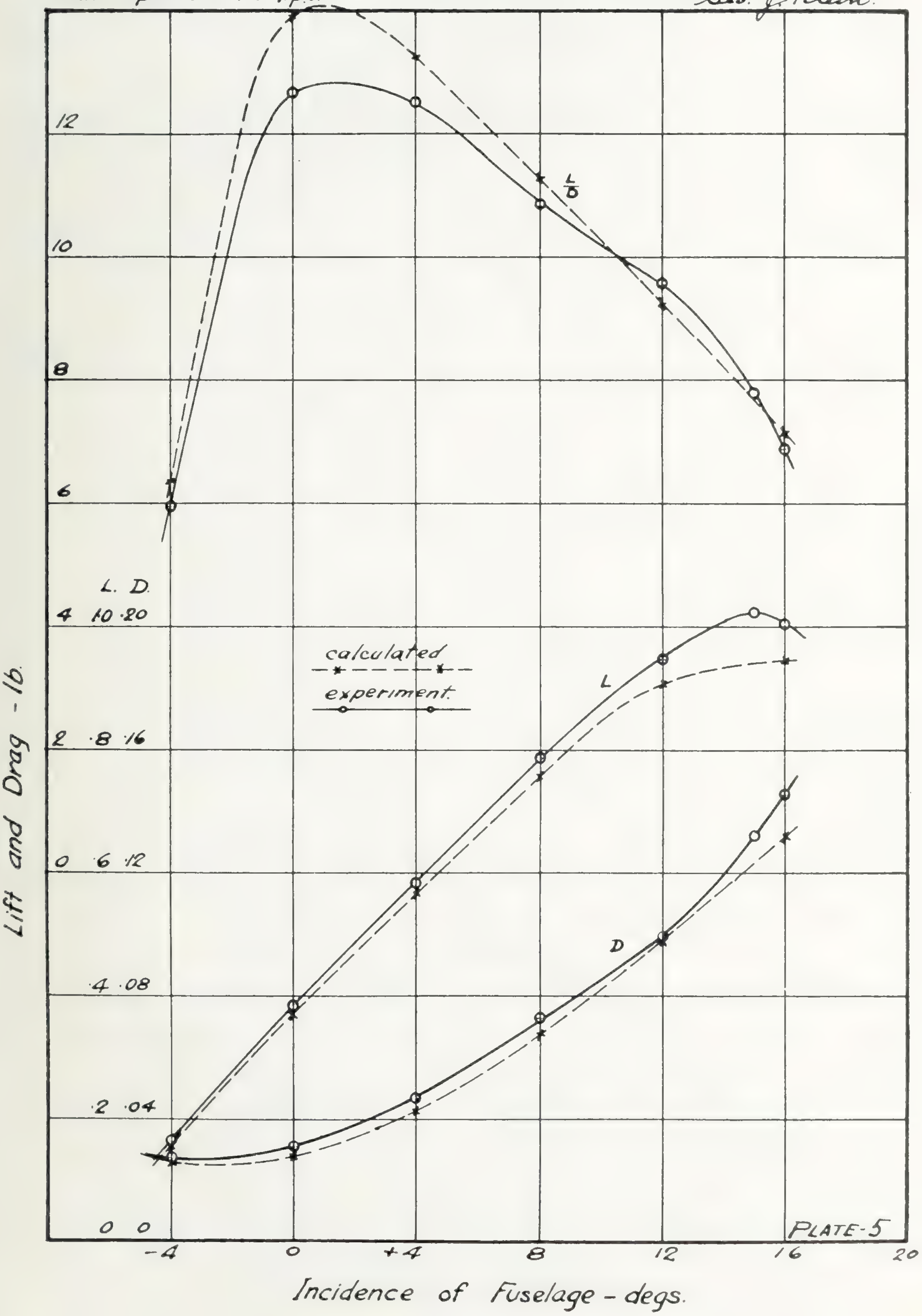
Monoplane 2, Fuselage A, Aerofoil - USA - 27
Angle between Aerofoil chord and Axis of Fuselage = 2°0′
Air speed - 40 f.p.s.
Geo. J. Klein

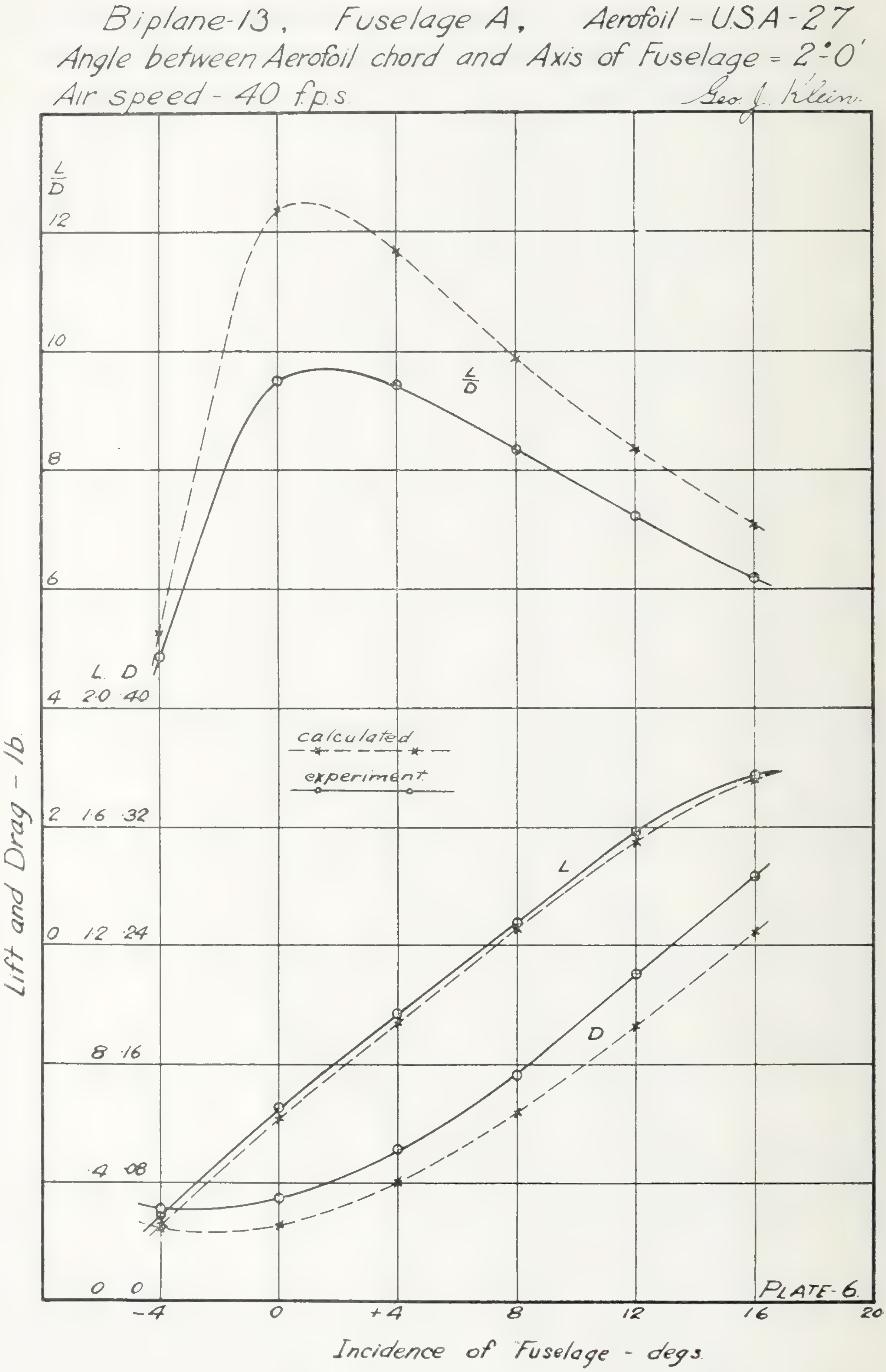



Monoplane-3, Fuselage-A, Aerofoil - USA-27.
Angle between Aerofoil chord and Axis of Fuselage = 2°0'
Air speed - 40 f.p.s.
Geo. J. Klein.
L/D
12
10
8
6
L/D
L. D.
4 1·0 20
2 ·8 ·16
0 ·6 ·12
·4 ·08
·2 ·04
0 0
calculated
experiment
L
D
Lift and Drag - lb
PLATE-4
-4
0
+4
8
12
16
20
Incidence of Fuselage - degs

Monoplane -3-F, Fuselage A, Aerofoil - U.S.A.-27
Angle between Aerofoil chord and Axis of Fuselage = 2°0'
Air speed - 40 f.p.s.

Geo. J. Klein.




Biplane-13, Fuselage A, Aerofoil - U.S.A - 27
Angle between Aerofoil chord and Axis of Fuselage = 2°0'
Air speed - 40 f.p.s.
Geo. J. Klein.
L/D
12
10
8
6
4
L. D
2.0 .40
2 1.6 .32
0 1.2 .24
8 .16
.4 .08
0 0
L/D
calculated
experiment
L
D
Lift and Drag - lb.
-4
0
+4
8
12
16
20
Incidence of Fuselage - degs.
PLATE - 6.

Monoplane-1, Fuselage A, Aerofoil- Gött-387
Angle between Aerofoil chord and Axis of Fuselage = 2°-0'
Air speed - 40 f.p.s. Geo. J. Klein

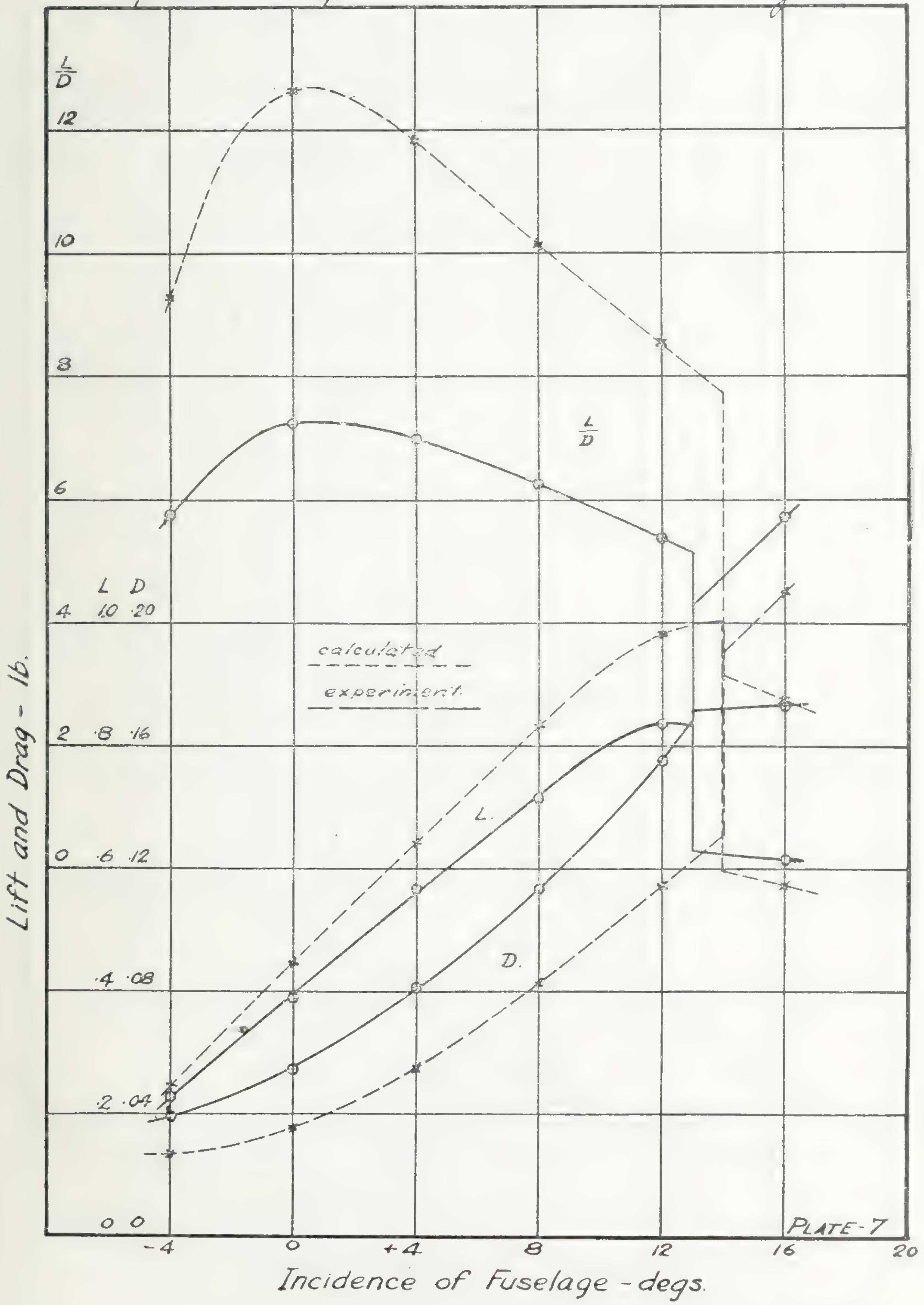
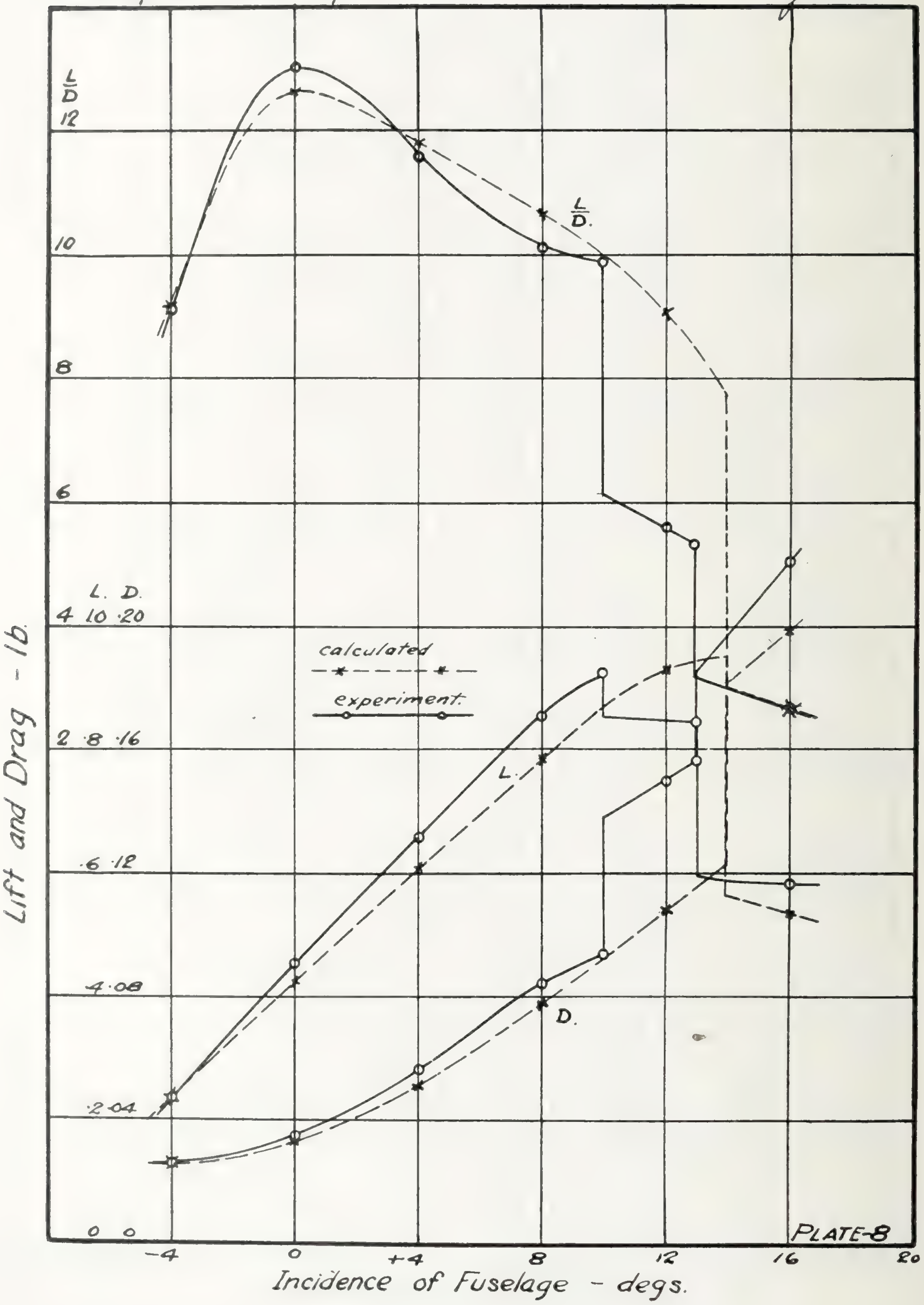


PLATE-7

Monoplane-2, Fuselage A, Aerofoil - Gött. 387.
Angle between Aerofoil chord and Axis of Fuselage = 2°0′
Air speed - 40 f.p.s.
Geo. J. Klein.



Monoplane 3, Fuselage A, Aerofoil - Gött 387.
Angle between Aerofoil chord and Axis of Fuselage = 2°0'
Air speed - 40 f.p.s. Geo. J. Klein.

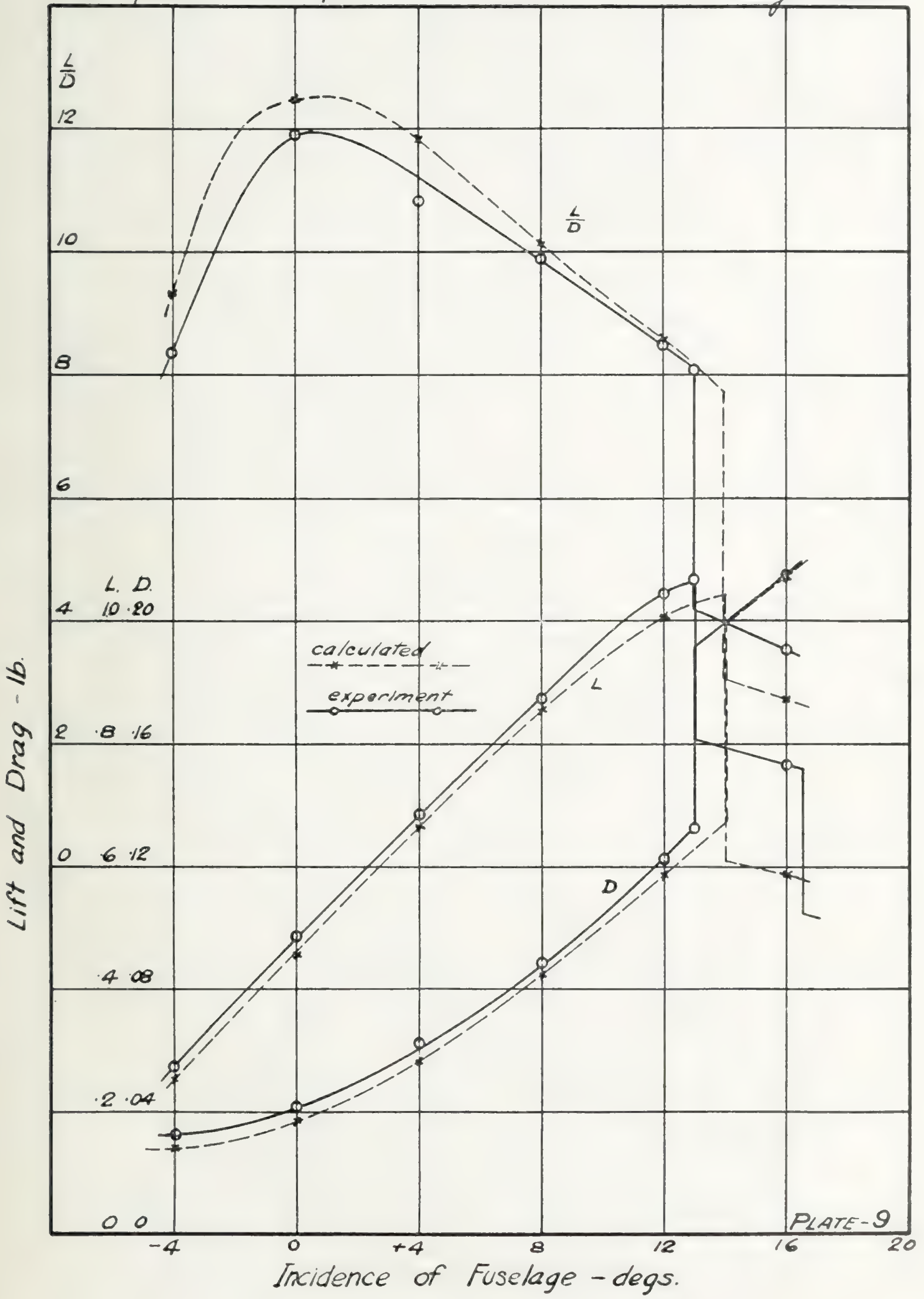


PLATE-9

Biplane -13, Fuselage A, Aerofoil - Gott 387
Angle between Aerofoil chord and Axis of Fuselage = 2°0′
Air speed - 40 f.p.s.

Geo. J. Klein

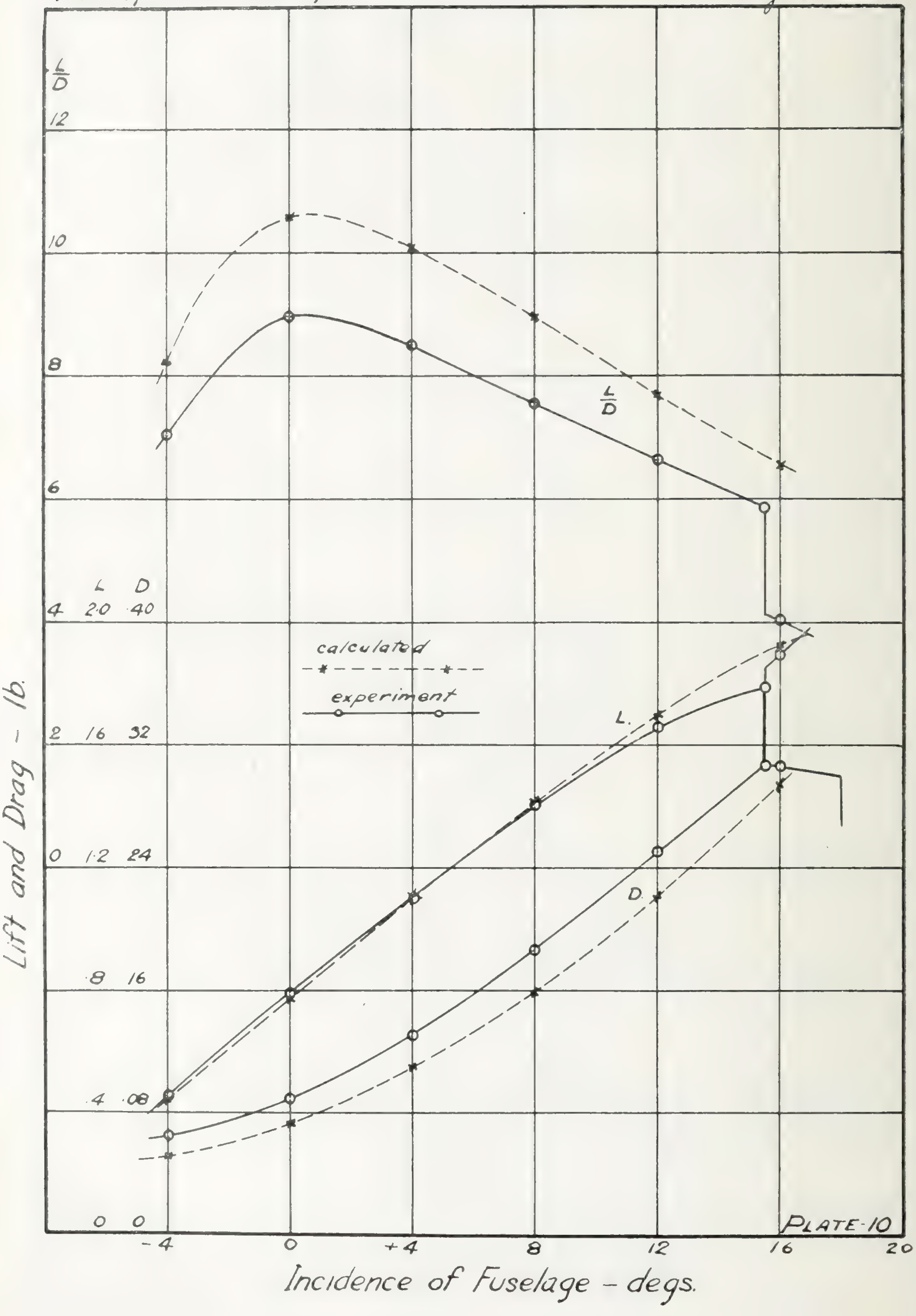
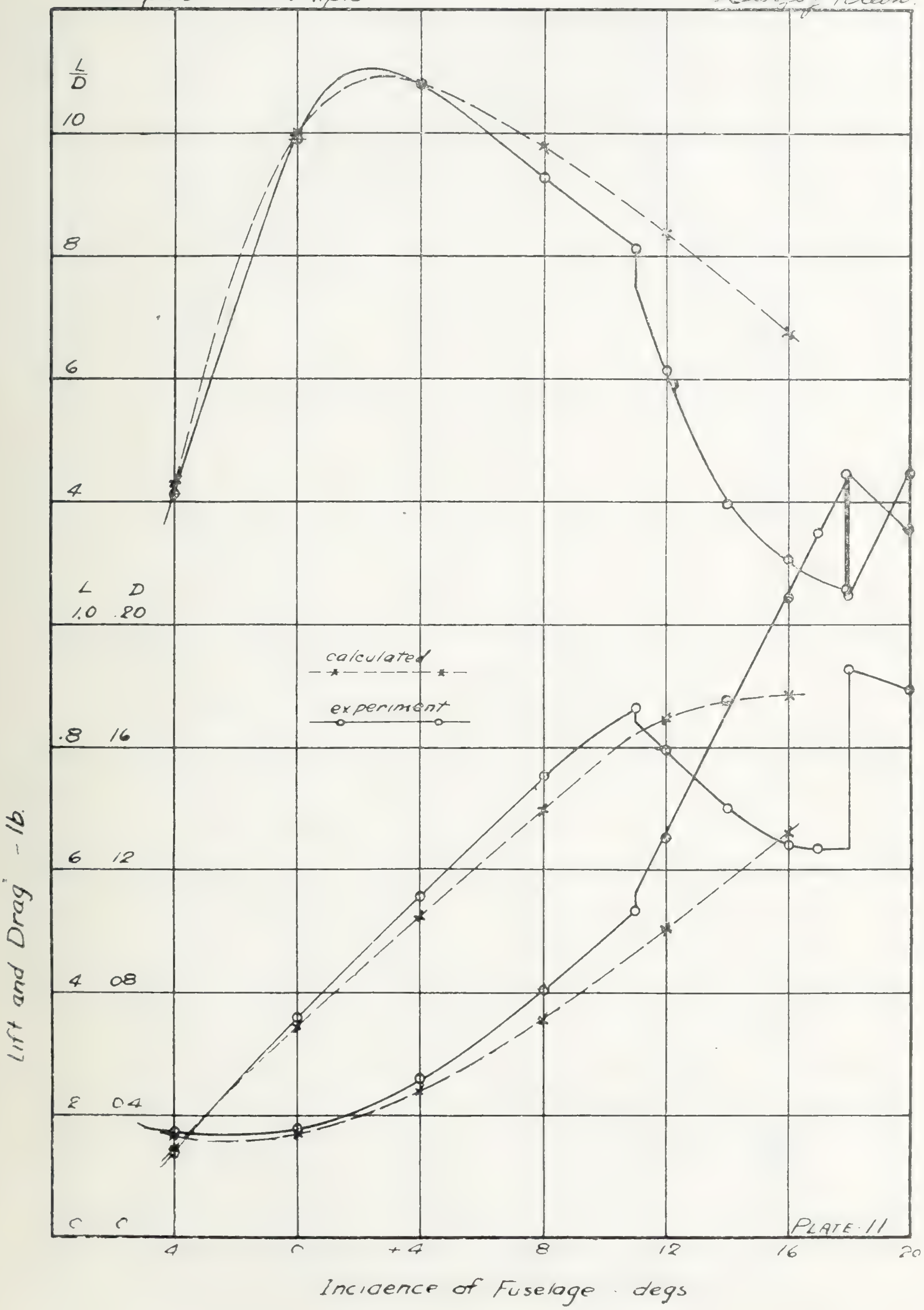
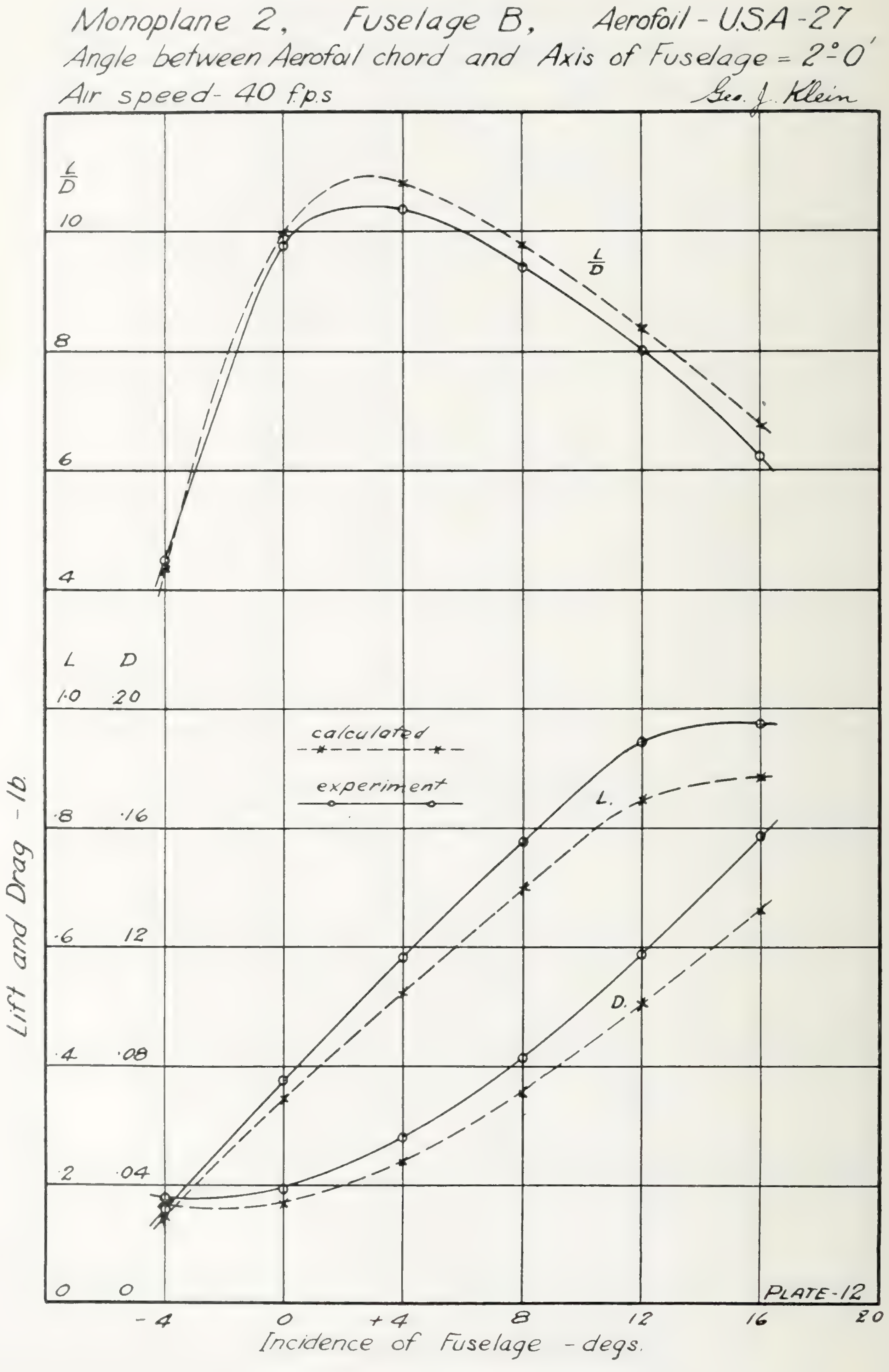


PLATE-10

Monoplane 1, Fuselage B, Aerofoil - USA - 27
Angle between Aerofoil chord and Axis of Fuselage = 2°-0'
Air speed - 40 f.p.s

George J. Klein




Monoplane 2, Fuselage B, Aerofoil - U.S.A -27
Angle between Aerofoil chord and Axis of Fuselage = 2°0′
Air speed - 40 f.p.s
Geo. J. Klein
L/D
10
8
6
4
L/D
L D
1·0 ·20
·8 ·16
·6 12
·4 ·08
·2 ·04
0 0
calculated
experiment
L.
D.
Lift and Drag - lb.
PLATE - 12
-4
0
+4
8
12
16
20
Incidence of Fuselage - degs.

Monoplane 3, Fuselage B, Aerofoil - USA -27
Angle between Aerofoil chord and Axis of Fuselage = 2°-0'
Air speed - 40 f.p.s.

Geo. J. Klein

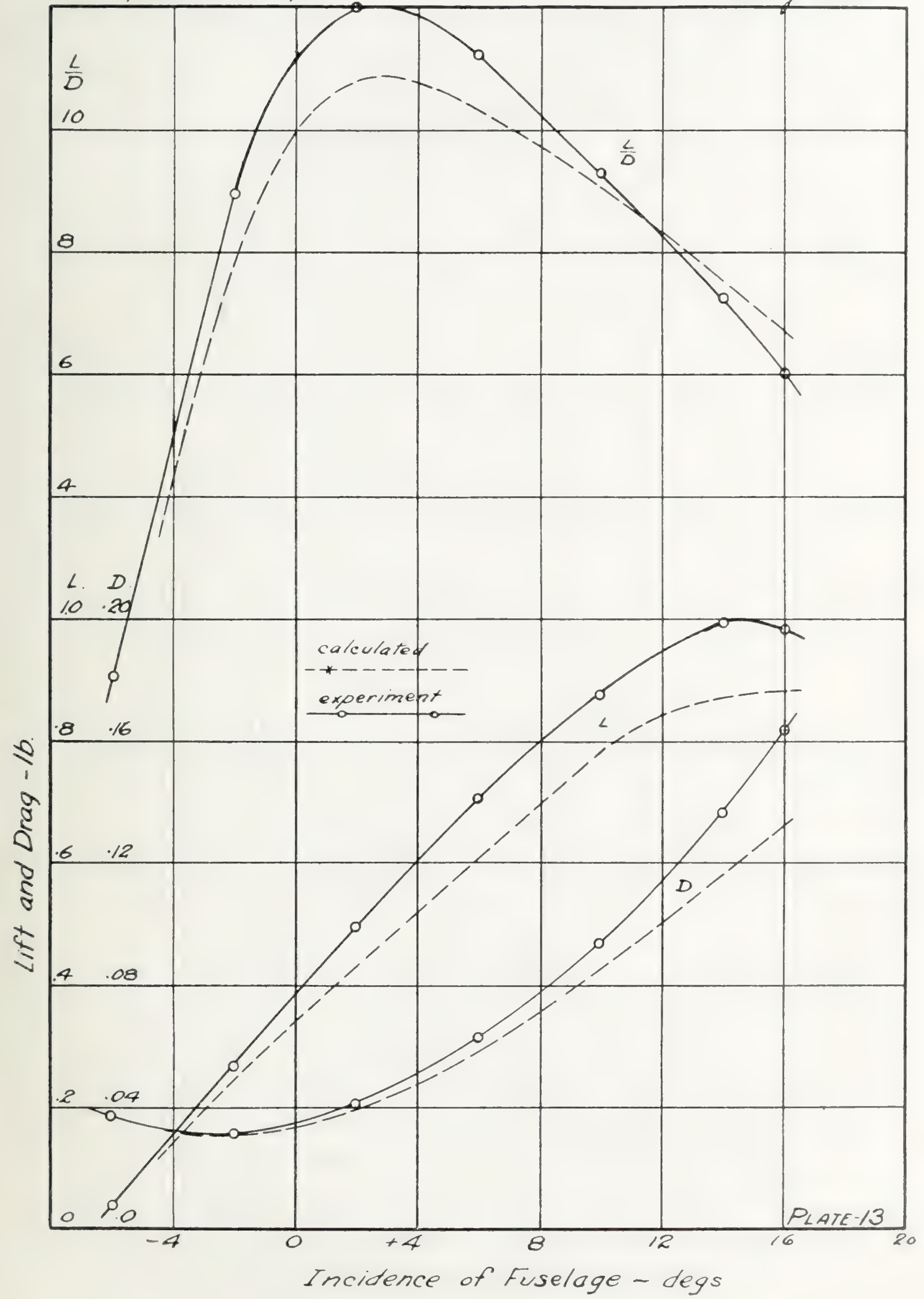


PLATE-13

Biplane-13, Fuselage B, Aerofoil-U.S.A.-27.
Angle between Aerofoil chord and Axis of Fuselage = 2°0'
Air speed - 40 f.p.s.
Geo. J. Klein.

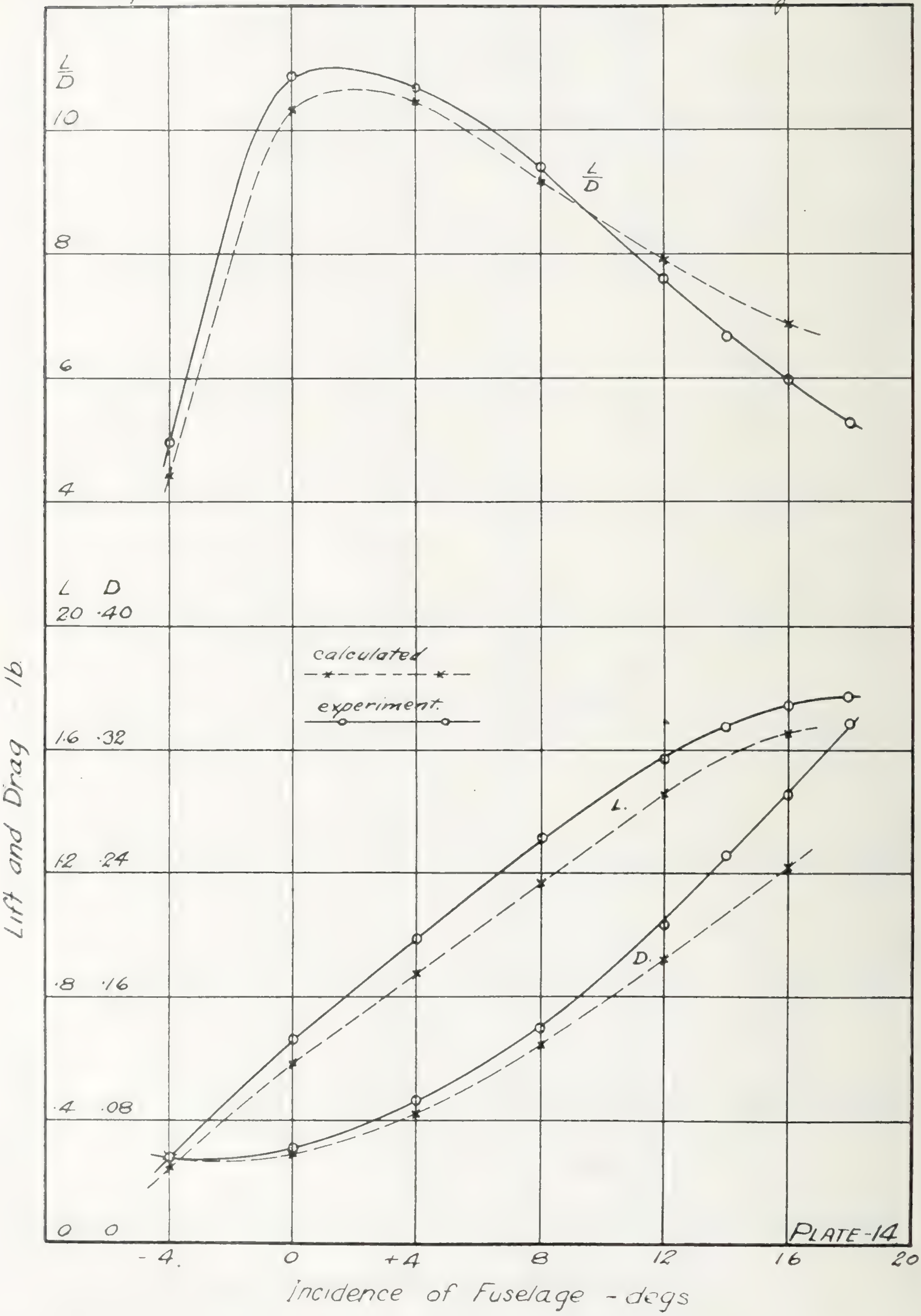


PLATE-14

Monoplane 1, Fuselage B, Aerofoil - Gött 387.
Angle between Aerofoil chord and Axis of Fuselage = 2°-0'
Air speed - 40 f.p.s.

George J. Klein.

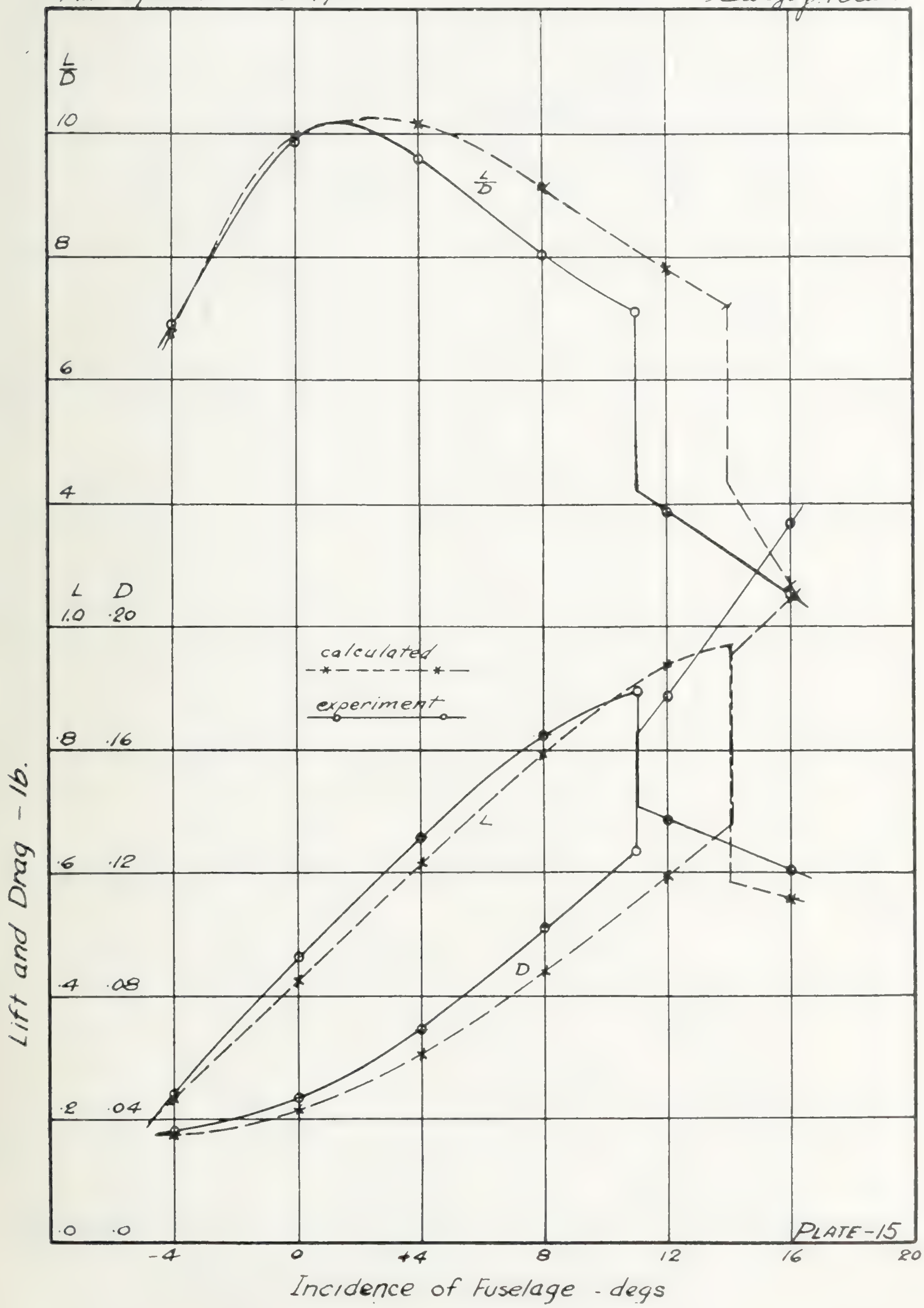
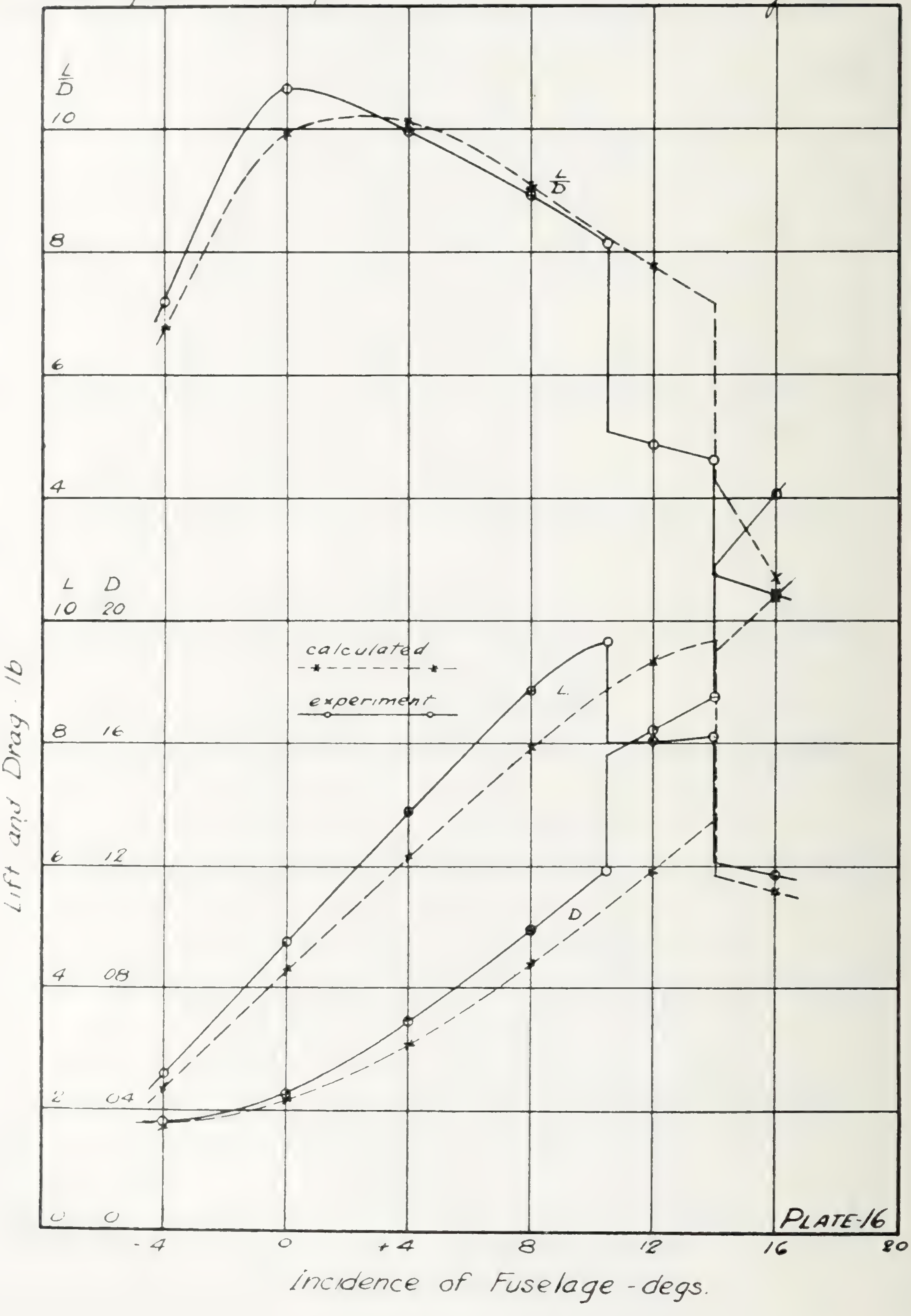
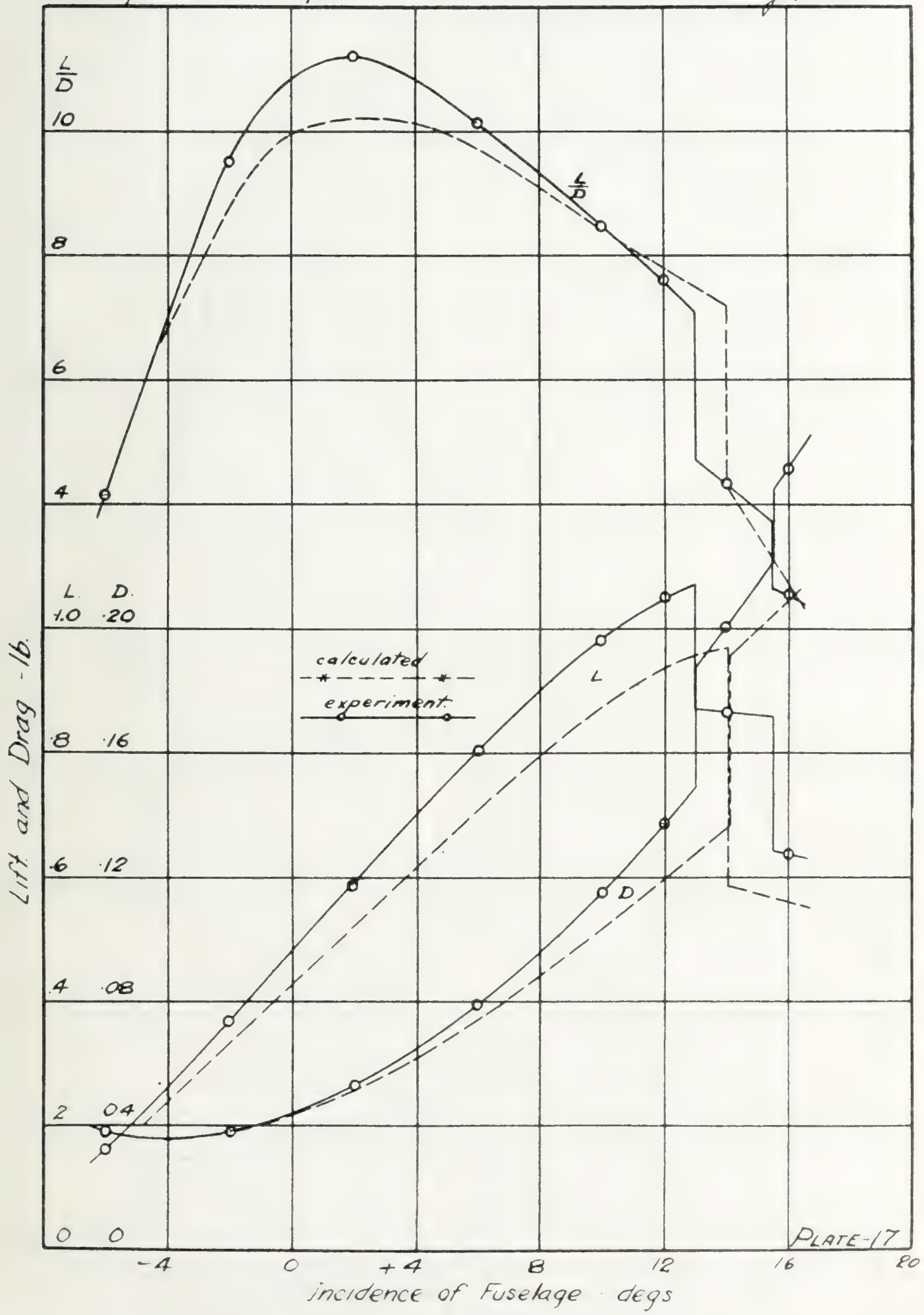


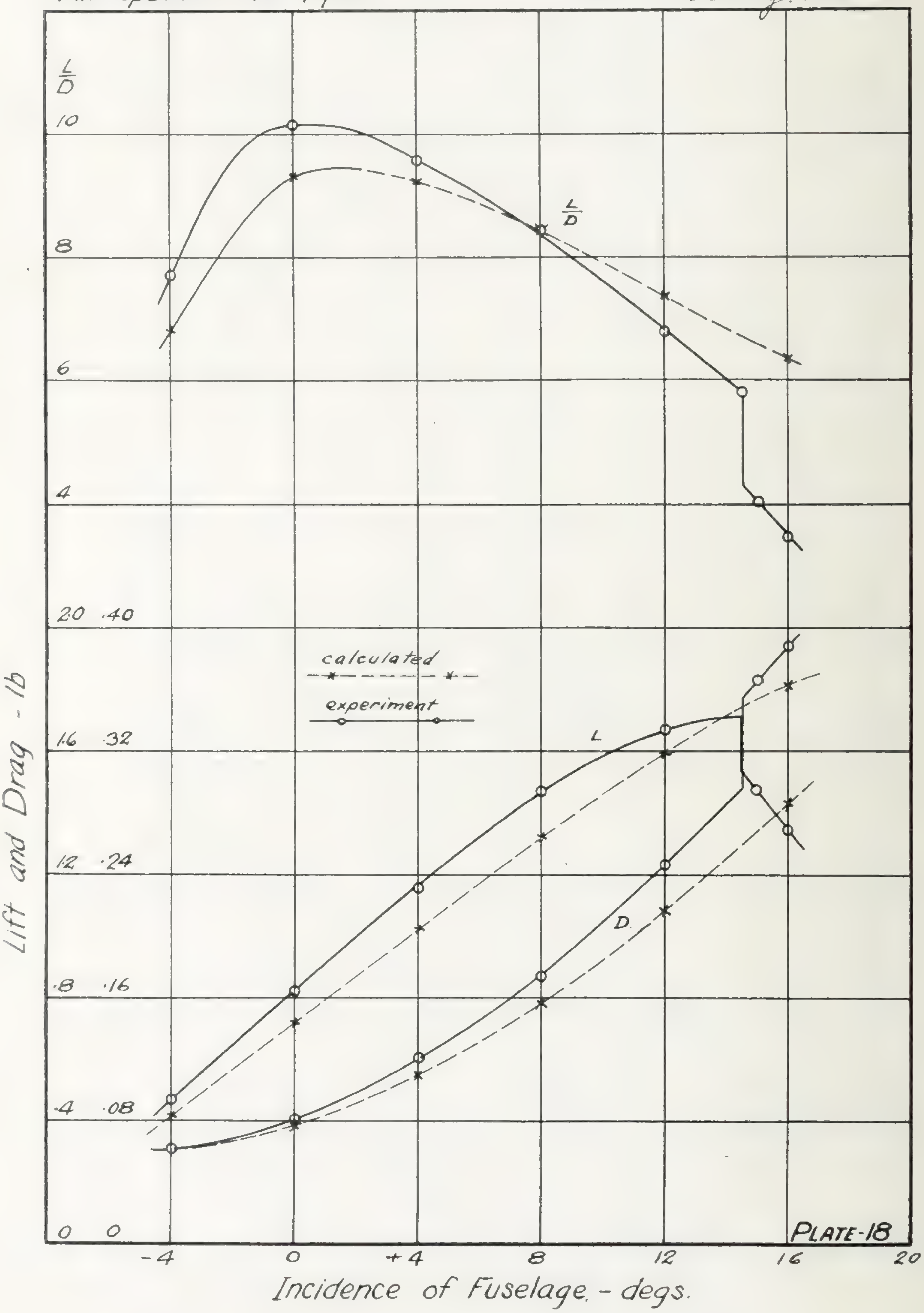
PLATE-15

Monoplane 2, Fuselage B, Aerofoil - Gott 387
Angle between Aerofoil chord and Axis of Fuselage = 2°0′
Air speed - 40 f.p.s.
Geo. J. Klein



Monoplane 3, Fuselage B, Aerofoil - Gött 387
Angle between Aerofoil chord and Axis of Fuselage = 2°0′
Air speed - 40 f.p.s.
Geo. J. Klein.

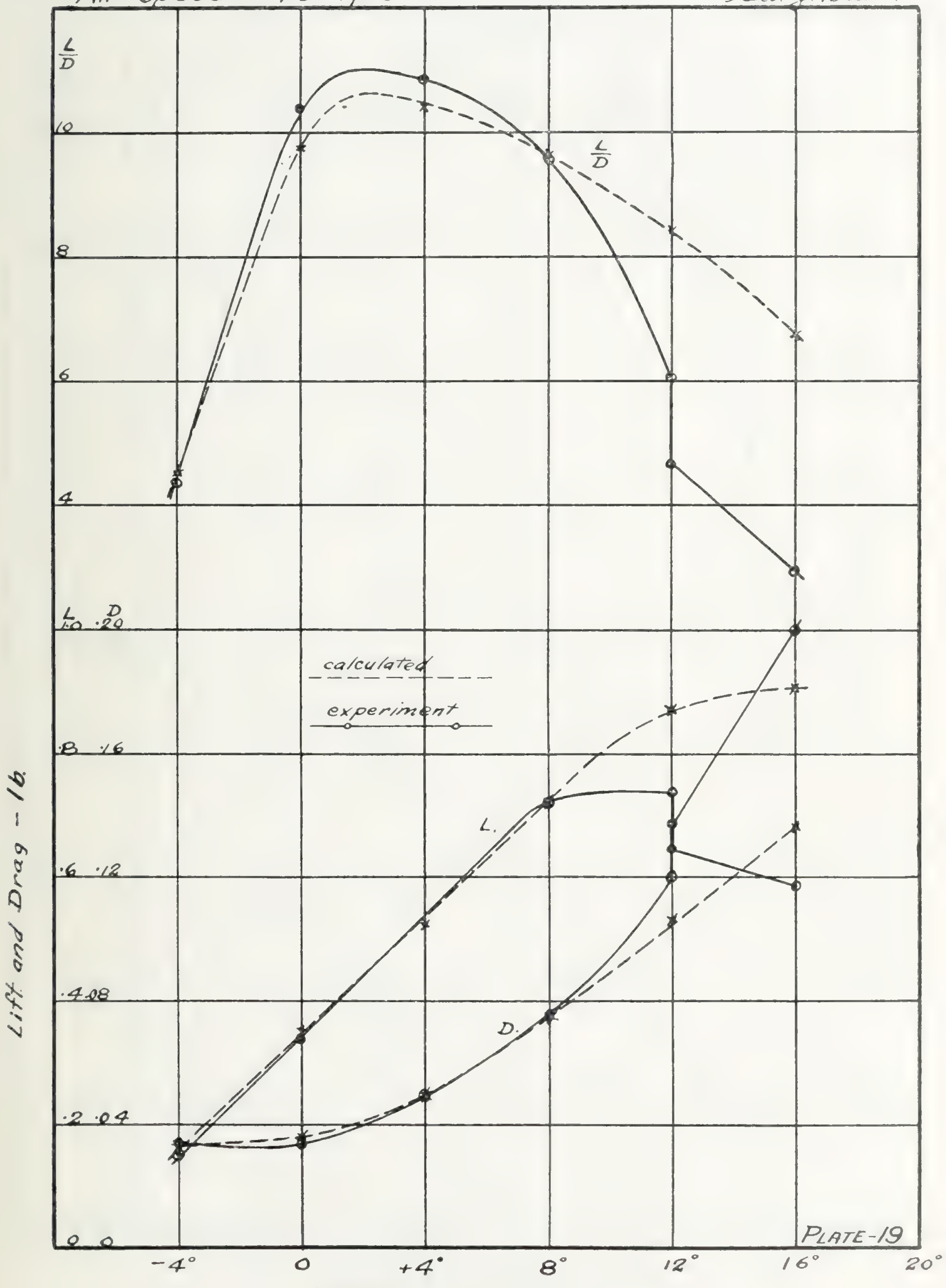


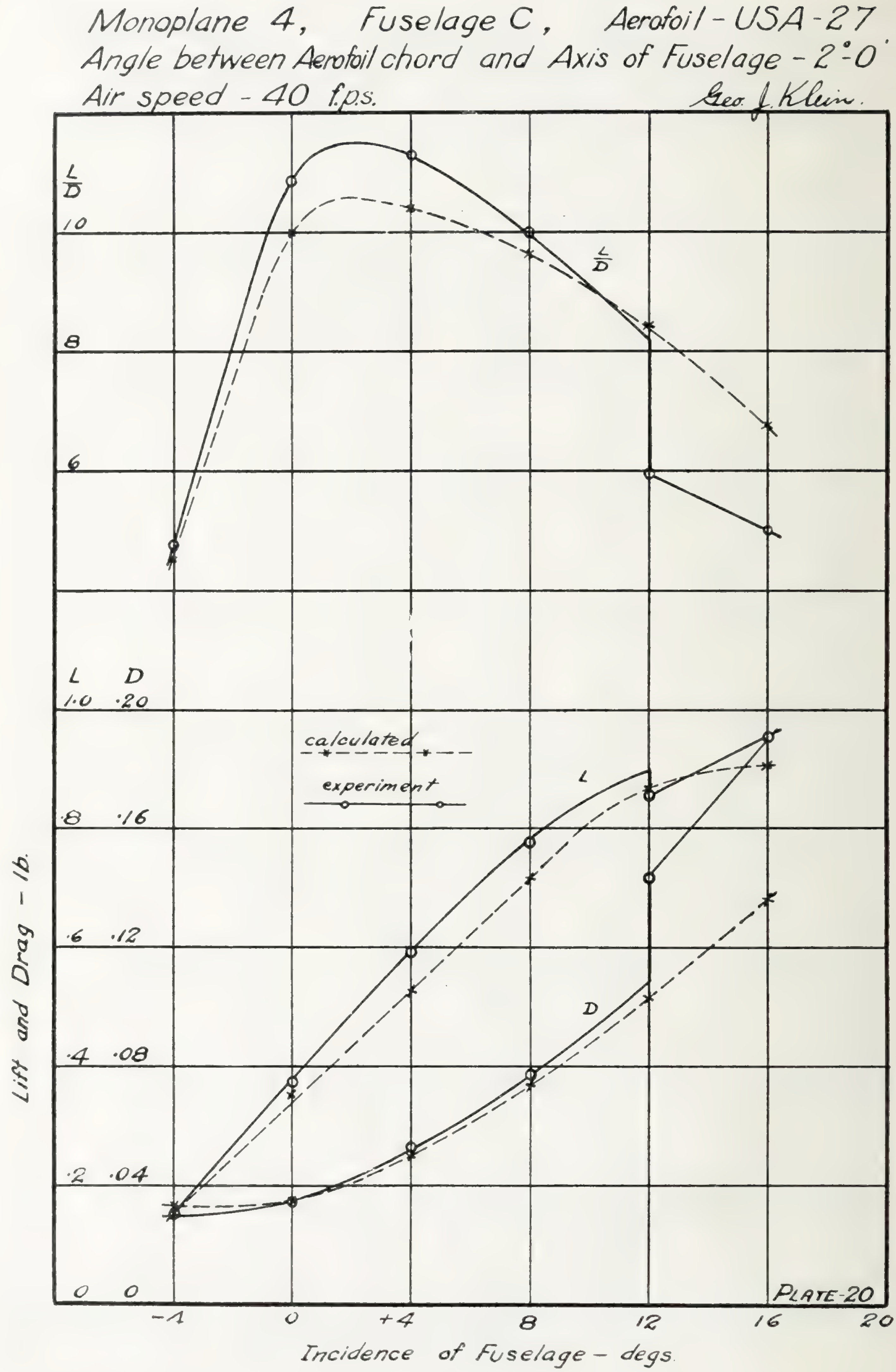


Biplane -13, Fuselage B, Aerofoil - Gött. 387
Angle between Aerofoil chord and Axis of Fuselage = 2°-0′
Air speed - 40 f.p.s.
Geo. J. Klein.

PLATE-18

Monoplane 3, Fuselage C, Aerofoil USA-27.
Angle between Aerofoil chord and Axis of Fuselage 2°-0′
Air speed - 40 f.p.s. Geo. J. Klein.




Monoplane 4, Fuselage C, Aerofoil - USA-27
Angle between Aerofoil chord and Axis of Fuselage - 2°-0'
Air speed - 40 f.p.s.
Geo. J. Klein.
L/D
10
8
6
L/D
L D
1.0 ·20
·8 ·16
·6 ·12
·4 ·08
·2 ·04
0 0
calculated
experiment
L
D
Lift and Drag - lb.
PLATE-20
-4
0
+4
8
12
16
20
Incidence of Fuselage - degs.

Monoplane 5, Fuselage C, Aerofoil - USA-27.
Angle between Aerofoil chord and Axis of Fuselage = 2° - 0′
Air speed - 40 fps.

Geo. J. Klein.

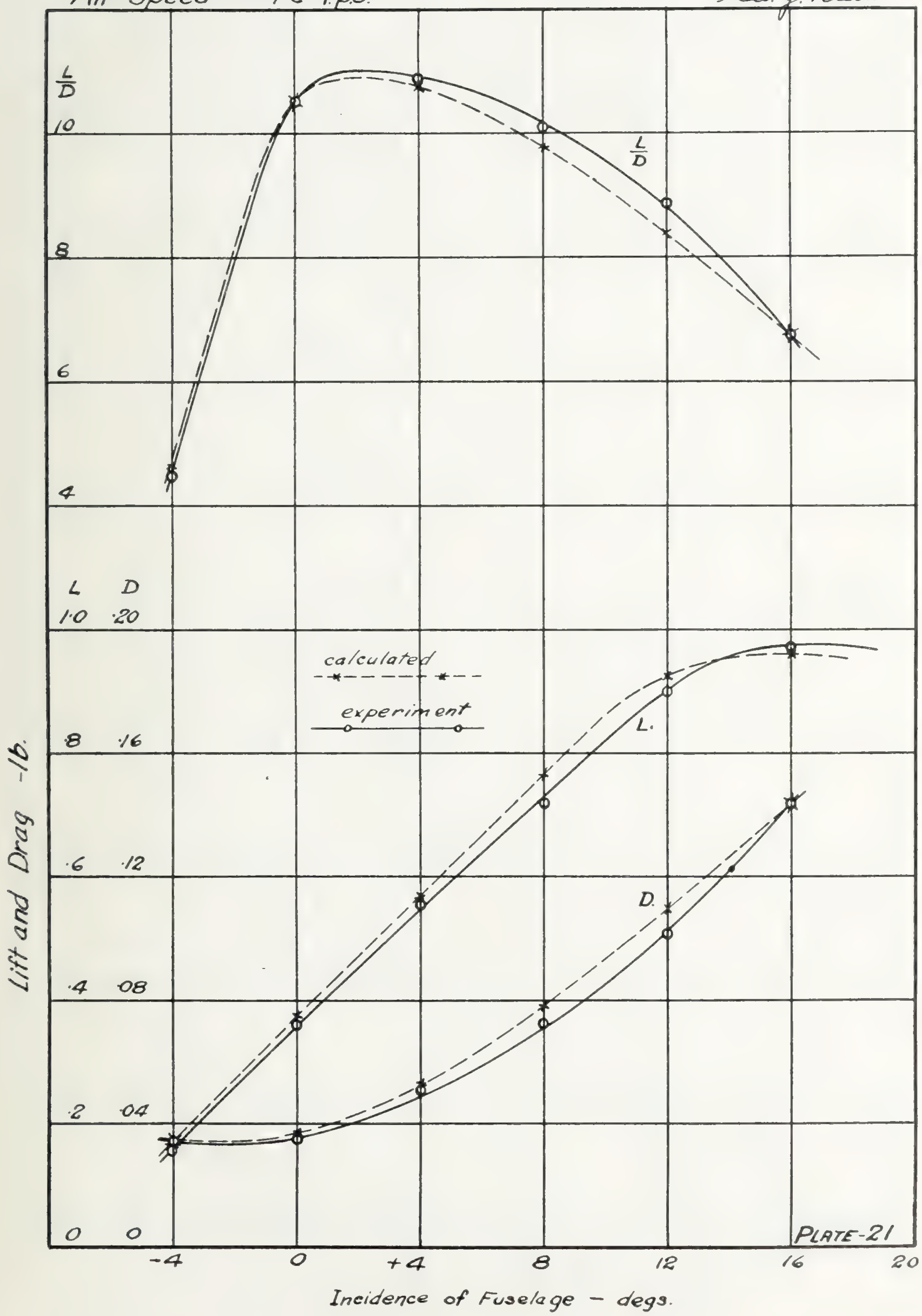


PLATE-21

Monoplane 5-F, Fuselage C, Aerofoil - USA-27
Angle between Aerofoil chord and Axis of Fuselage = 2° 0′
Air speed - 40 fps.
Geo. J. Klein.

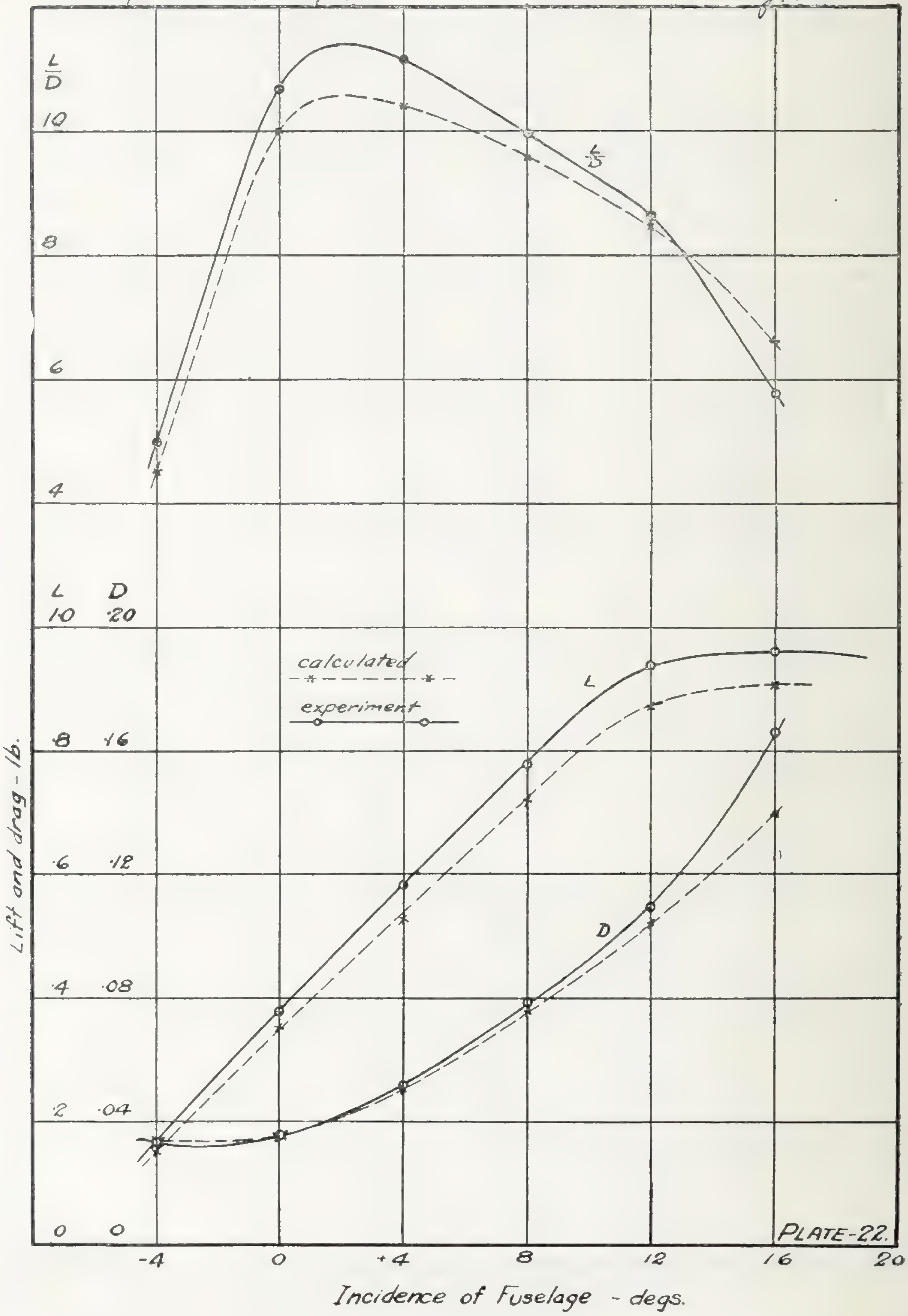


PLATE-22.

Monoplane 6, Fuselage C, Aerofoil - U.S.A.-27.
Angle between Aerofoil chord and Axis of Fuselage = 2°0
Air speed - 40 f.p.s.
Geo. J. Klein.

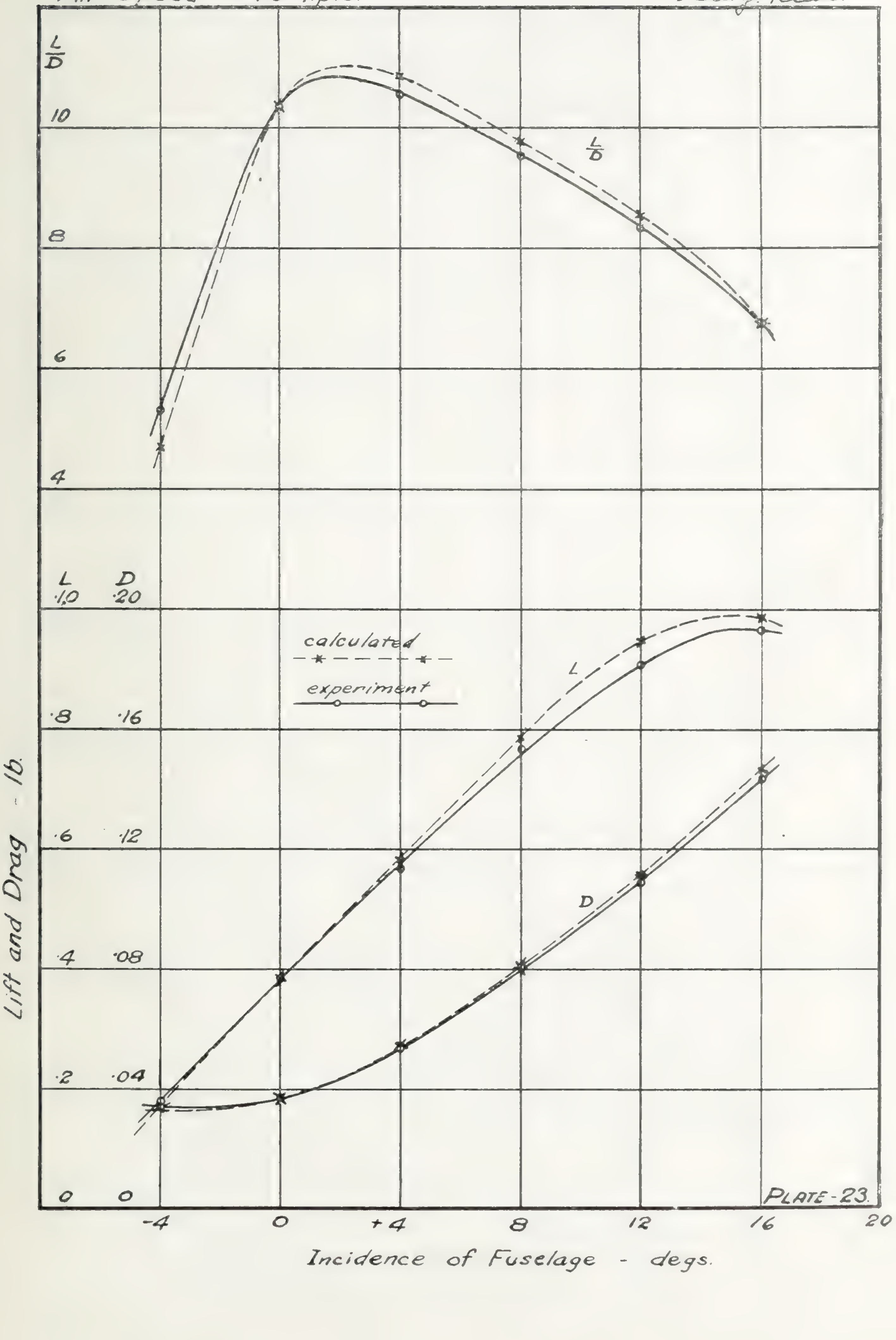
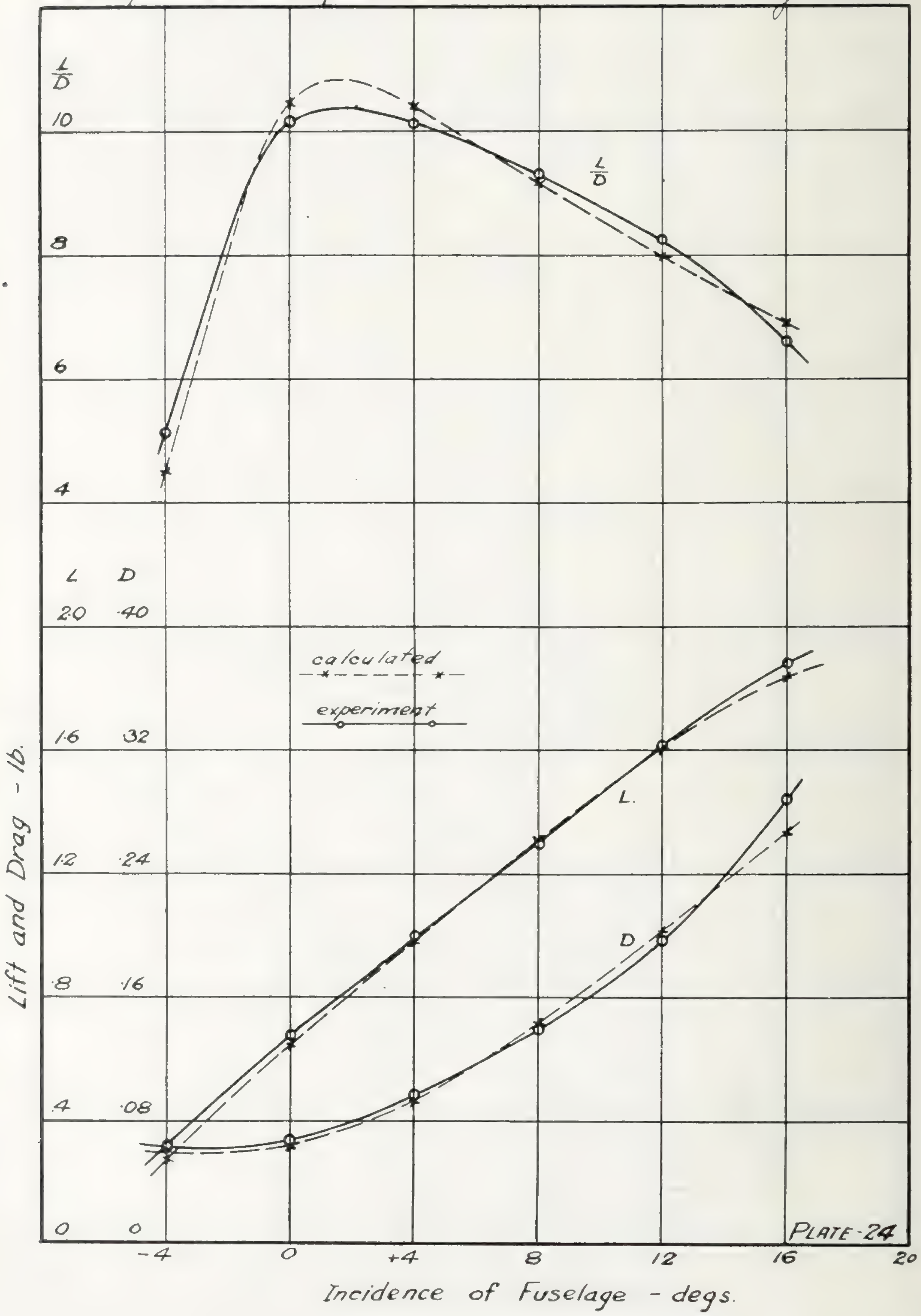
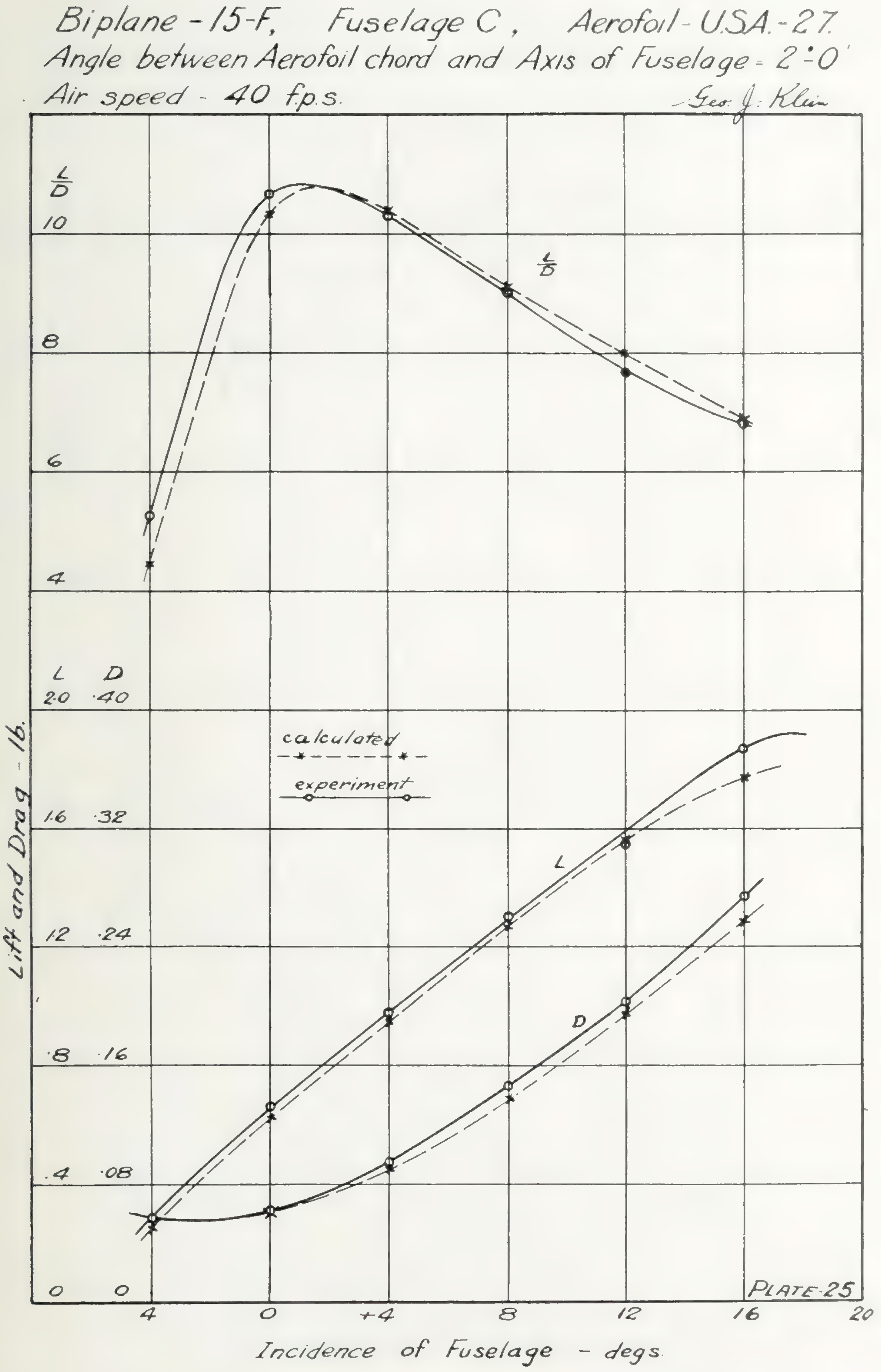


PLATE-23.

Biplane -15, Fuselage C, Aerofoil- U.S.A.-27.
Angle between Aerofoil chord and Axis of Fuselage = 2°-0′
Air speed - 40 f.p.s.
Geo. J. Klein.




Biplane - 15-F, Fuselage C, Aerofoil - U.S.A. - 27.
Angle between Aerofoil chord and Axis of Fuselage = 2°-0'
Air speed - 40 f.p.s.
Geo. J. Klein
L/D
10
8
6
4
L D
2.0 ·40
1.6 ·32
1.2 ·24
·8 ·16
.4 ·08
0 0
calculated
experiment
L
D
Lift and Drag - lb.
4
0
+4
8
12
16
20
PLATE-25
Incidence of Fuselage - degs.

Biplane - 26, Fuselage C, Aerofoil - U.S.A.-27.
Angle between Aerofoil chord and Axis of Fuselage = 2°-0′
Air speed - 40 f.p.s.
Geo. J. Klein.

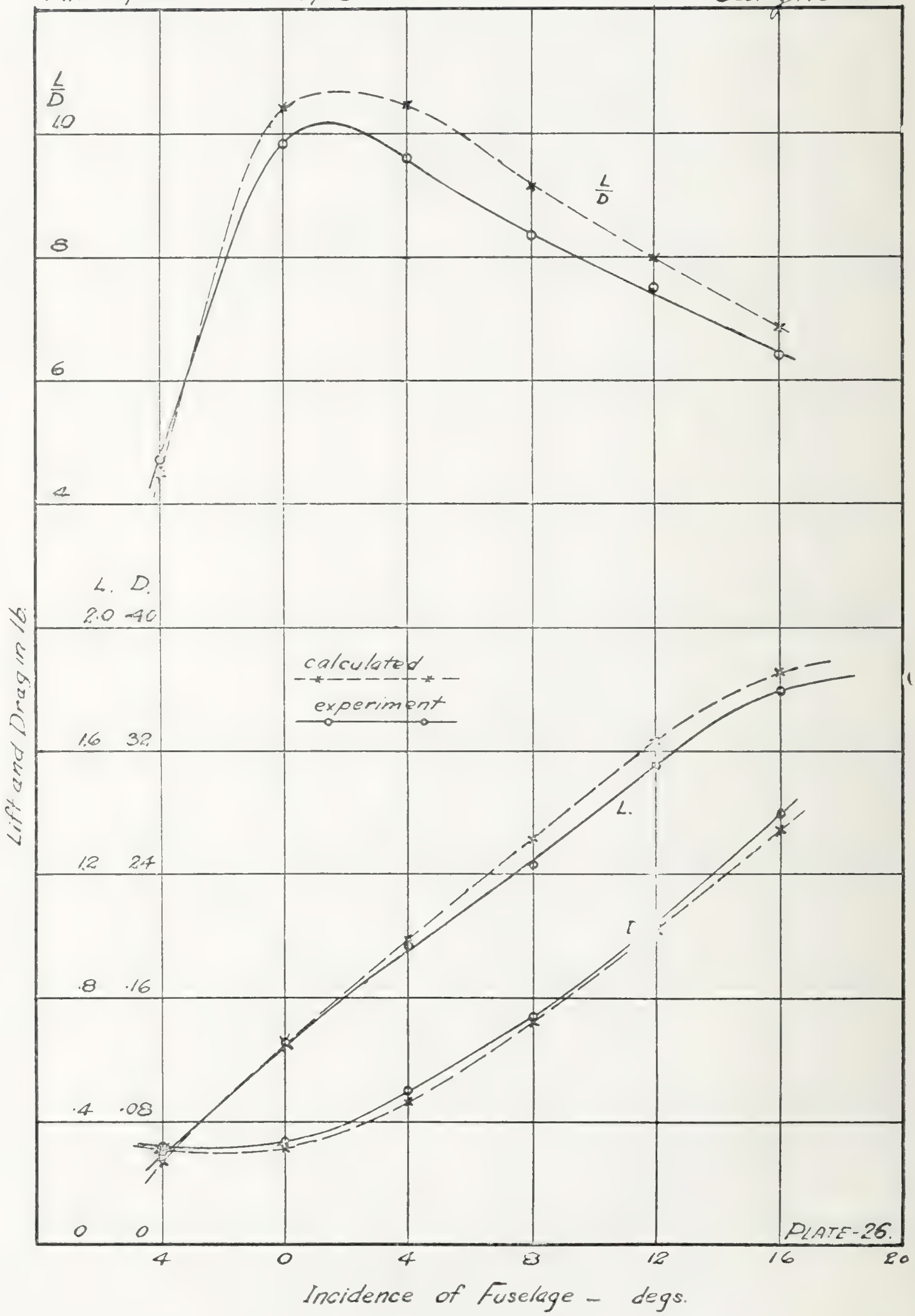
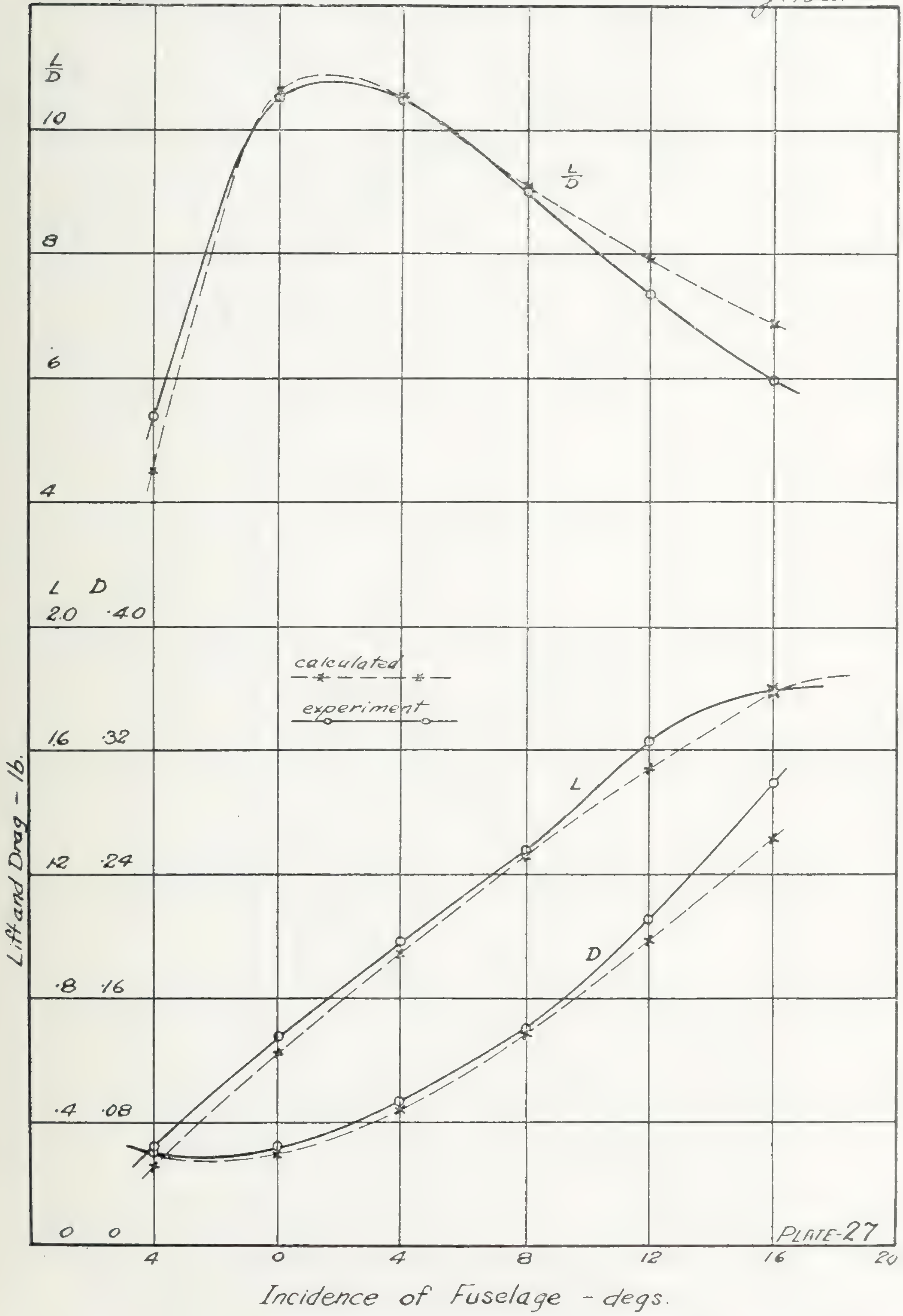
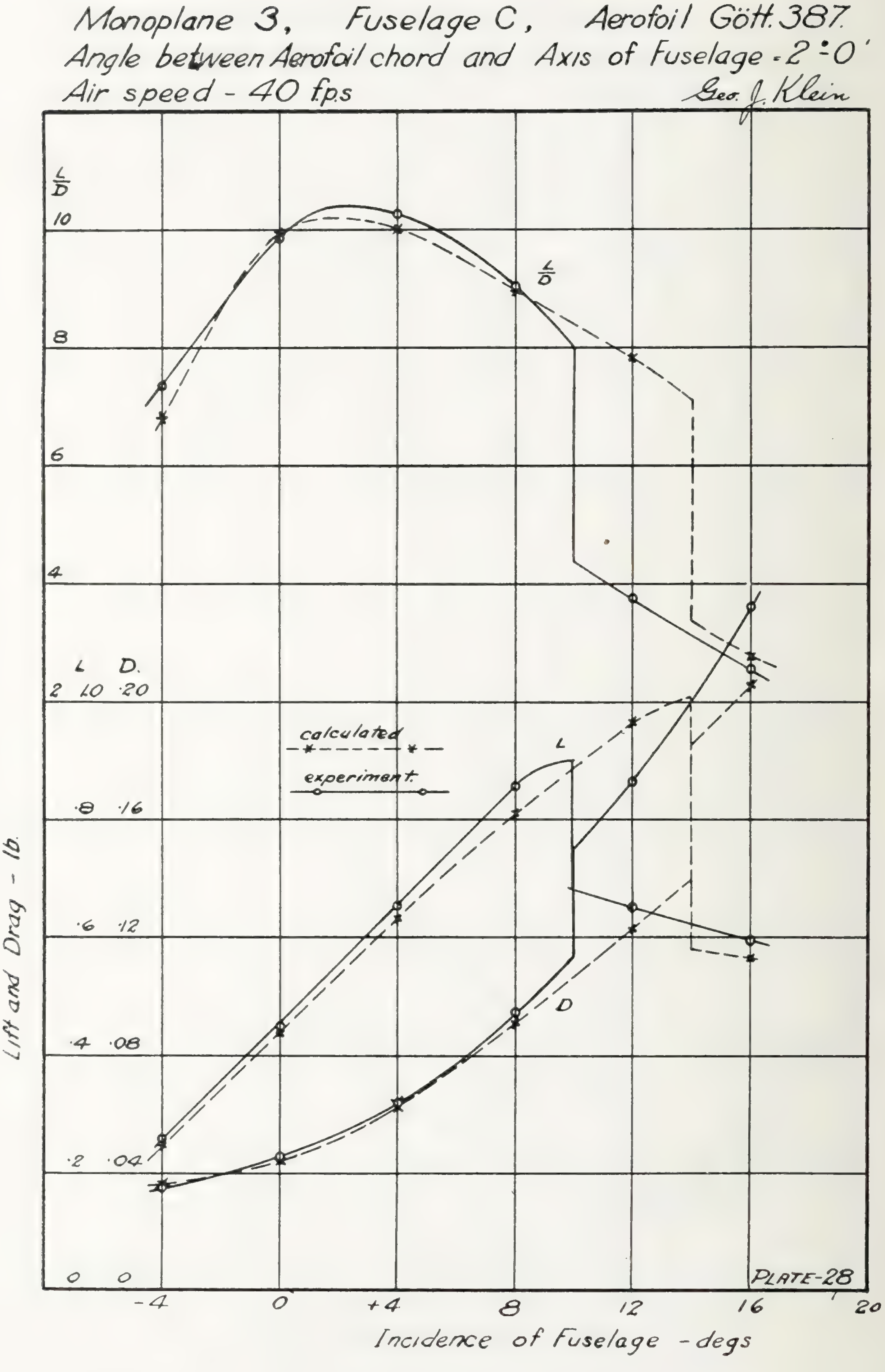


PLATE-26.

Biplane - 37, Fuselage C, Aerofoil: U.S.A.-27
Angle between Aerofoil chord and Axis of Fuselage = 2°0′
Air speed 40 fp.s.
Geo. J. Klein.





Monoplane 3, Fuselage C, Aerofoil Gött. 387.
Angle between Aerofoil chord and Axis of Fuselage = 2°0'
Air speed - 40 f.p.s.
Geo. J. Klein

PLATE-28

Monoplane-4, Fuselage-C, Aerofoil-Gött 387.
Angle between Aerofoil chord and Axis of Fuselage = 2°-0'
Air speed - 40 f.p.s.
Geo. J. Klein.

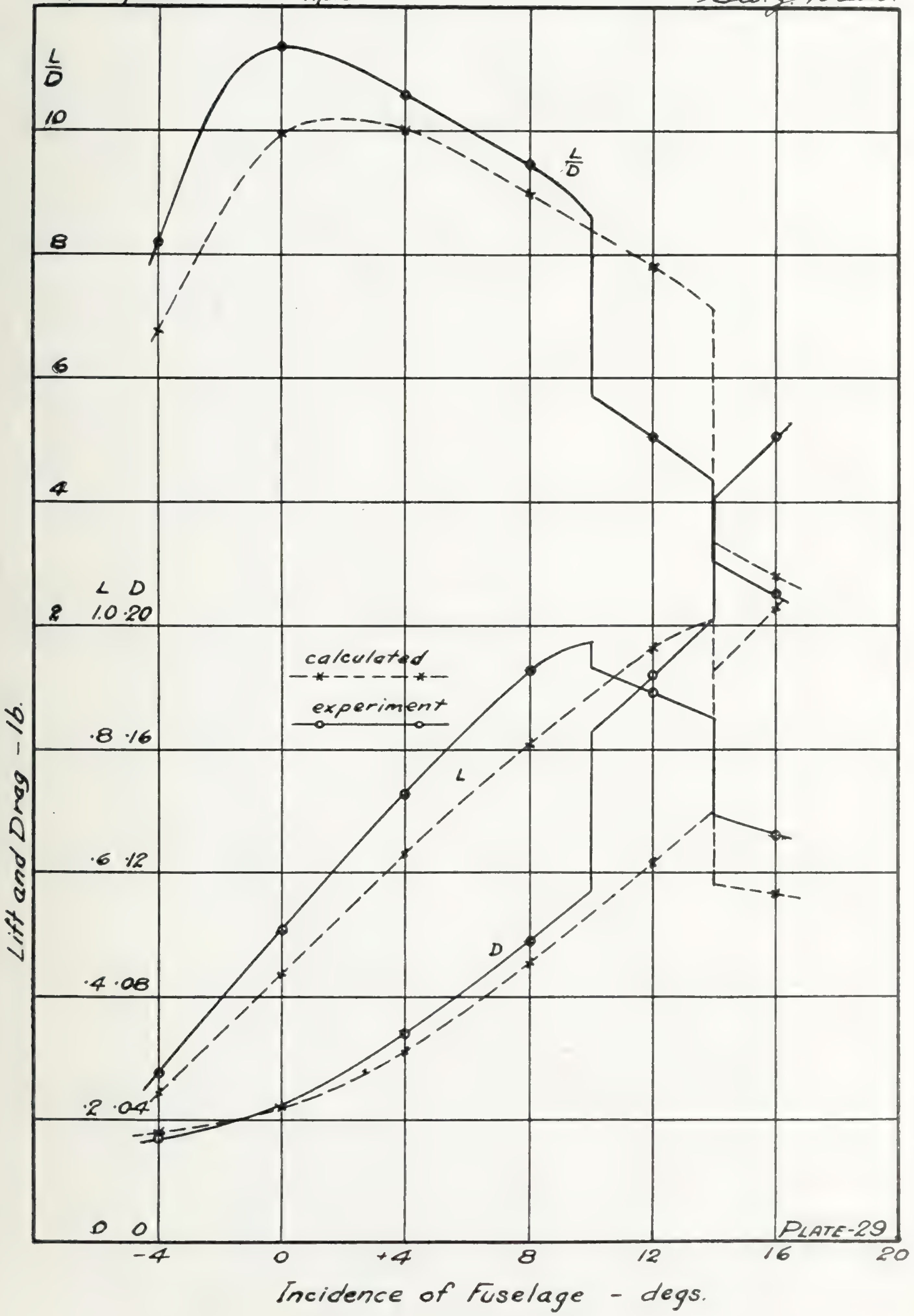
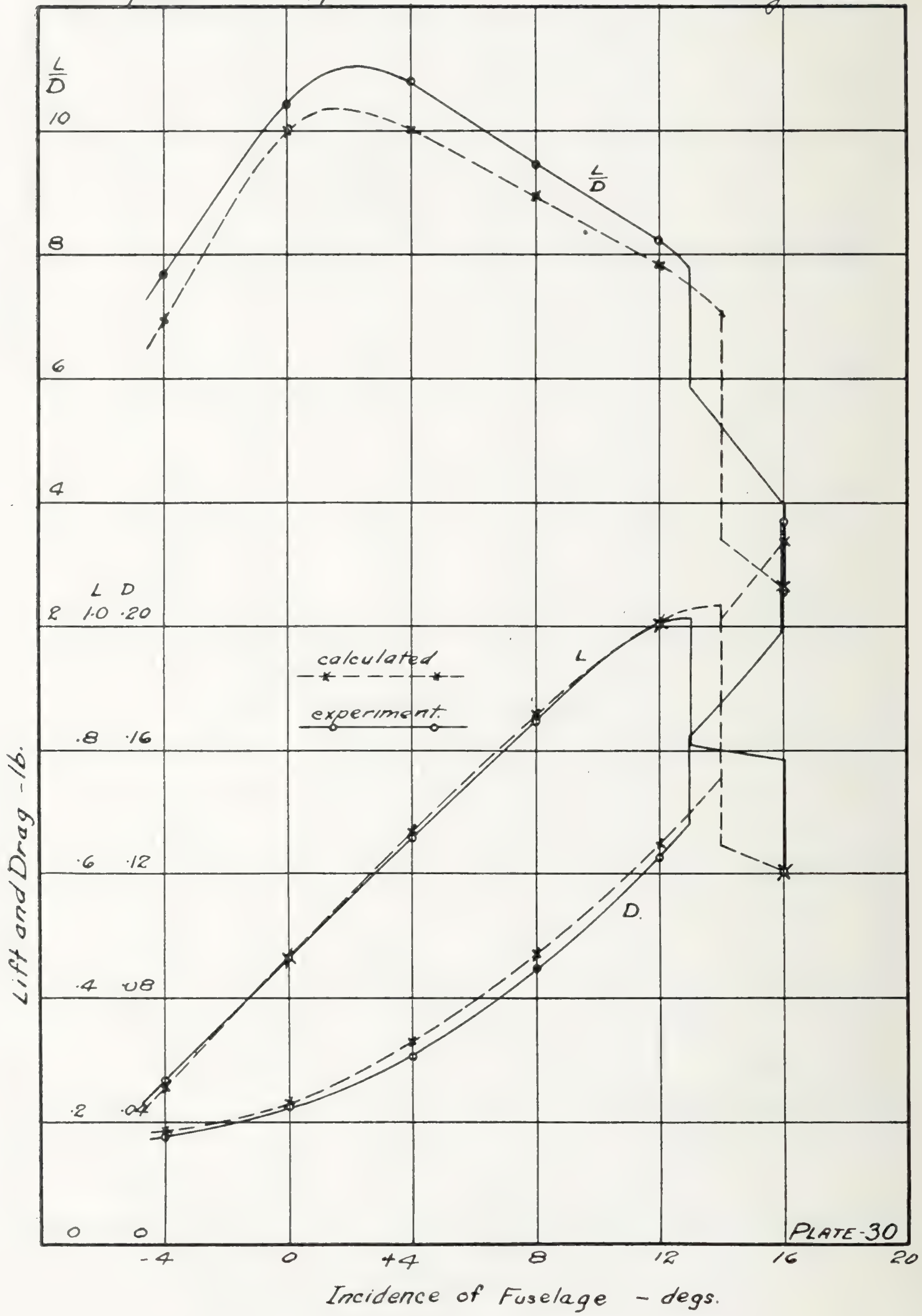


PLATE-29

Monoplane 5, Fuselage C, Aerofoil - Gött. 387.
Angle between Aerofoil chord and Axis of Fuselage = 2°0'
Air speed 40 f.p.s.
Geo. J. Klein.



Monoplane-5-F, Fuselage C, Aerofoil-Gött. 387.
Angle between Aerofoil chord and Axis of Fuselage = 2°-0′
Air speed - 40 f.p.s.

Geo. J. Klein.

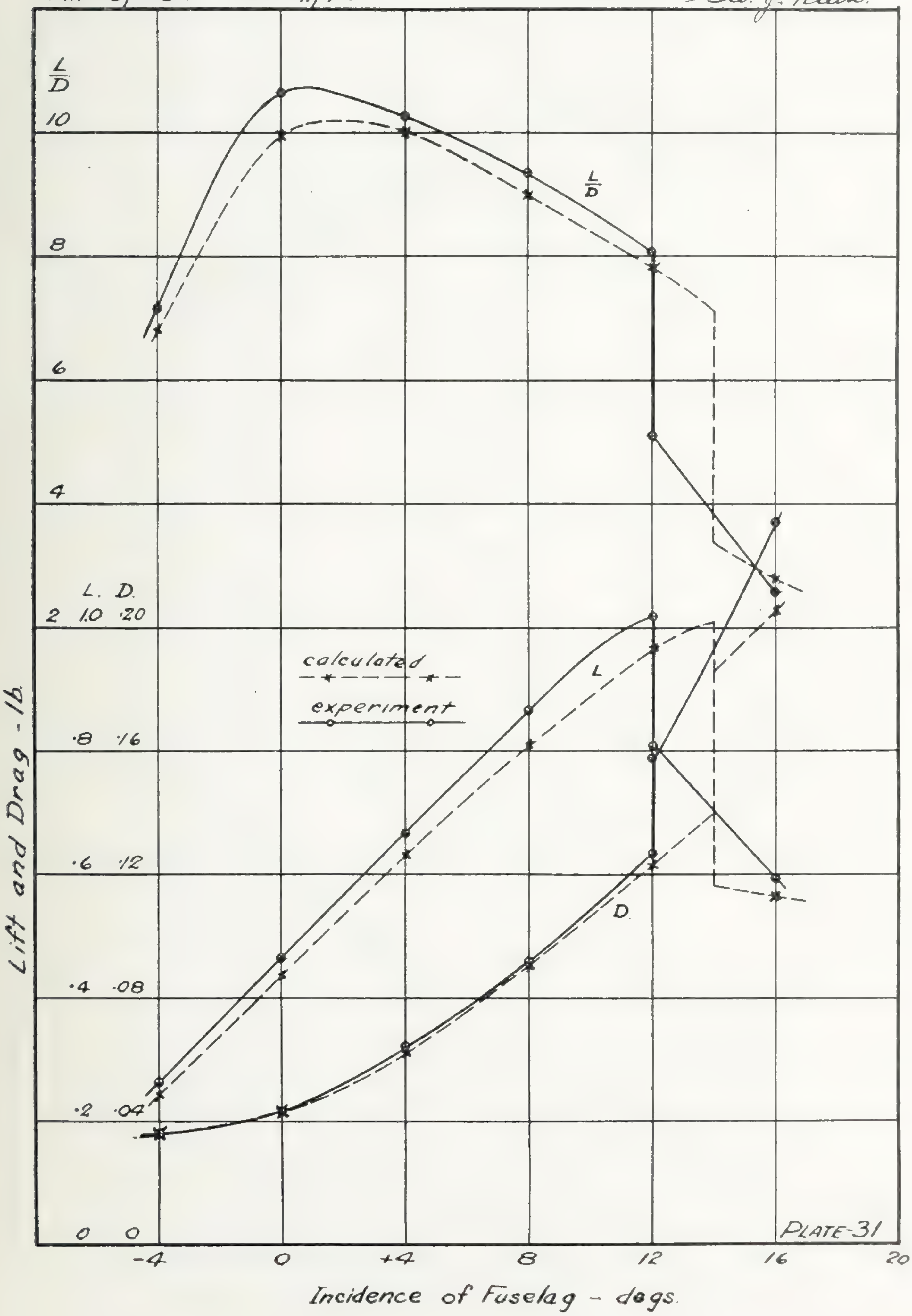


PLATE-31

Monoplane 6, Fuselage C, Aerofoil - Gött. 387.
Angle between Aerofoil chord and Axis of Fuselage = 2°0′
Air speed - 40 f.p.s.
Geo. J. Klein.

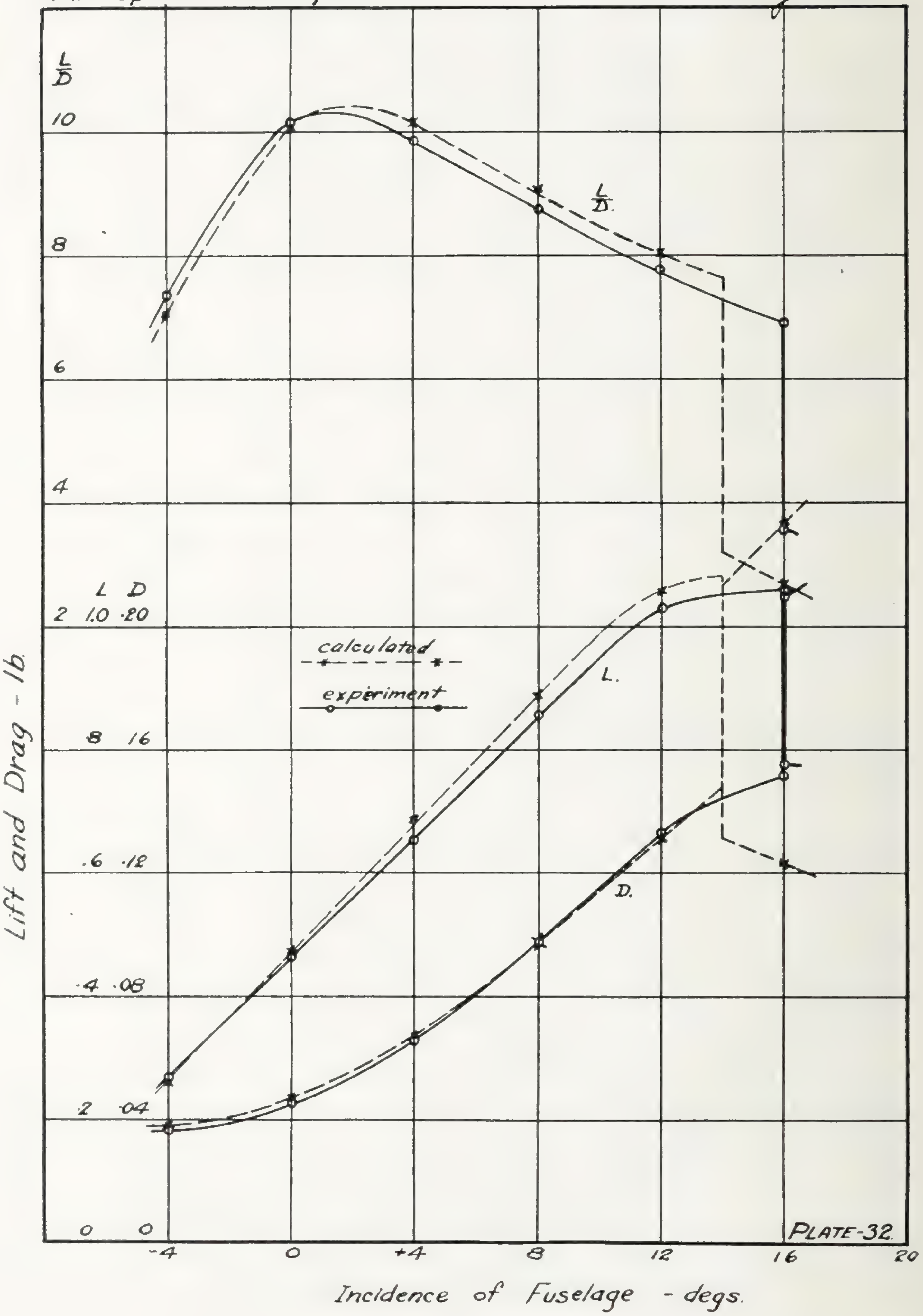
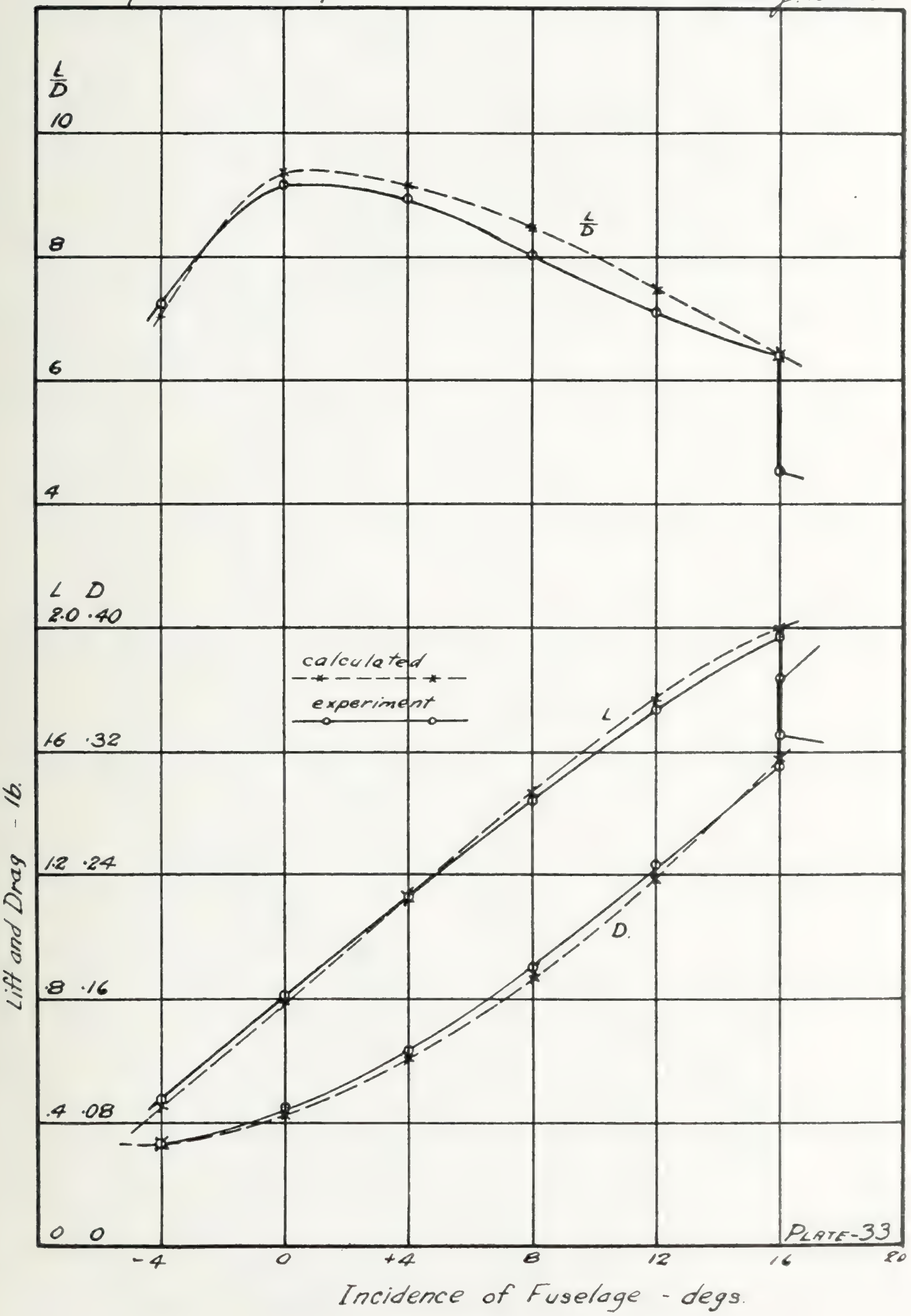
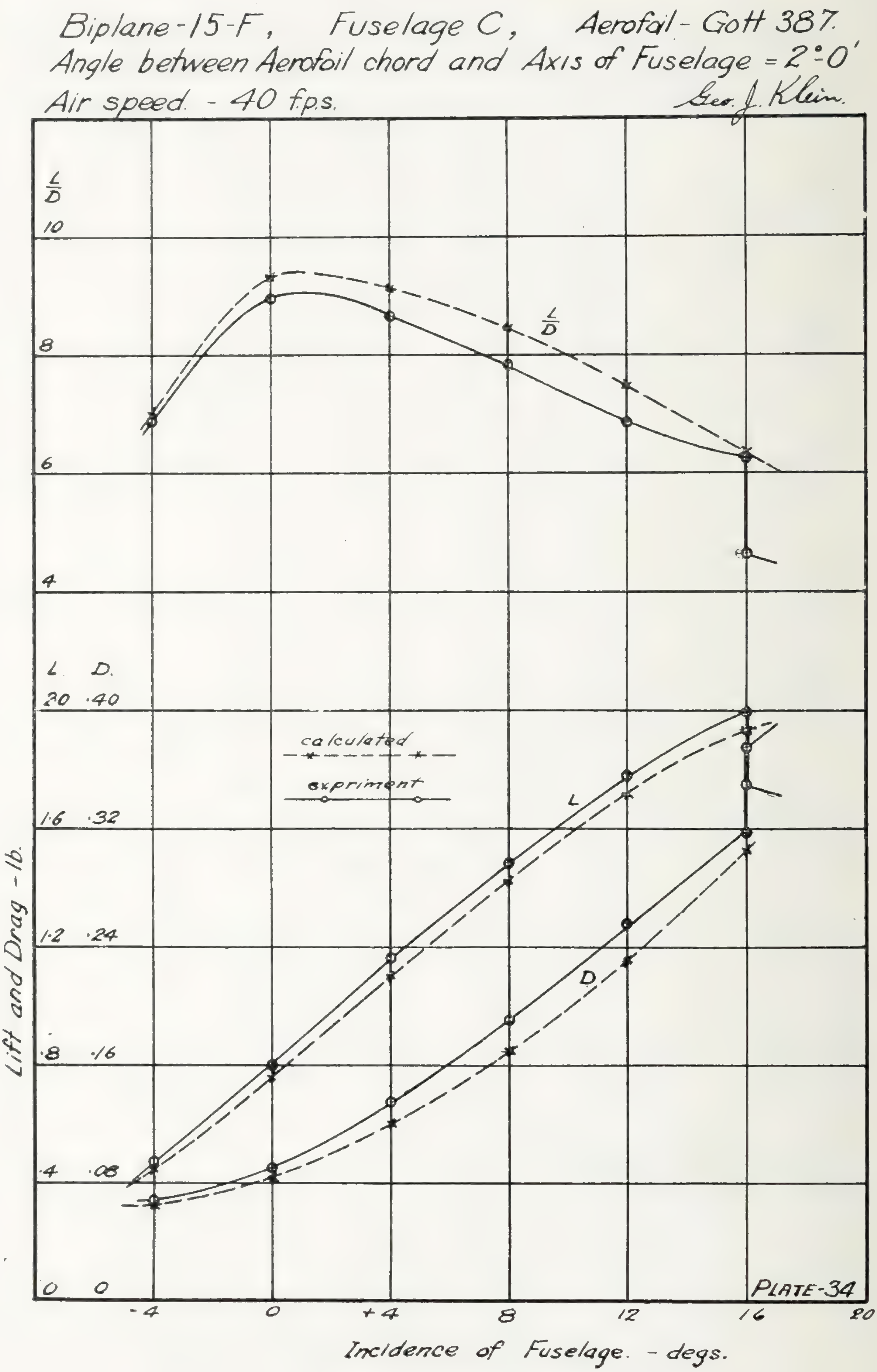


PLATE-32.

Biplane - 15, Fuselage C, Aerofoil - Gött. 387.
Angle between Aerofoil chord and Axis of Fuselage = 2° 0′
Air speed - 40 f.p.s.
Geo. J. Klein.





Biplane-15-F, Fuselage C, Aerofoil - Gott 387.
Angle between Aerofoil chord and Axis of Fuselage = 2°-0'
Air speed - 40 f.p.s.
Geo. J. Klein.

Biplane - 26, Fuselage C, Aerofoil - Gött. 387.
Angle between Aerofoil chord and Axis of Fuselage = 2°-0′
Air speed - 40 f.p.s.

Geo. J. Klein.

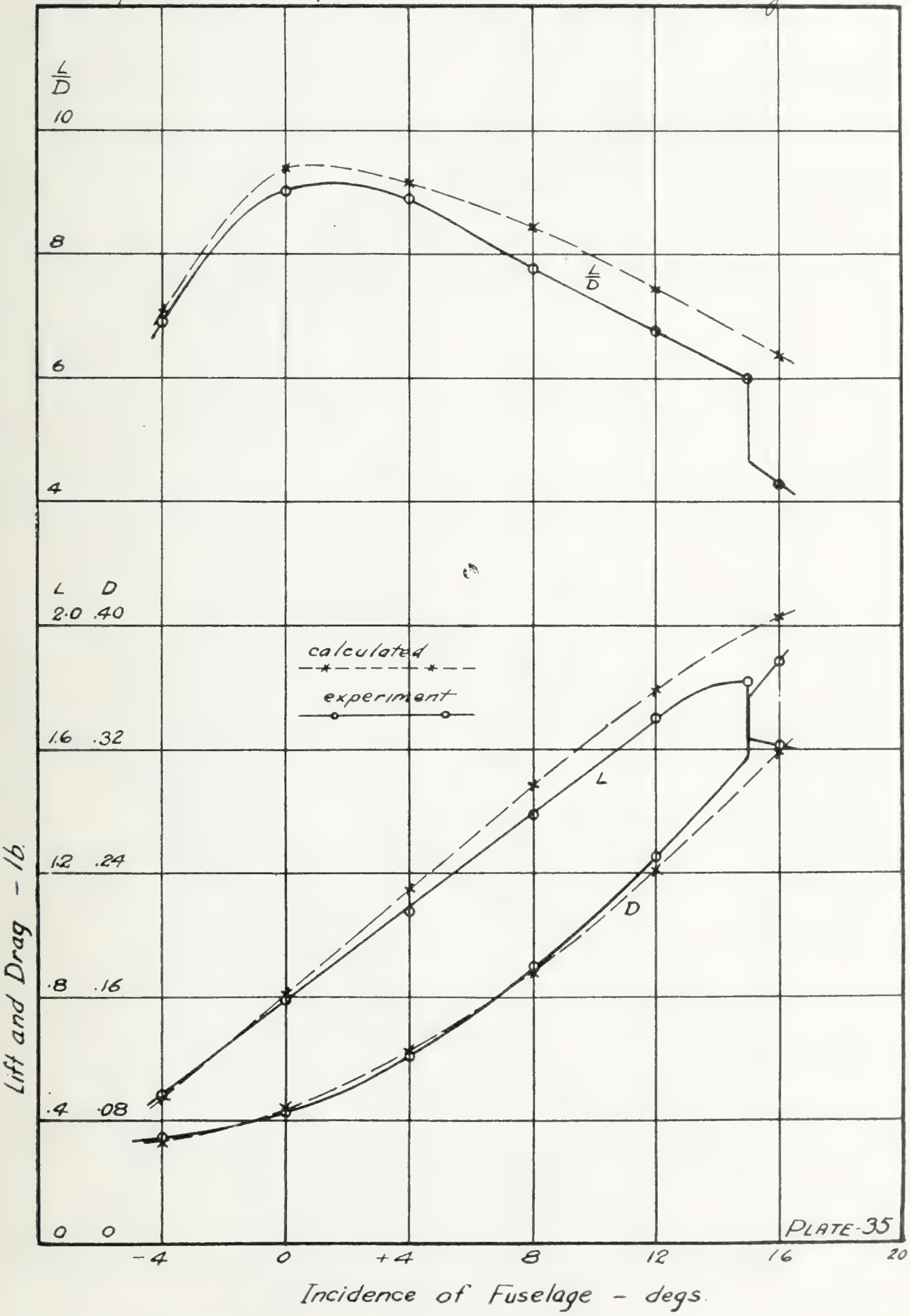


PLATE-35

Biplane 37, Fuselage C, Aerofoil - Gött. 387.
Angle between Aerofoil chord and Axis of Fuselage = 2°-0'
Air speed - 40 f.p.s.

Geo. J. Klein.

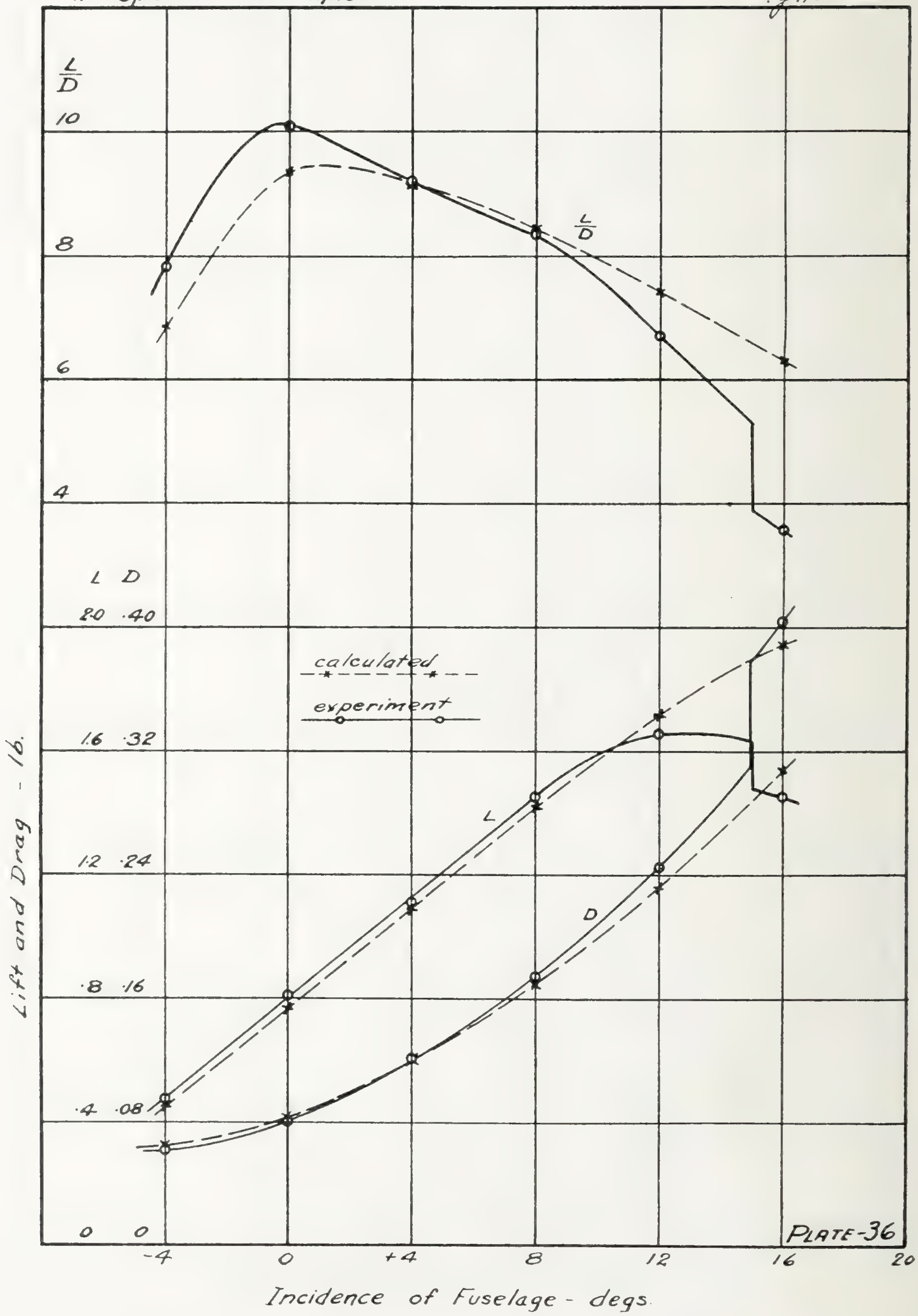


PLATE-36

Monoplane 1, Fuselage A, Aerofoil - R.A.F.-15.
Angle between Aerofoil chord and Axis of Fuselage = 3°-0′
Air speed - 40 f.p.s.
Geo. J. Klein.

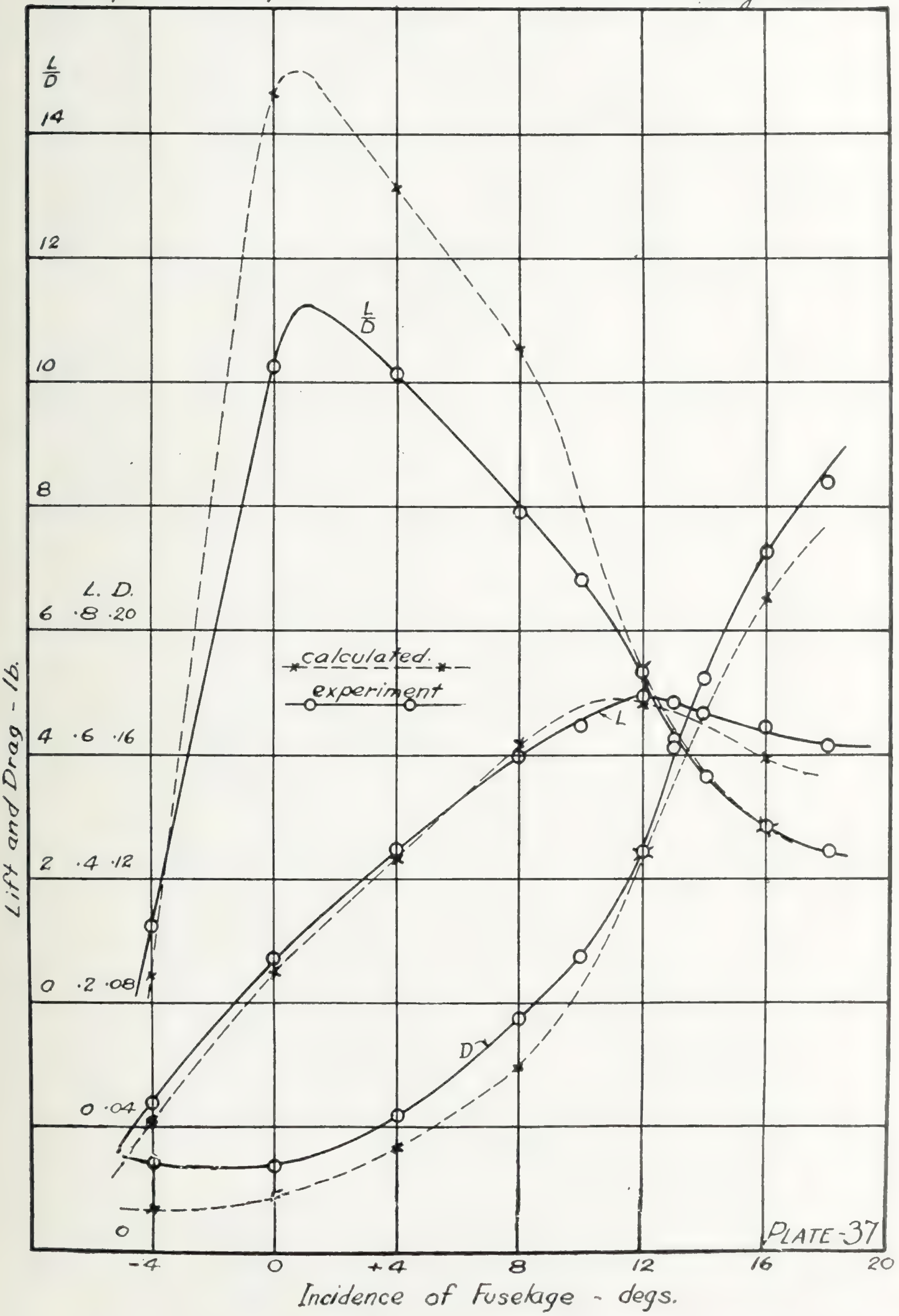


PLATE-37

Monoplane I-F, Fuselage A, Aerofoil - RAF-15
Angle between Aerofoil chord and Axis of Fuselage = 3°-0′
Air speed 40 f.p.s. Geo. J. Klein.

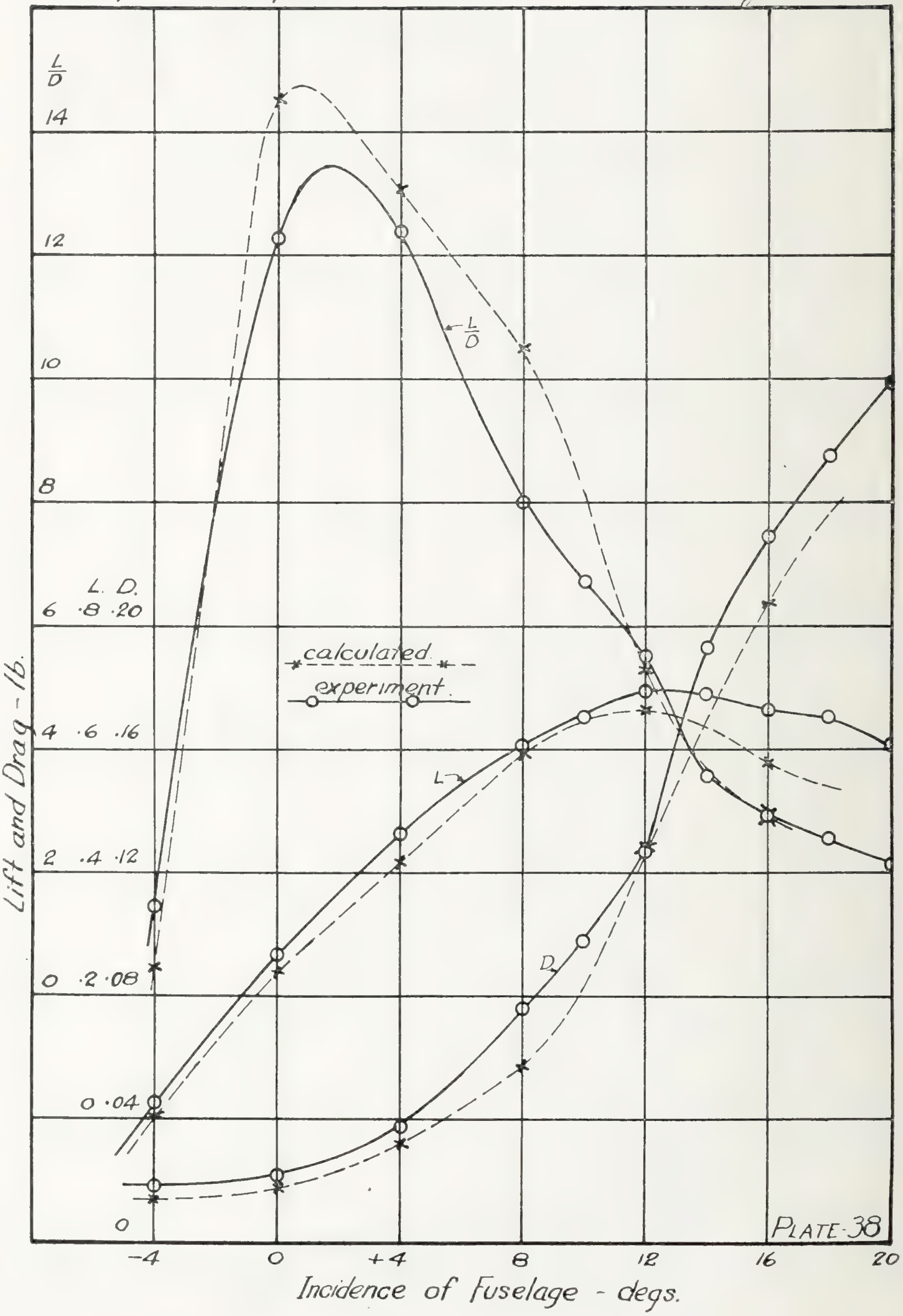
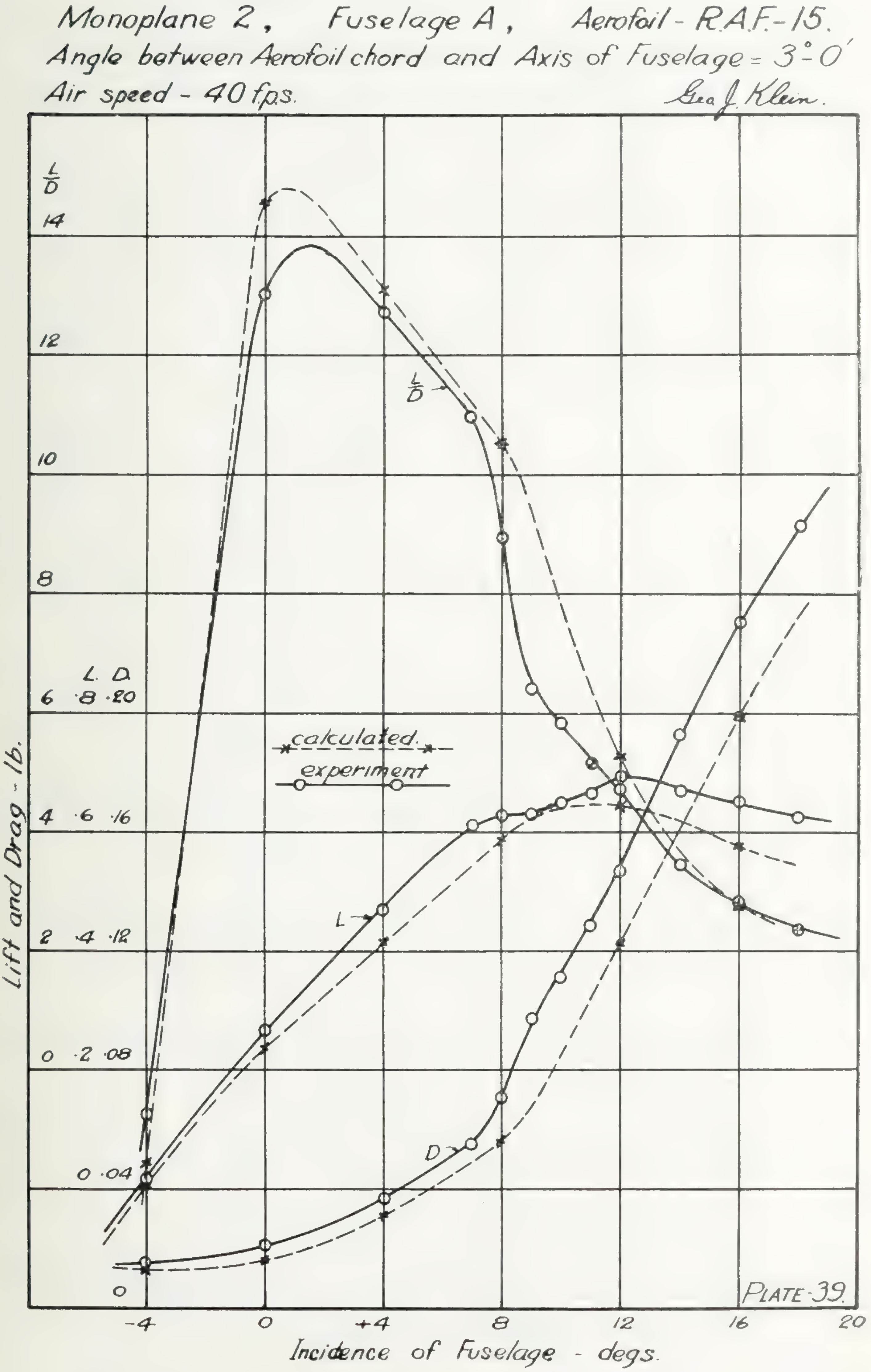
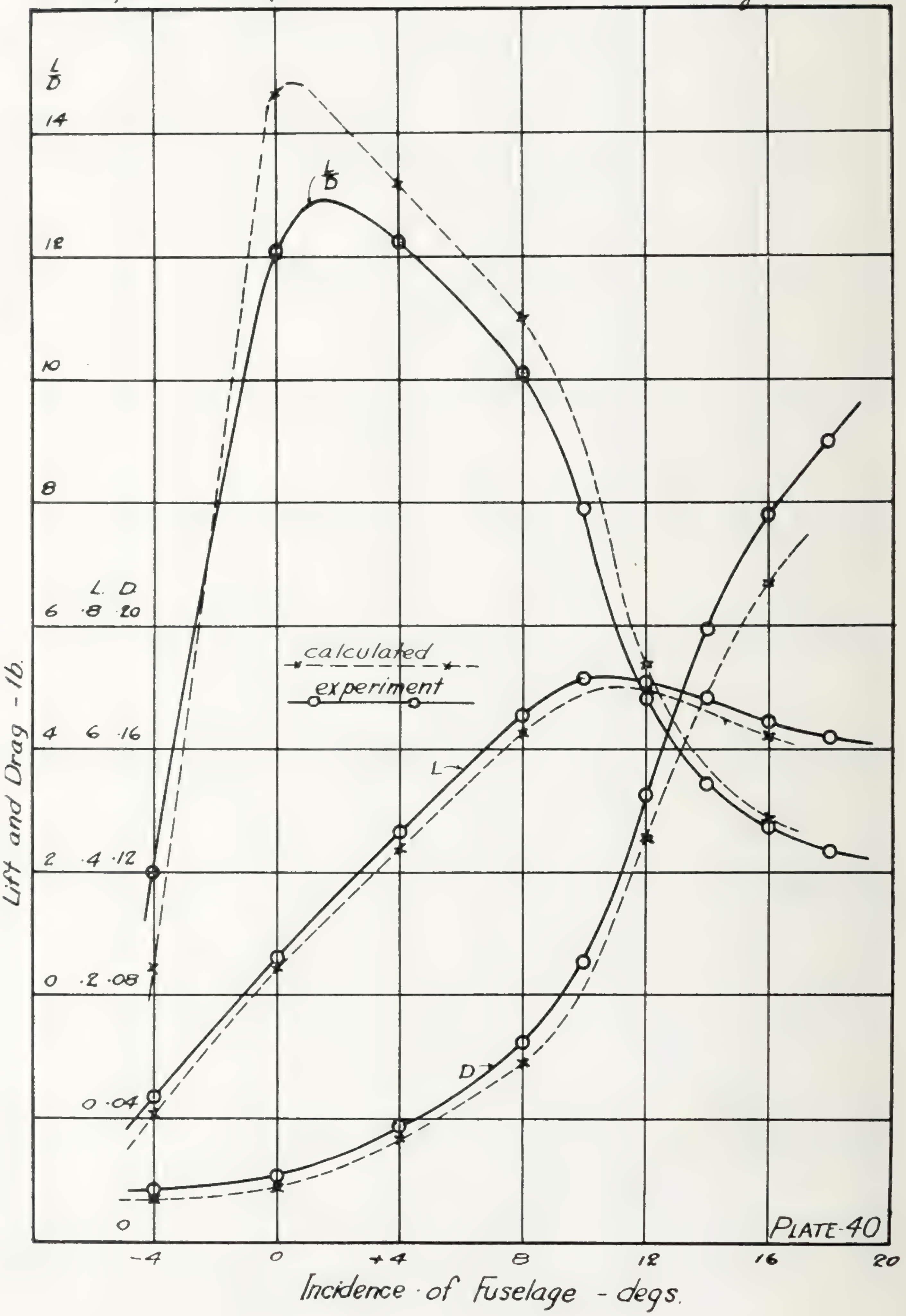


PLATE-38


Monoplane 2, Fuselage A, Aerofoil - R.A.F.-15.
Angle between Aerofoil chord and Axis of Fuselage = 3°-0′
Air speed - 40 f.p.s.
Geo J. Klein.
L/D
14
12
10
8
L. D.
6 ·8 ·20
4 ·6 ·16
2 ·4 ·12
0 ·2 ·08
0 ·04
0
Lift and Drag - lb.
calculated
experiment
L/D
L
D
-4
0
+4
8
12
16
20
Incidence of Fuselage - degs.
PLATE-39.

Monoplane 3, Fuselage A, Aerofoil - R.A.F.-15.
Angle between Aerofoil chord and Axis of Fuselage = 3°-0′
Air speed - 40 fps. Geo. J. Klein.



Monoplane 3-F, Fuselage A, Aerofoil - R.A.F.-15.
Angle between Aerofoil chord and Axis of Fuselage = 3°-0′
Air speed - 40 f.p.s. Geo J Klein.

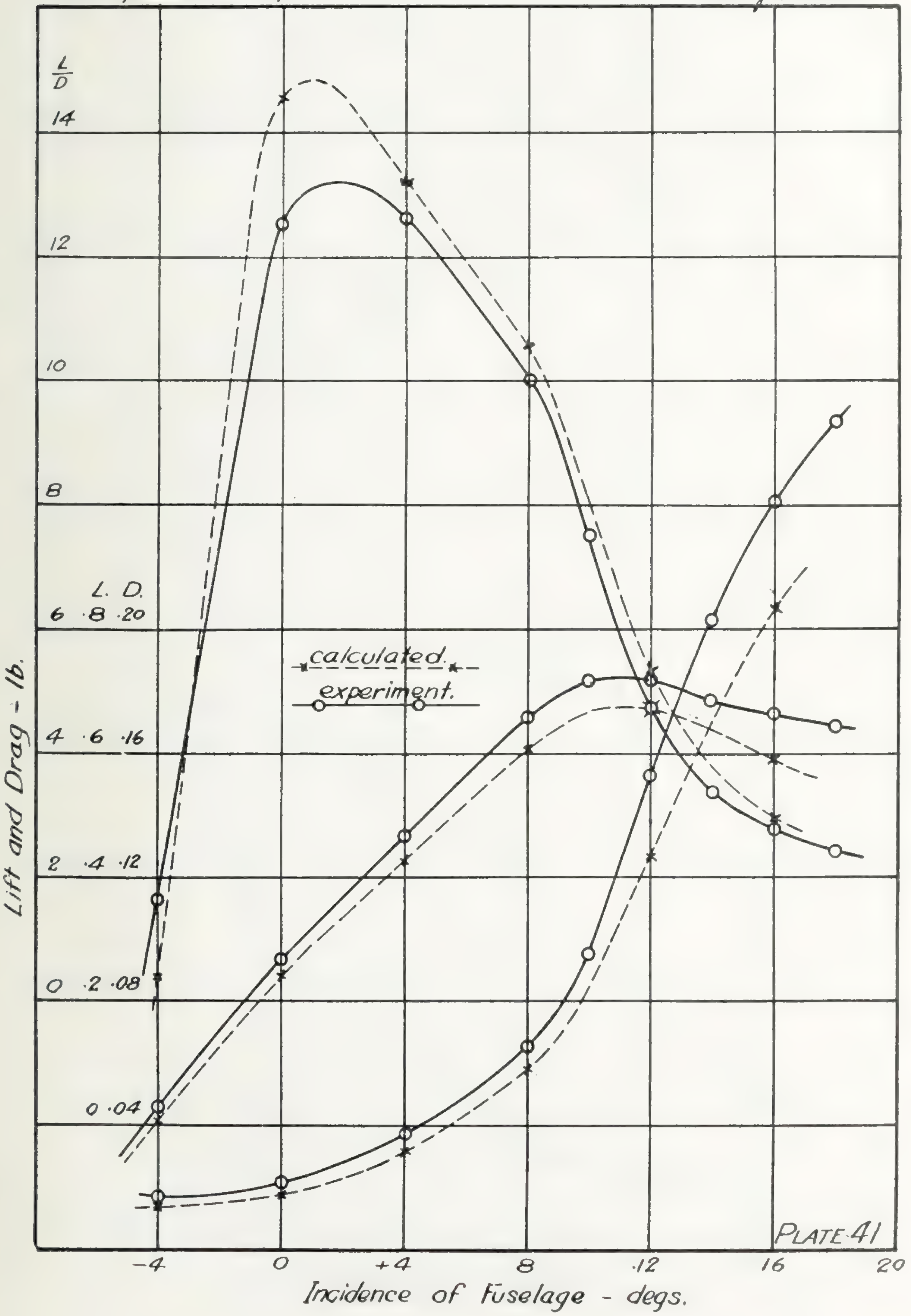


PLATE 41

Biplane-13, Fuselage A, Aerofoil R.A.F.-15
Angle between Aerofoil chord and Axis of Fuselage = 3°-0'
Air speed - 40 f.p.s.

Geo J. Klein.

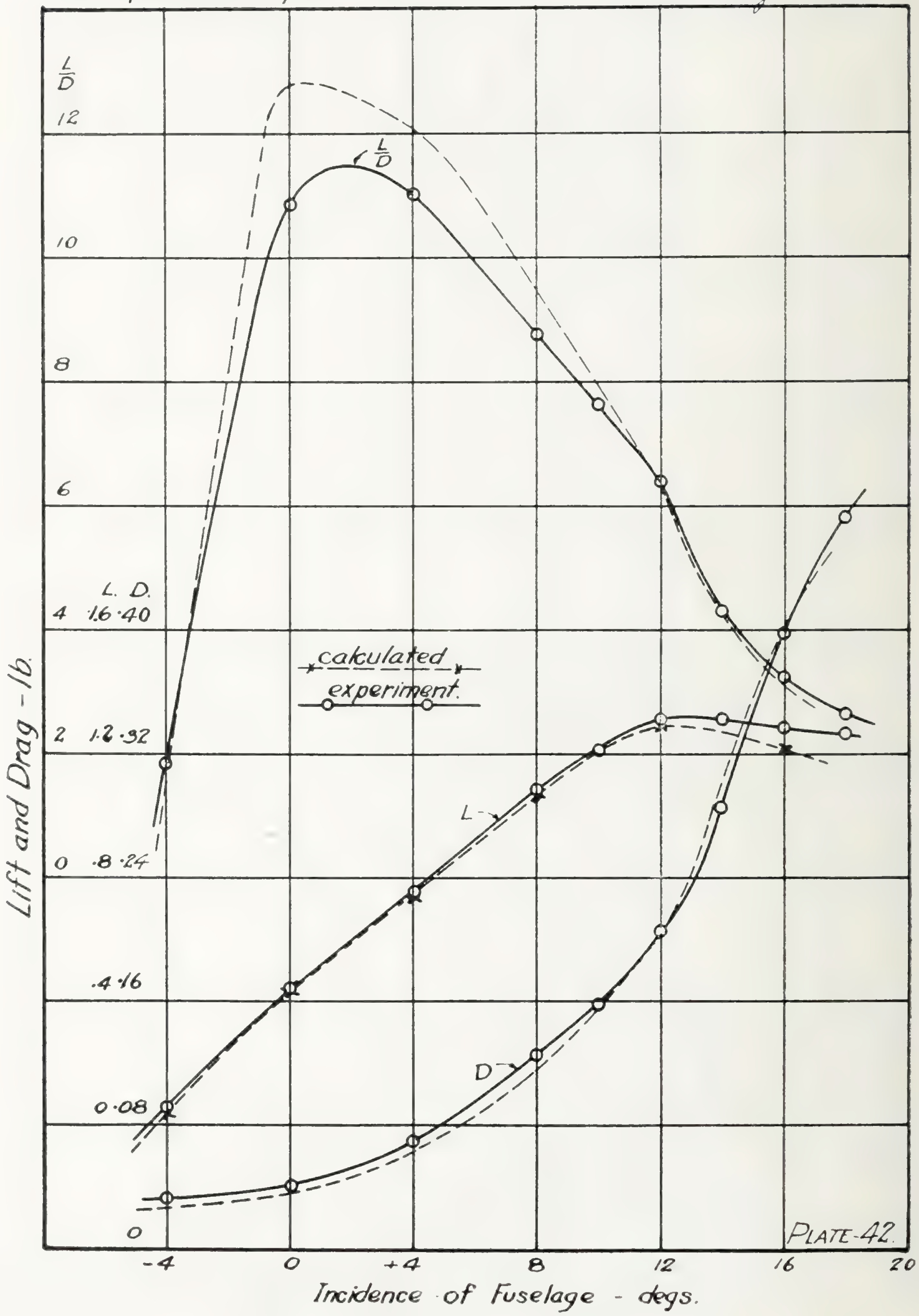


PLATE-42.

Biplane - 13-F, Fuselage A, Aerofoil - R.A.F.-15.
Angle between Aerofoil chord and Axis of Fuselage = 3°-0'
Air speed - 40 fps.
Geo. J. Klein.

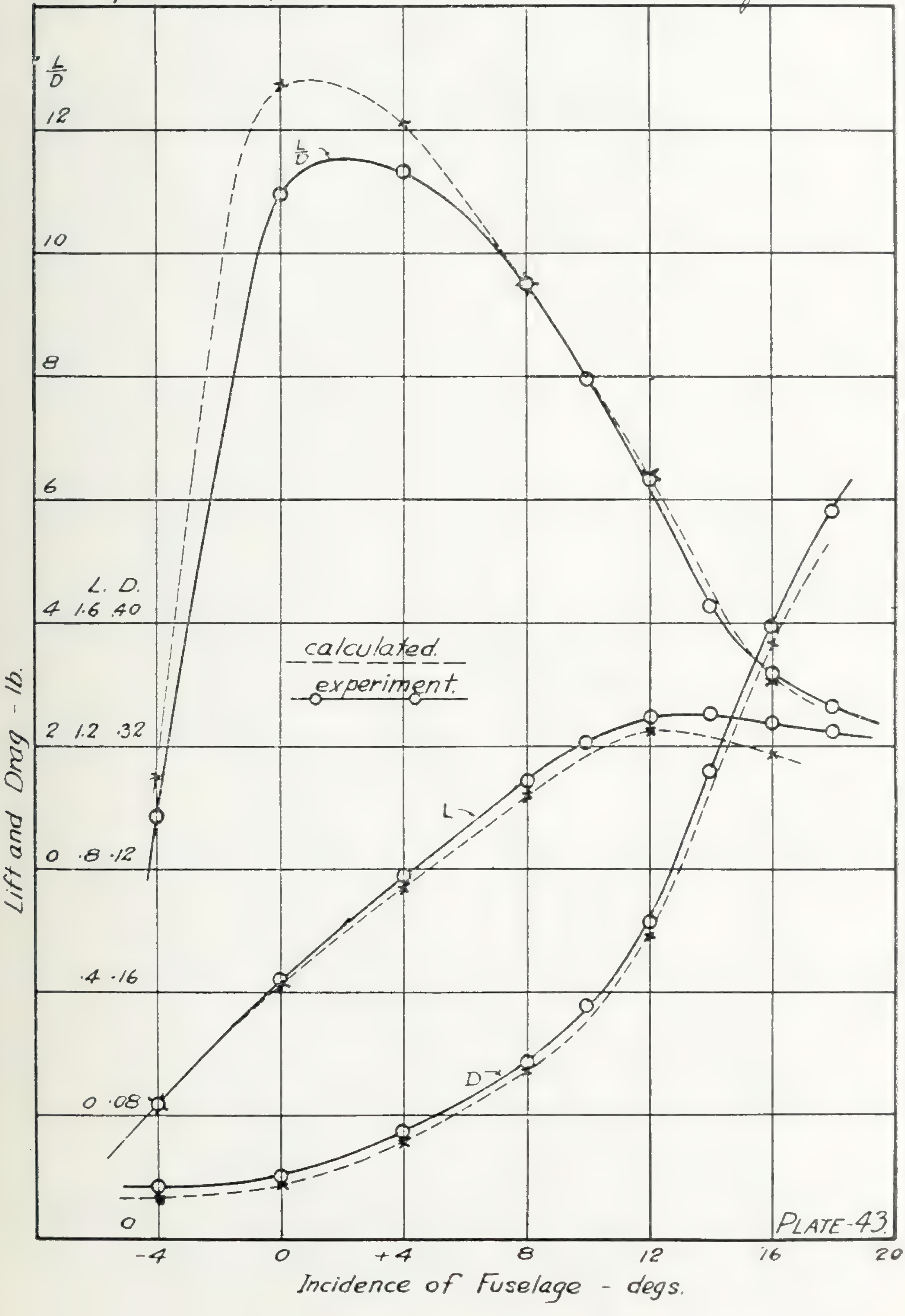


PLATE-43.

Monoplane 1, Fuselage B, Aerofoil - R.A.F.-15.
Angle between Aerofoil chord and Axis of Fuselage = 3°-0′
Air speed - 40 f.p.s.
Geo. J. Klein.

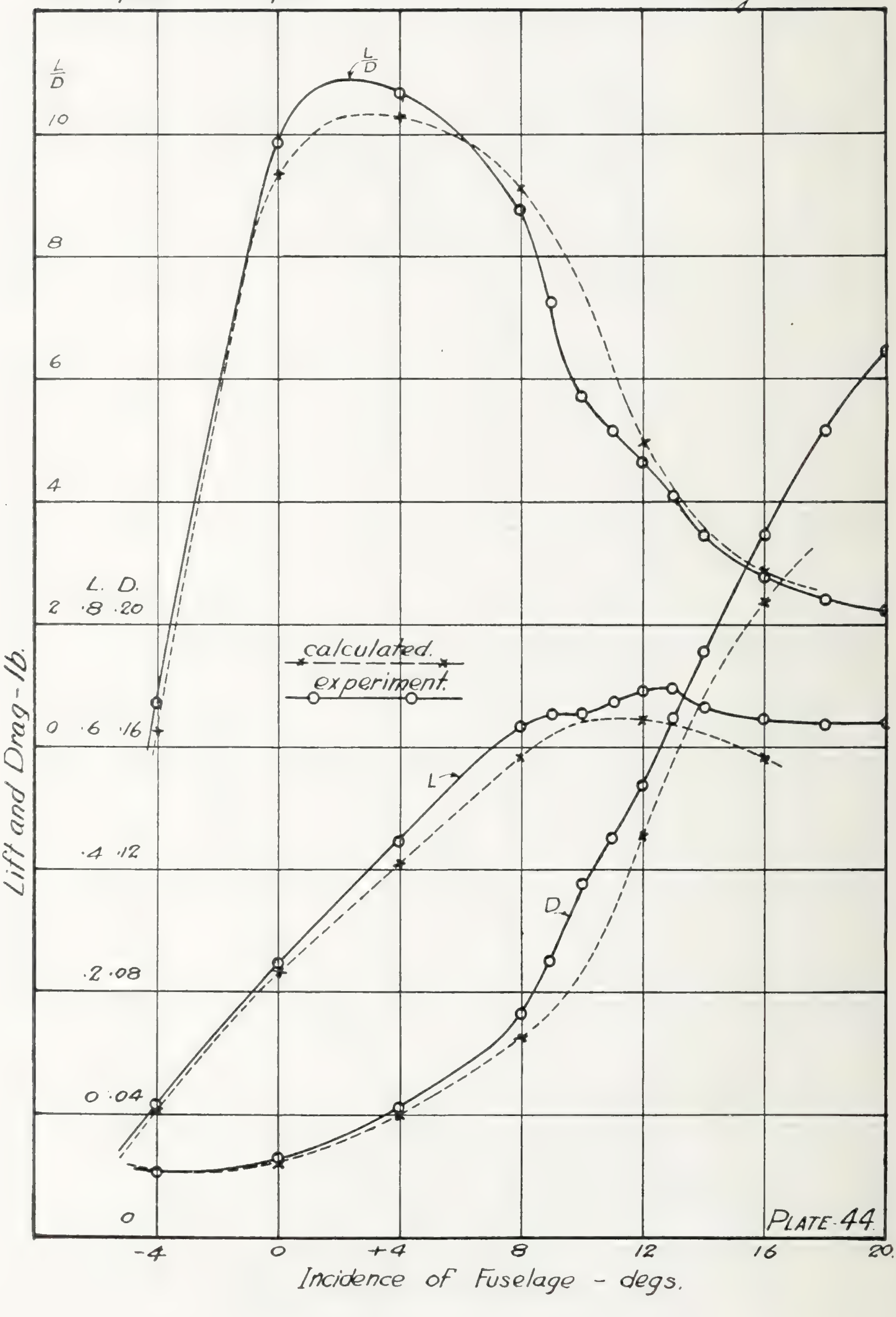


PLATE 44.

Monoplane 2, Fuselage B, Aerofoil - R.A.F.-15
Angle between Aerofoil chord and Axis of Fuselage = 3°-0'
Air speed - 40 f.p.s. Geo. J. Klein.

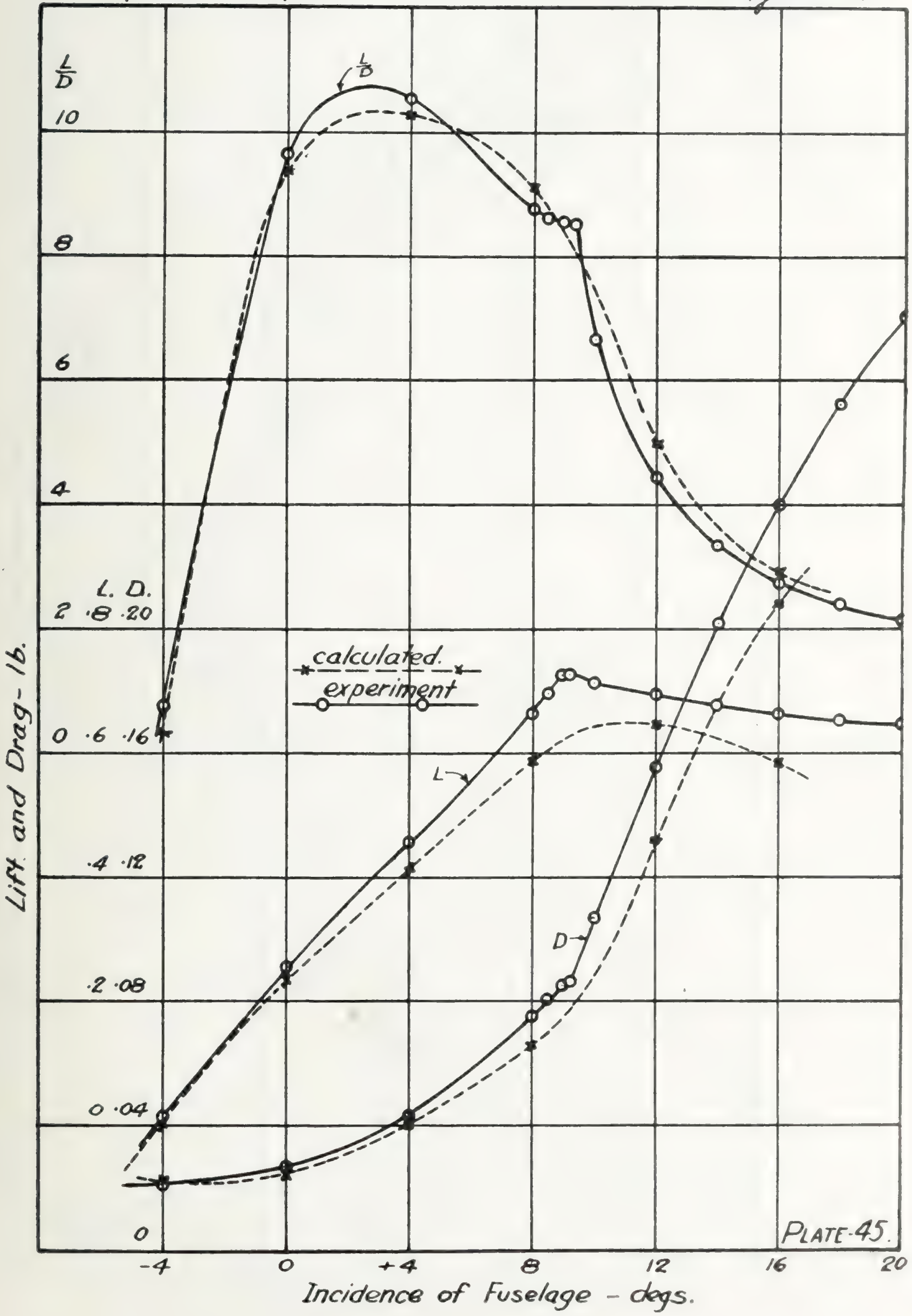


PLATE 45.

Monoplane 3, Fuselage B, Aerofoil - R.A.F. - 15.
Angle between Aerofoil chord and Axis of Fuselage = 3°-0′
Air speed - 40 f.p.s. Geo. J. Klein.

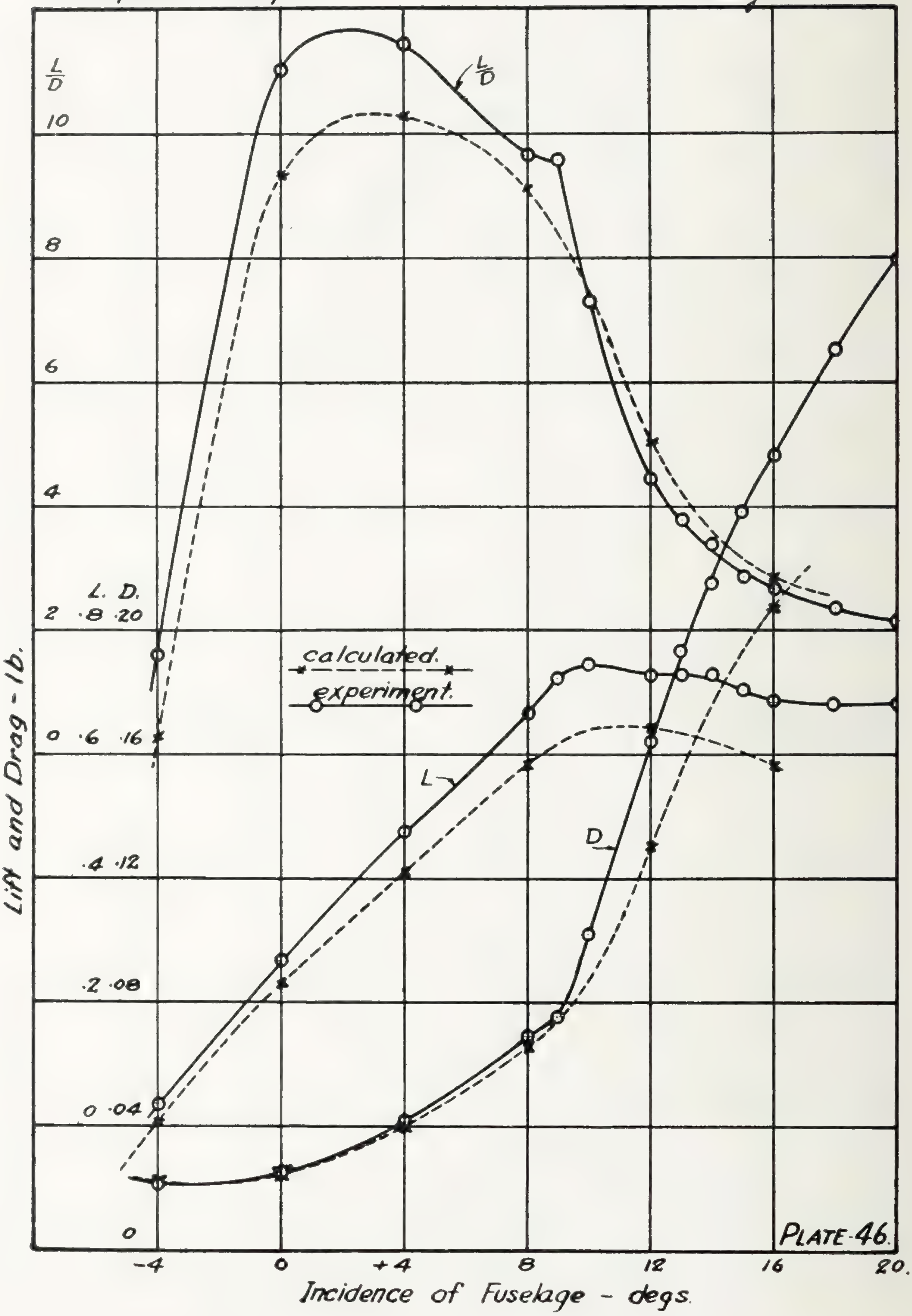


PLATE 46.

Biplane-13, Fuselage B, Aerofoil - R.A.F.-15.
Angle between Aerofoil chord and Axis of Fuselage = 3°-0'
Air speed - 40 f.p.s.

Geo. J. Klein.

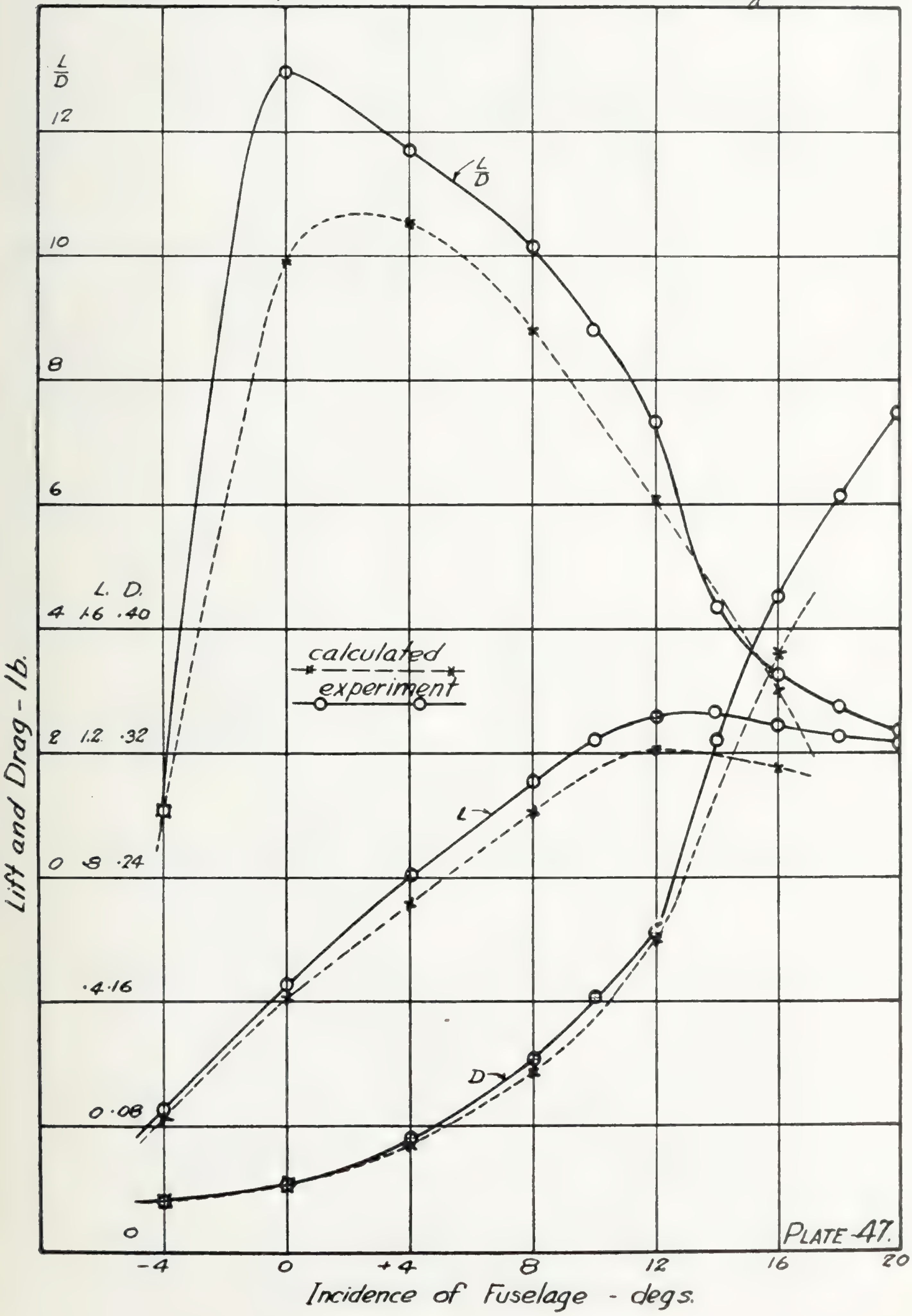


PLATE 47.

Monoplane 3, Fuselage C, Aerofoil - R.A.F.-15.
Angle between Aerofoil chord and Axis of Fuselage = 3°-0'
Air speed - 40 f.p.s. Geo J. Klein.

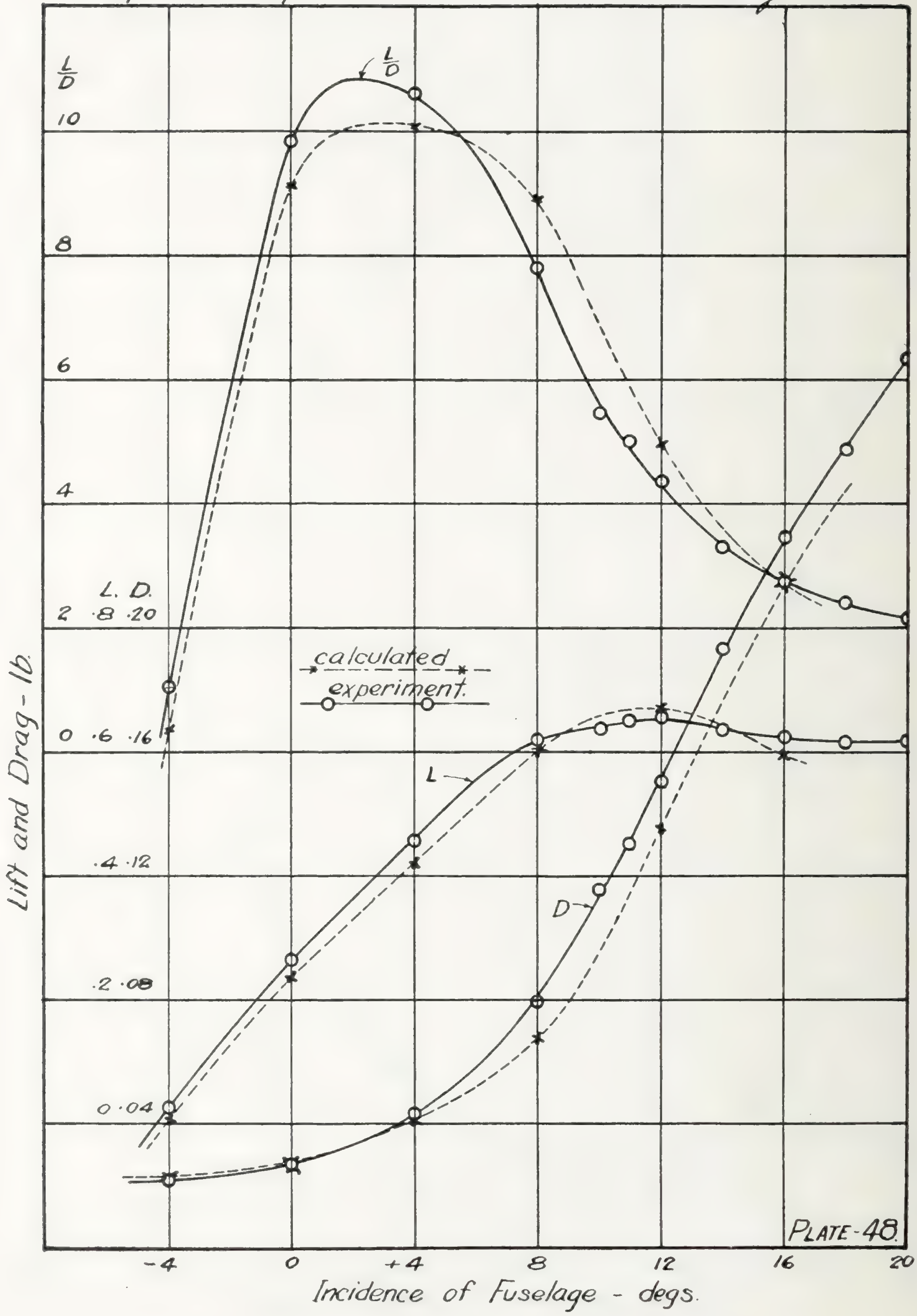


PLATE-48.

Monoplane 4, Fuselage C, Aerofoil - R.A.F.-15
Angle between Aerofoil chord and Axis of Fuselage = 3° - 0'
Air speed - 40 f.p.s. Geo. J. Klein.

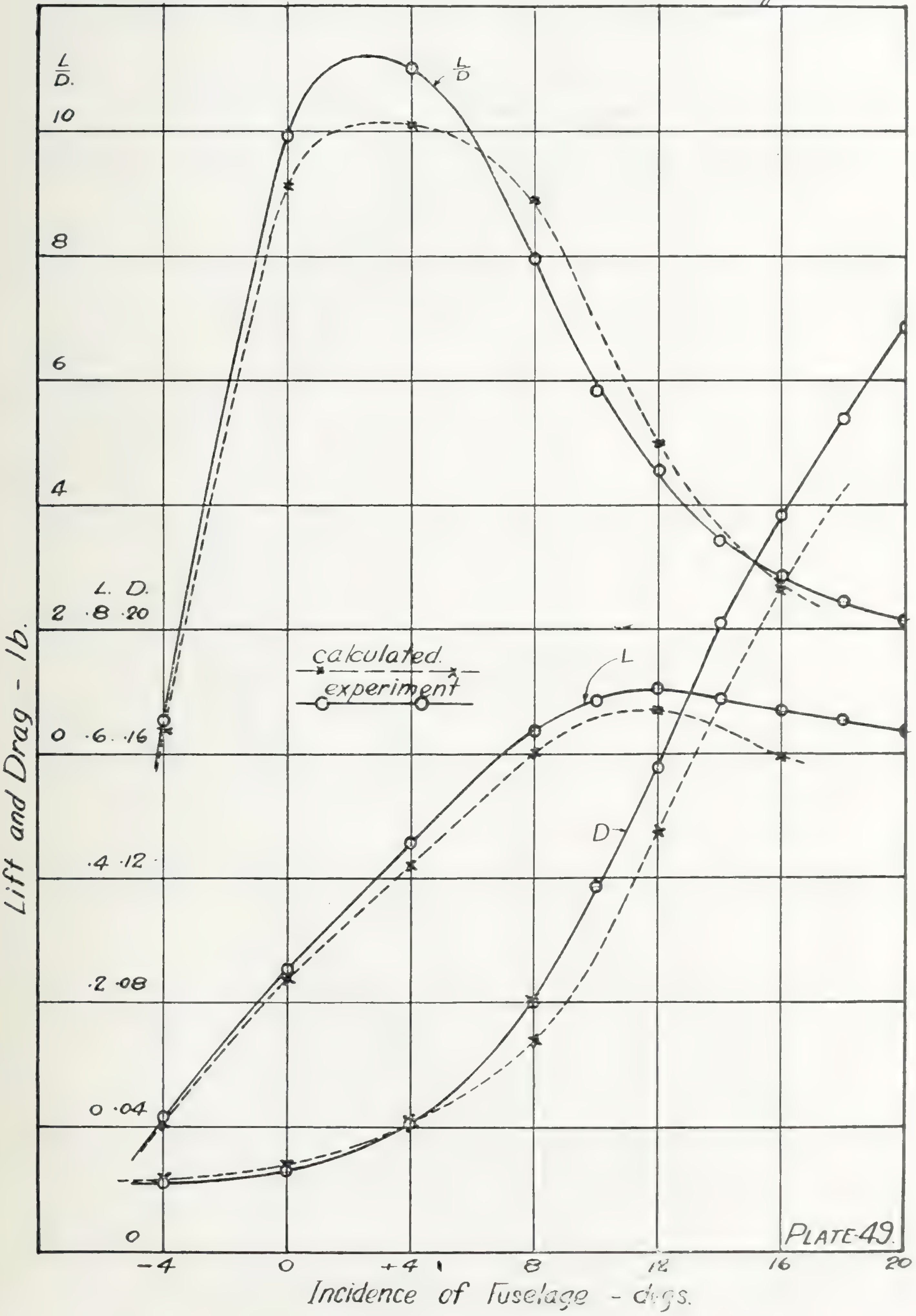
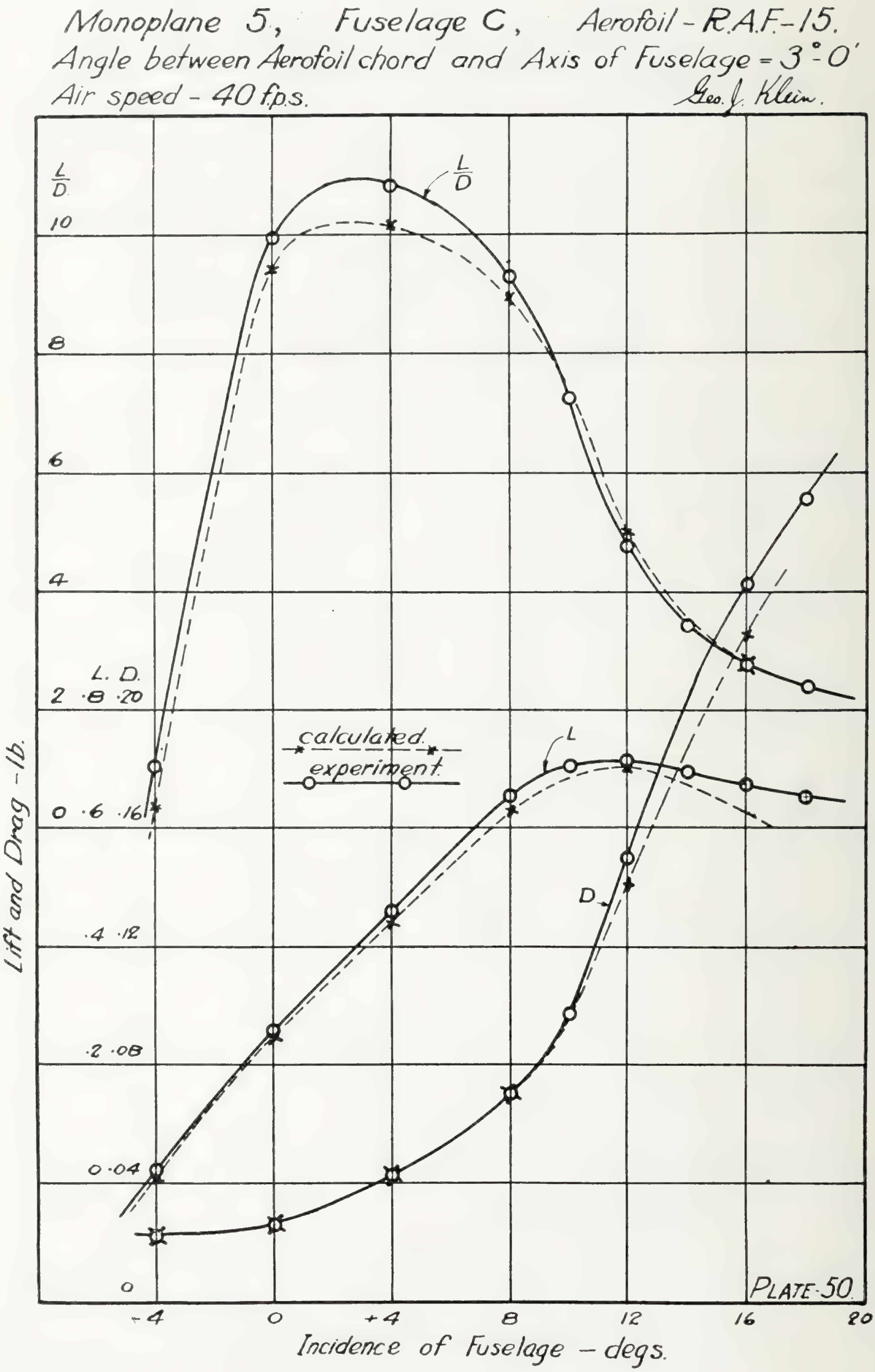
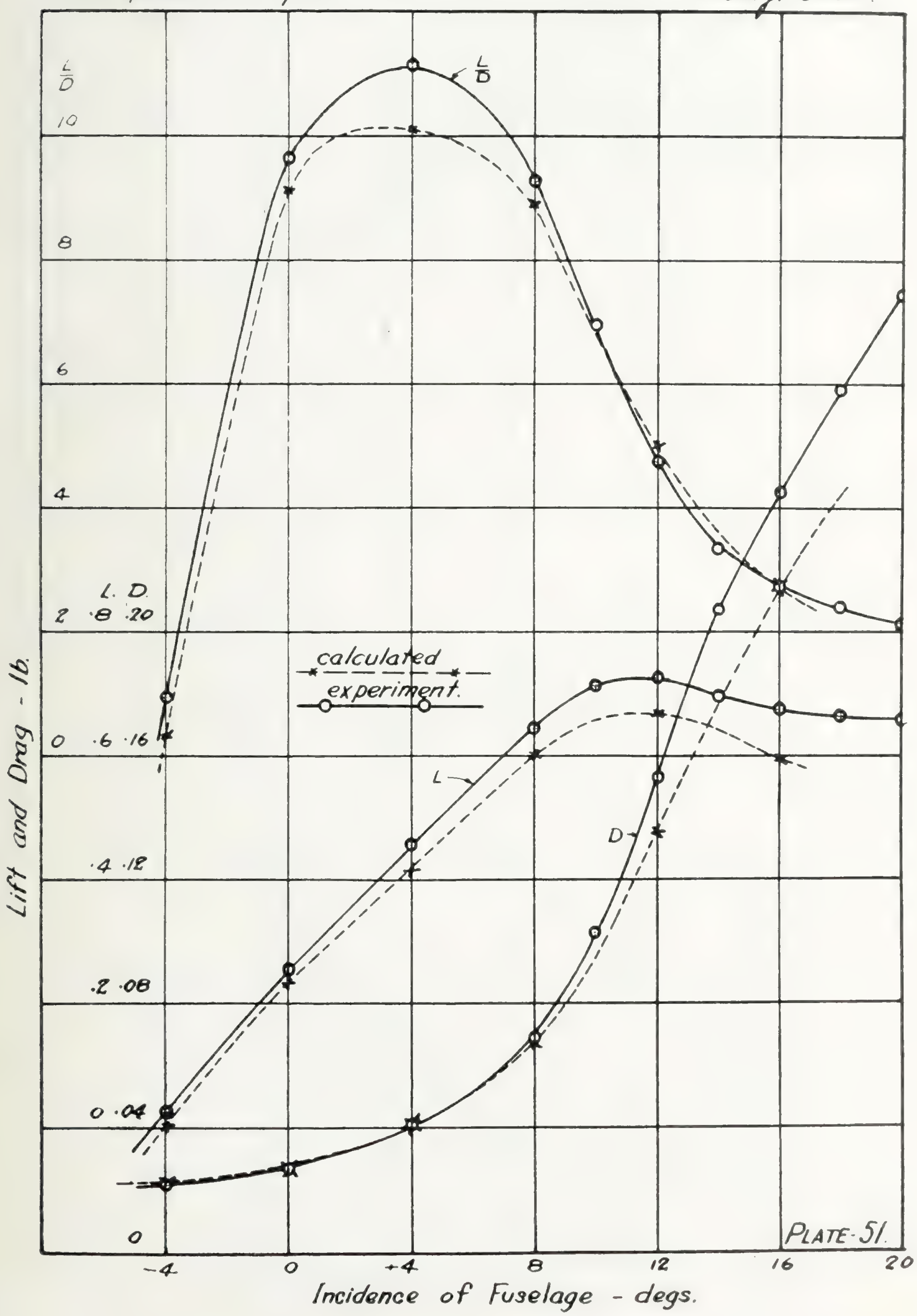


PLATE 49.


Monoplane 5, Fuselage C, Aerofoil - R.A.F.-15.
Angle between Aerofoil chord and Axis of Fuselage = 3°-0'
Air speed - 40 f.p.s.
Geo. J. Klein.
L/D
10
8
6
4
L. D.
2 ·8 ·20
0 ·6 ·16
·4 ·12
·2 ·08
0 ·04
0
calculated
experiment
L/D
L
D
Lift and Drag - lb.
-4
0
+4
8
12
16
20
Incidence of Fuselage - degs.
PLATE-50.

Monoplane 5-F, Fuselage C, Aerofoil - R.A.F.-15
Angle between Aerofoil chord and Axis of Fuselage = 3°-0'
Air speed - 40 f.p.s.
Geo. J. Klein.



Monoplane 6, Fuselage C, Aerofoil - R.A.F.-15.
Angle between Aerofoil chord and Axis of Fuselage = 3°-0′
Air speed - 40 f.p.s.
Geo. J. Klein.

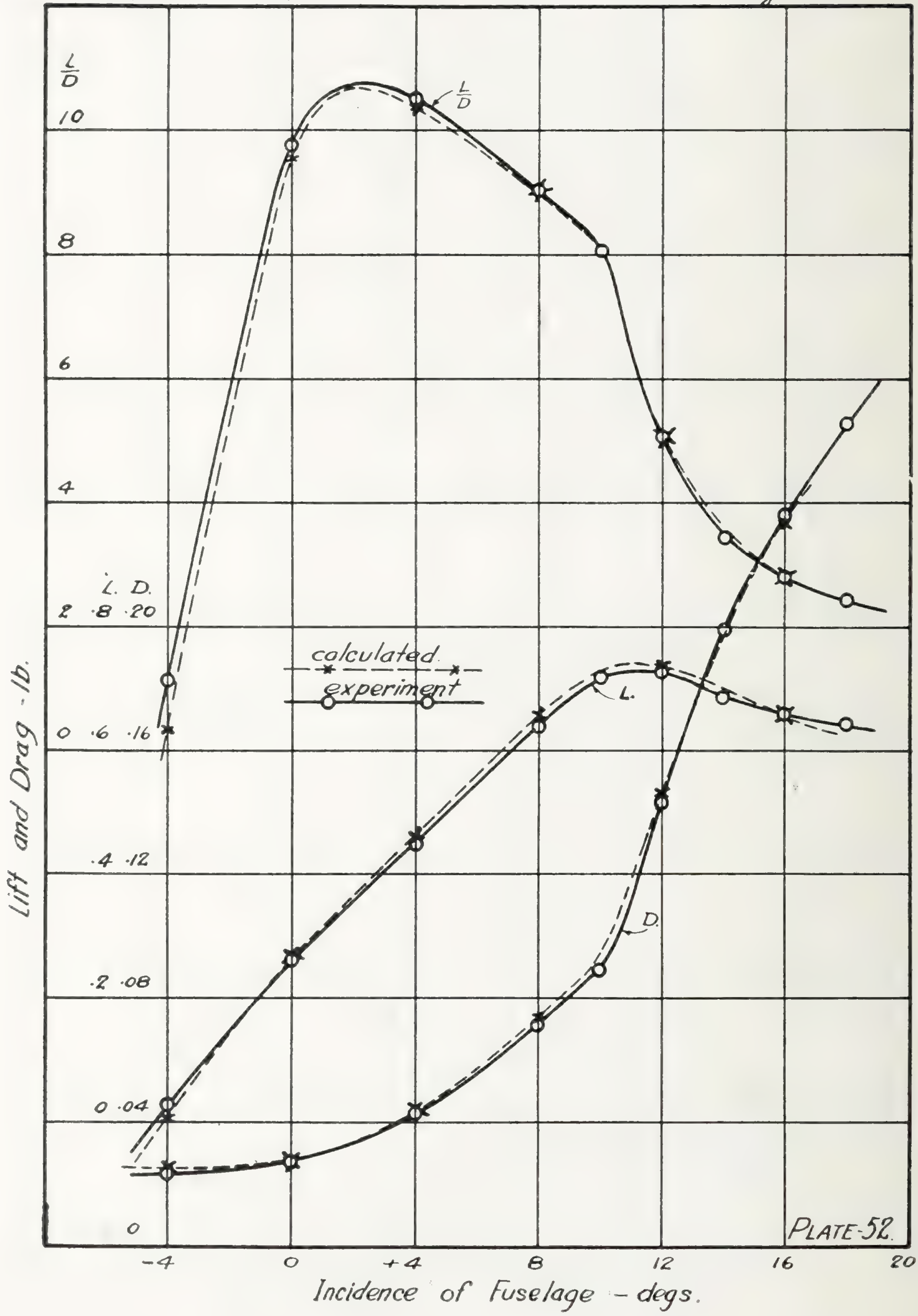
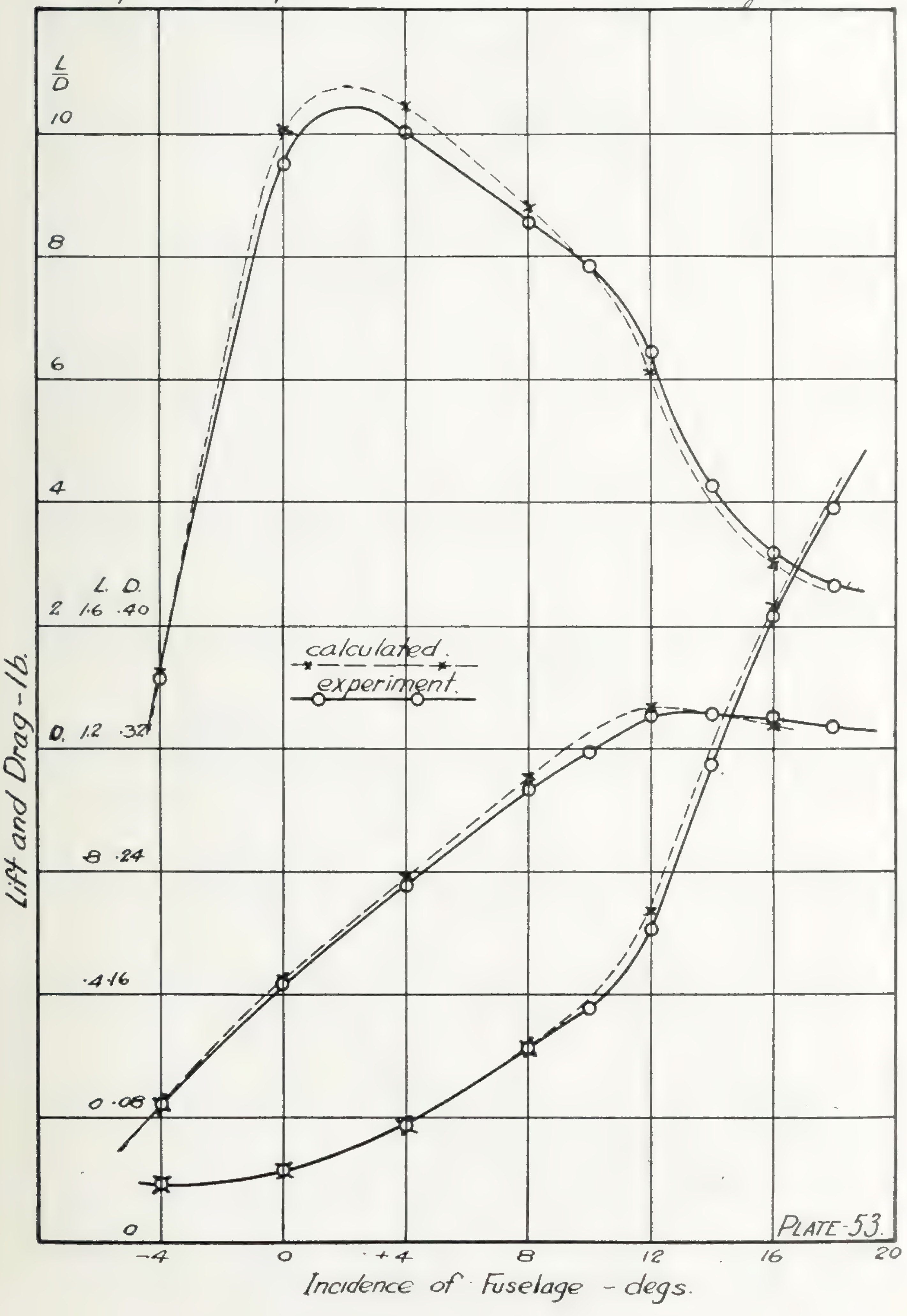


PLATE-52.

Biplane-15, Fuselage C, Aerofoil - R.A.F.-15.
Angle between Aerofoil chord and Axis of Fuselage = 3°-0′
Air speed - 40 f.p.s. Geo. J. Klein.



Biplane - 15-F, Fuselage C, Aerofoil - R.A.F. - 15

Angle between Aerofoil chord and Axis of Fuselage = 3°-0'

Air speed - 40 f.p.s.

Geo. J. Klein

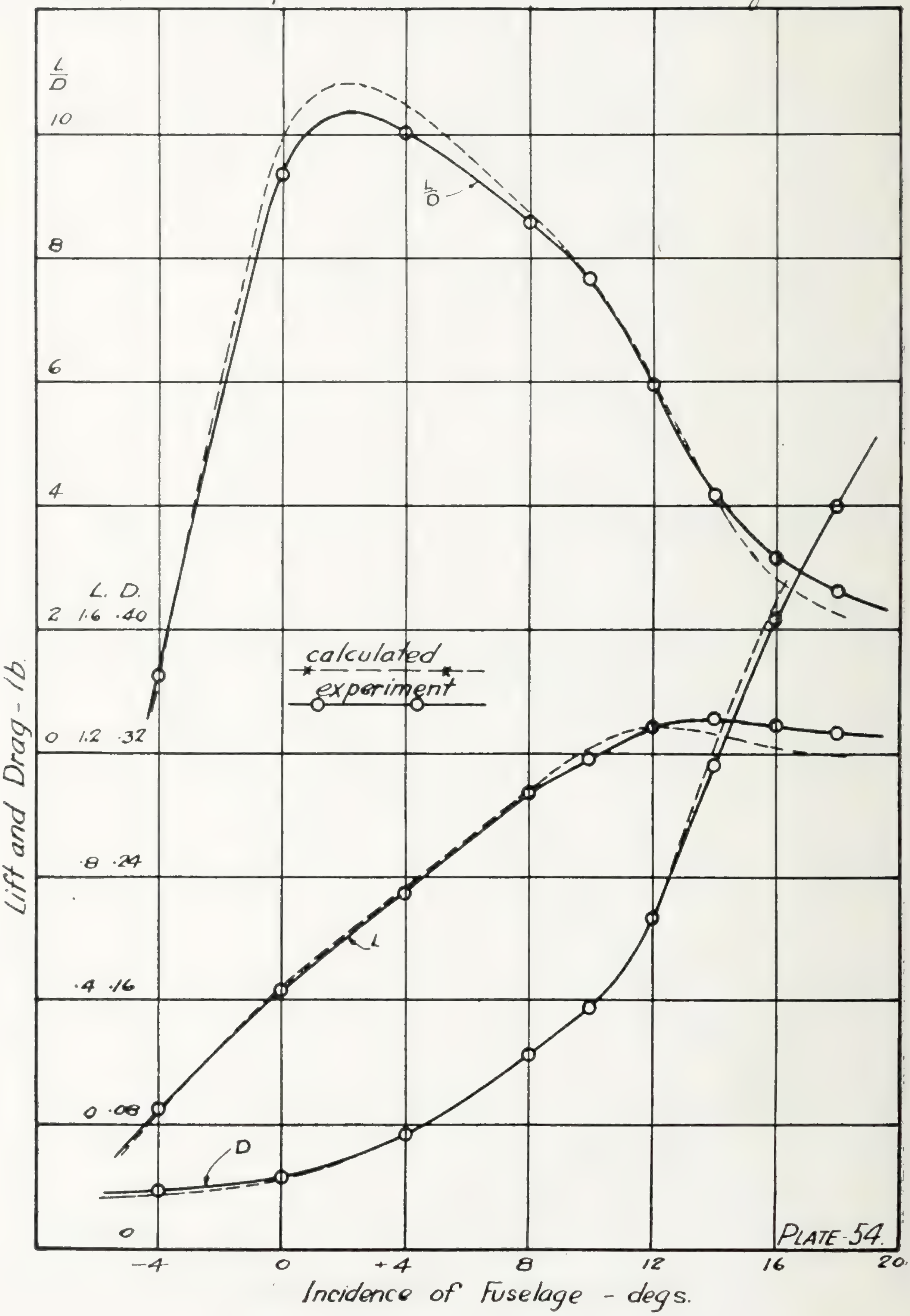
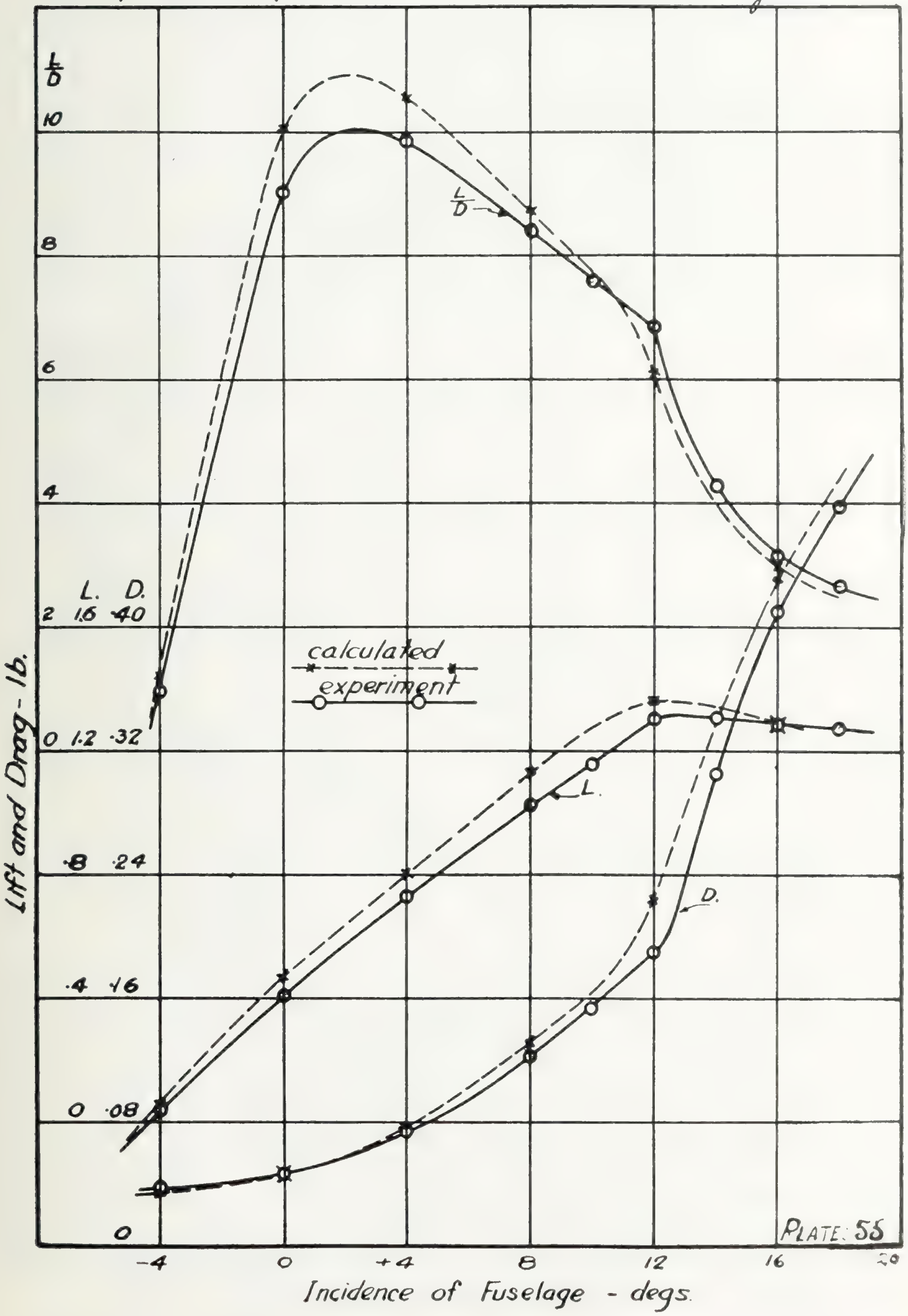
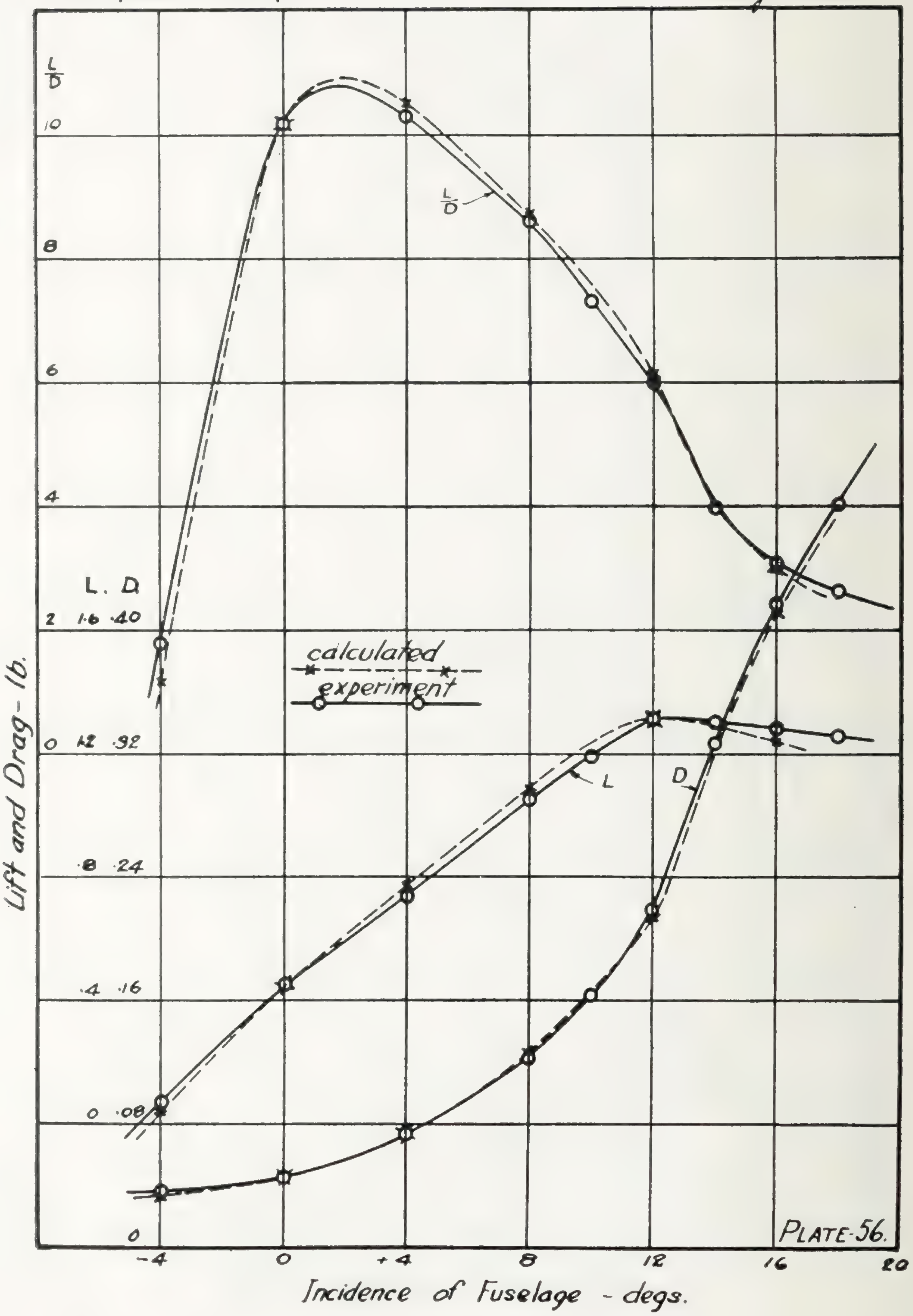


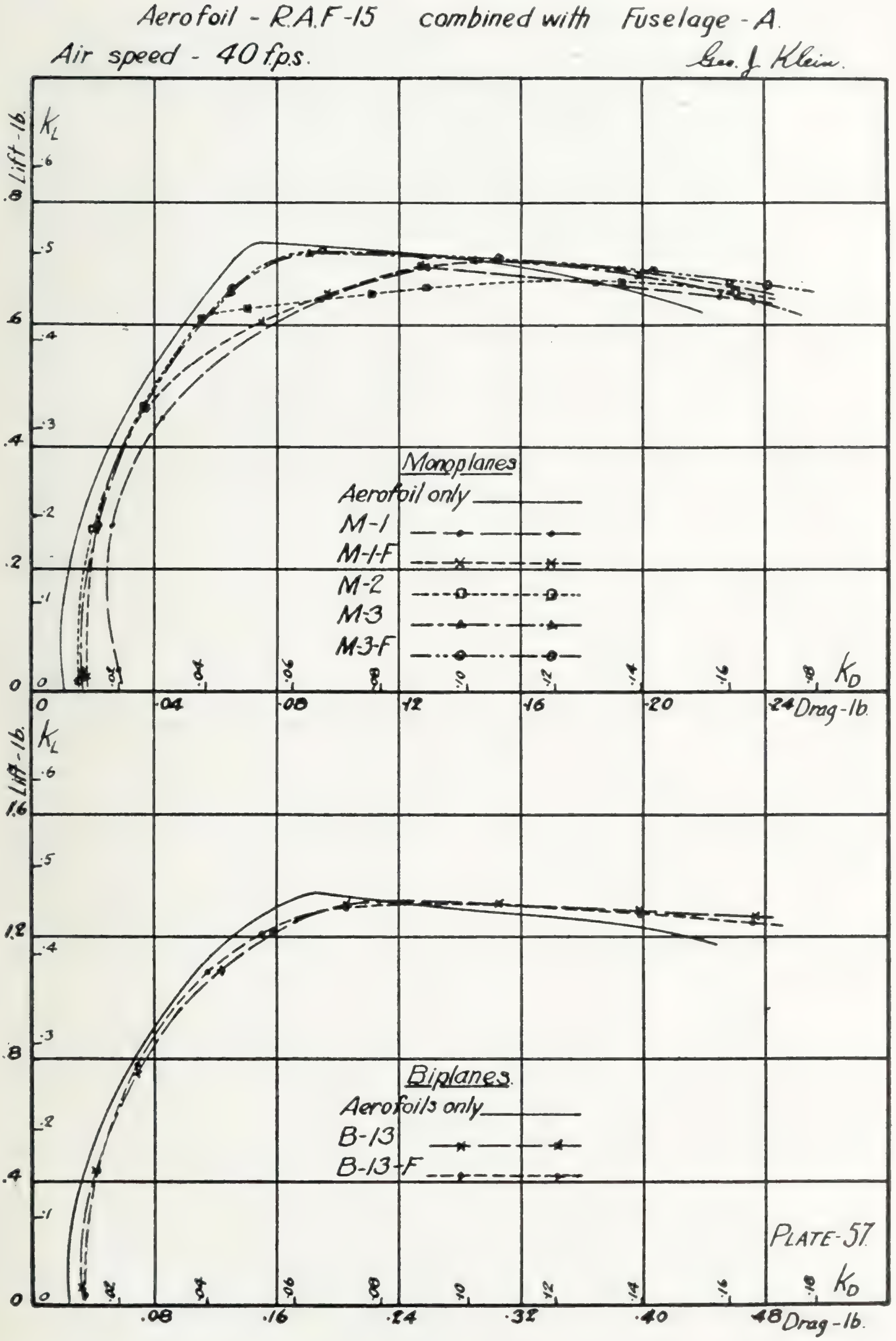
PLATE - 54.

Biplane-26, Fuselage C, Aerofoil - R.A.F.-15.
Angle between Aerofoil chord and Axis of Fuselage = 3°-0′
Air speed - 40 f.p.s.
Geo. J. Klein.



Biplane-37 , Fuselage C, Aerofoil - R.A.F.-15.
Angle between Aerofoil chord and Axis of Fuselage = 3°-0'
Air speed - 40 f.p.s. Geo. J. Klein.




Aerofoil - R.A.F-15 combined with Fuselage - A.
Air speed - 40 fps.
Geo. J. Klein.
Lift-lb.
K_L
.8
.6
.4
.2
0
·1
·2
·3
·4
·5
·6
Monoplanes
Aerofoil only
M-1
M-1-F
M-2
M-3
M-3-F
·02
·04
·06
·08
·10
·12
·14
·16
·18
K_D
0
·04
·08
·12
·16
·20
·24
Drag-lb.
Lift-lb.
K_L
1.6
1.2
.8
.4
0
Biplanes
Aerofoils only
B-13
B-13-F
PLATE-57.
K_D
·08
·16
·24
·32
·40
·48
Drag-lb.

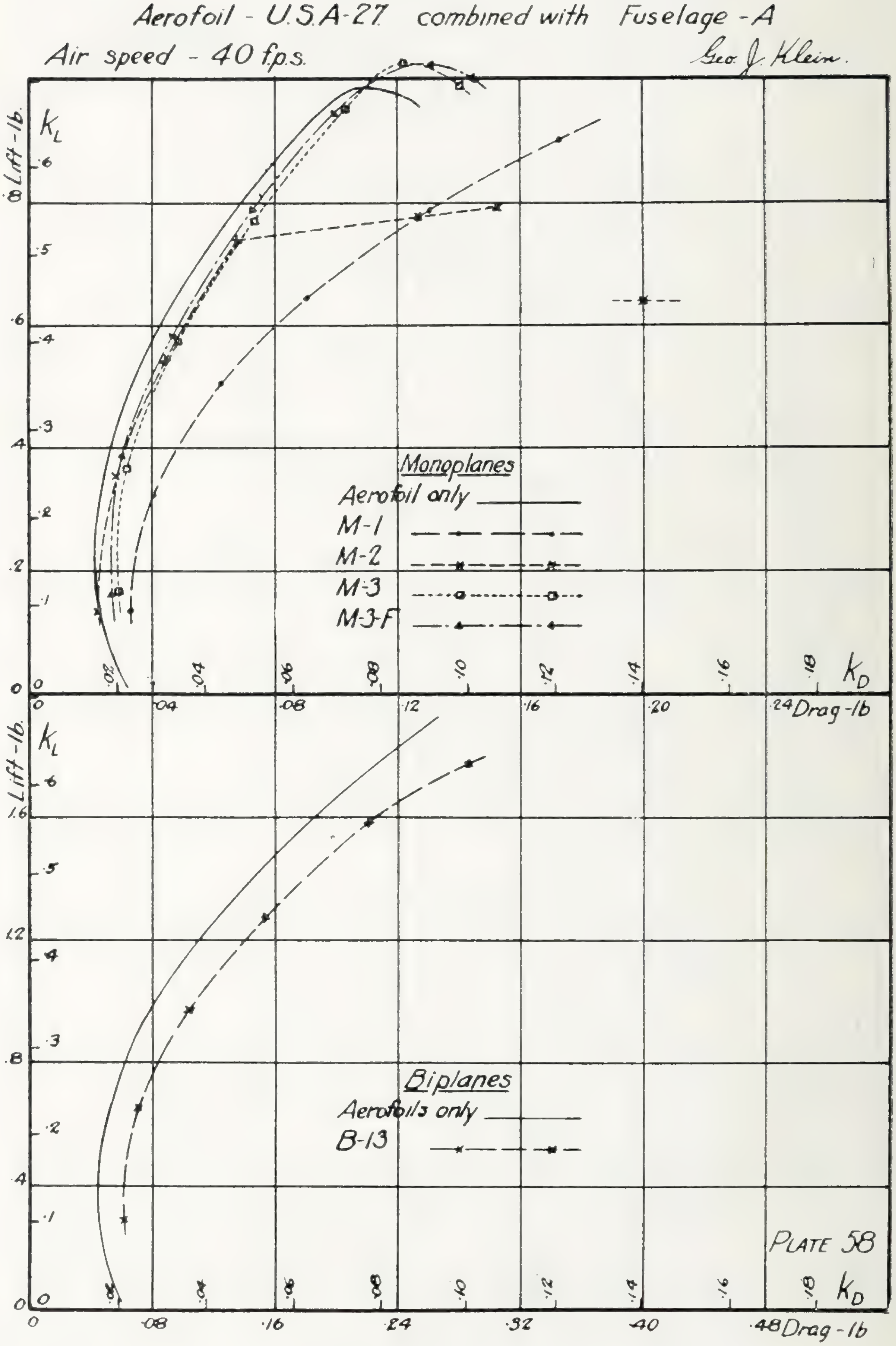

Aerofoil - U.S.A-27 combined with Fuselage -A
Air speed - 40 f.p.s.
Geo. J. Klein.
K_L
Lift - lb.
·8
·6
·4
·2
0
·6
·5
·4
·3
·2
·1
0
Monoplanes
Aerofoil only
M-1
M-2
M-3
M-3-F
·02
·04
·06
·08
·10
·12
·14
·16
·18
K_D
0
·04
·08
·12
·16
·20
·24 Drag - lb
K_L
Lift - lb.
1.6
1.2
·8
·4
0
·6
·5
·4
·3
·2
·1
0
Biplanes
Aerofoils only
B-13
·02
·04
·06
·08
·10
·12
·14
·16
·18
K_D
0
·08
·16
·24
·32
·40
·48 Drag - lb

PLATE 58

Aerofoil - Gött-387 combined with Fuselage - A

Air speed - 40 f.p.s.

Geo. J. Klein

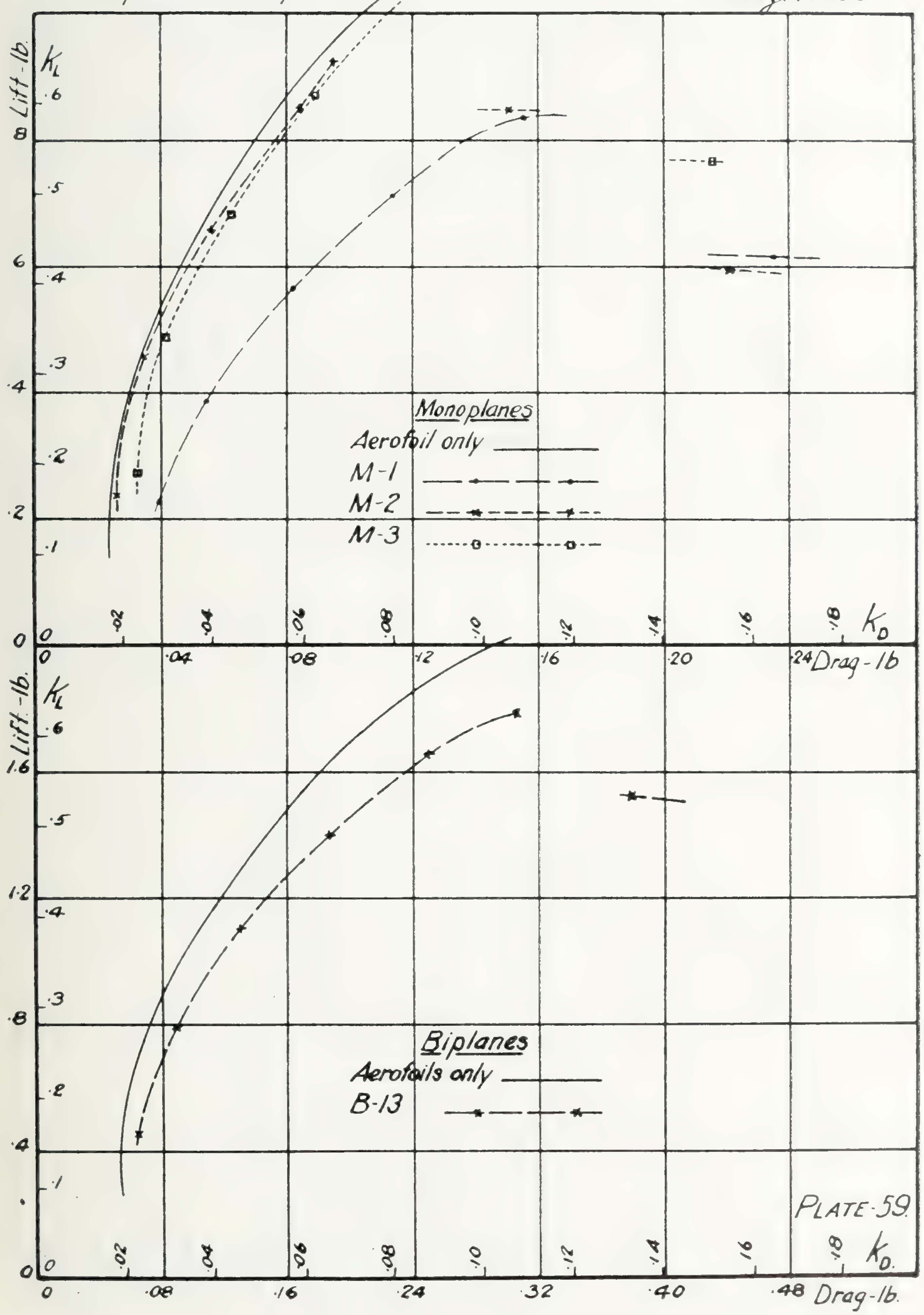


PLATE-59.

Aerofoil - R.A.F-15 combined with Fuselage - B.
Air speed - 40 fps.
Geo J Klein.

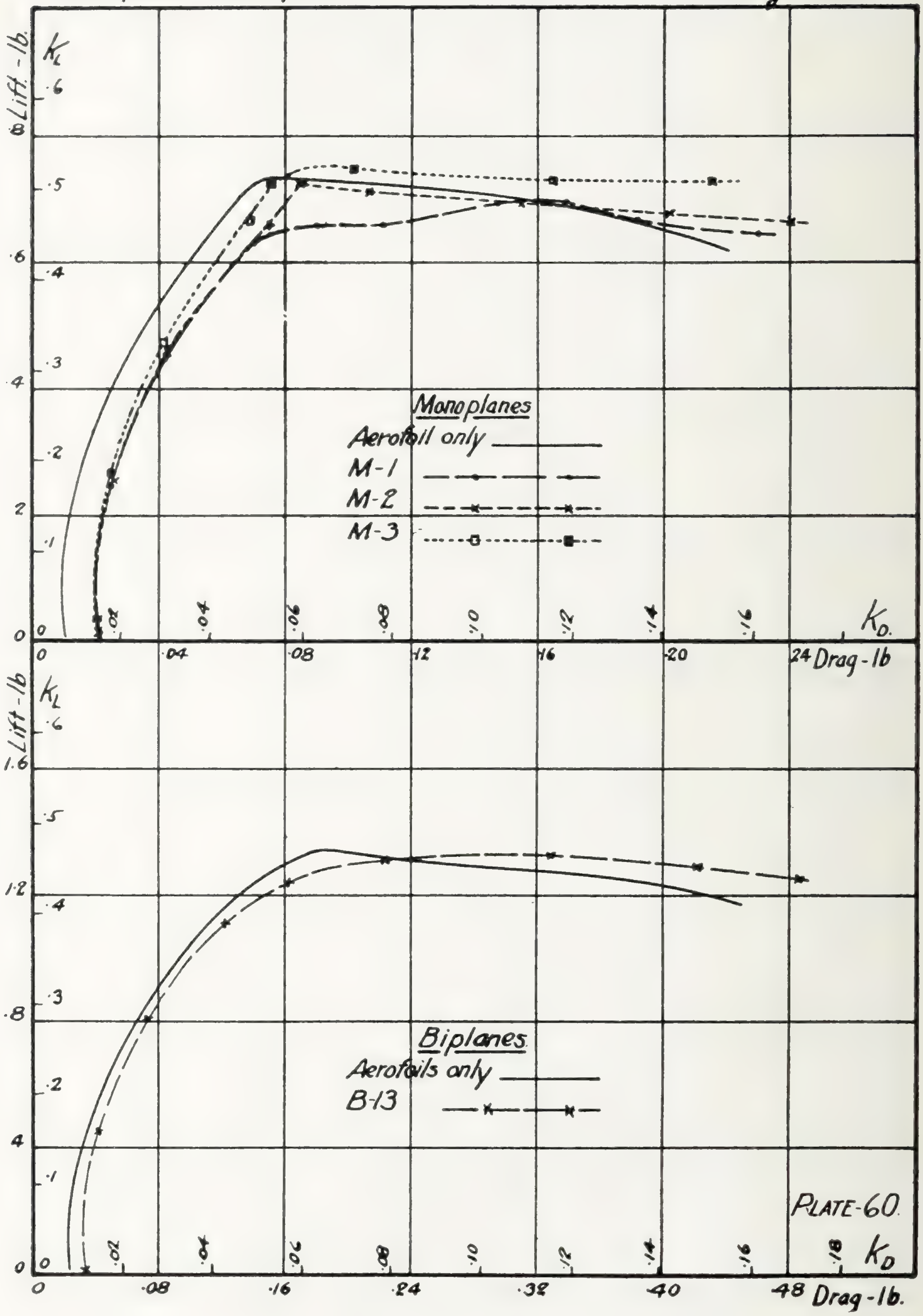


PLATE-60.

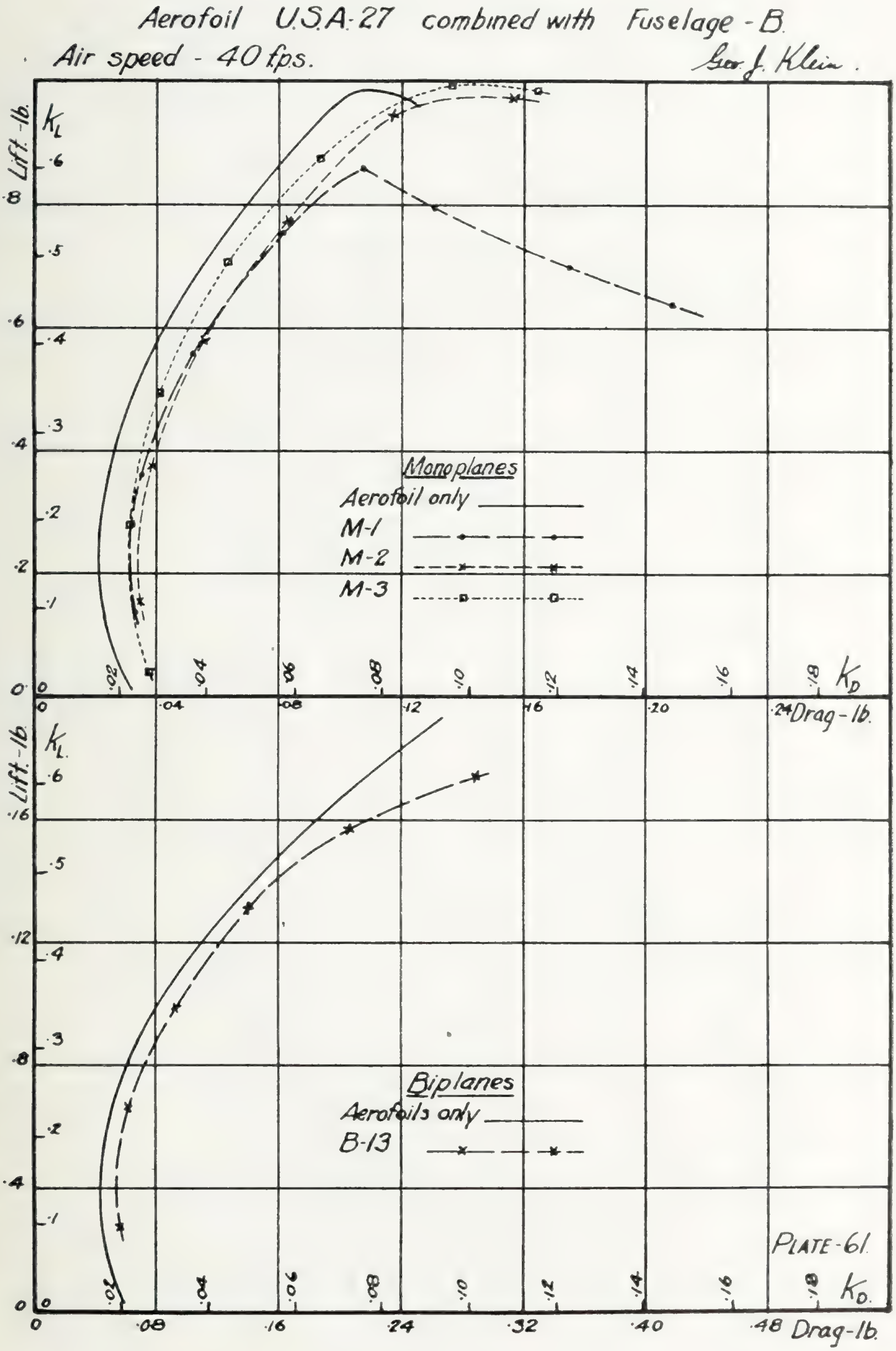

Aerofoil U.S.A-27 combined with Fuselage-B.
Air speed - 40 fps.
Geo. J. Klein.
Lift-lb.
K_L
Monoplanes
Aerofoil only
M-1
M-2
M-3
K_D
Drag-lb.
0
·02
·04
·06
·08
·10
·12
·14
·16
·18
·1
·2
·3
·4
·5
·6
·8
·12
·16
·20
·24
·32
·40
·48
Biplanes
Aerofoils only
B-13

PLATE-61.

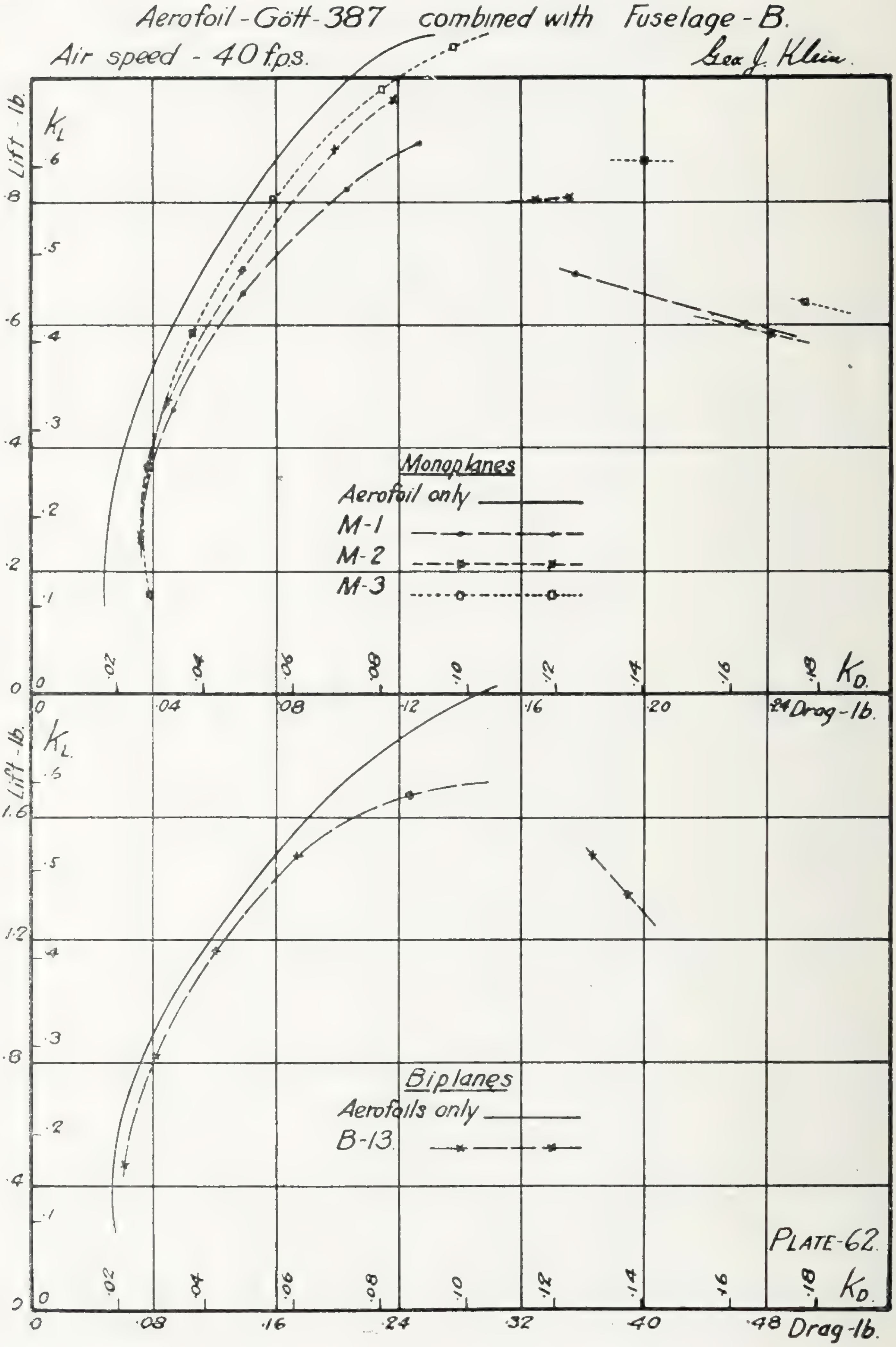

Aerofoil - Gött - 387 combined with Fuselage - B.
Air speed - 40 f.p.s.
Geo J. Klein.
Lift - lb.
K_L
Monoplanes
Aerofoil only
M-1
M-2
M-3
0
·2
·4
·6
·8
·1
·2
·3
·4
·5
·6
·02
·04
·06
·08
·10
·12
·14
·16
·18
K_D
0
·04
·08
·12
·16
·20
·24 Drag - lb.
Lift - lb.
K_L
Biplanes
Aerofoils only
B-13
0
·4
·8
1·2
1·6
·1
·2
·3
·4
·5
·6
·02
·04
·06
·08
·10
·12
·14
·16
·18
K_D
0
·08
·16
·24
·32
·40
·48 Drag - lb.

PLATE-62.

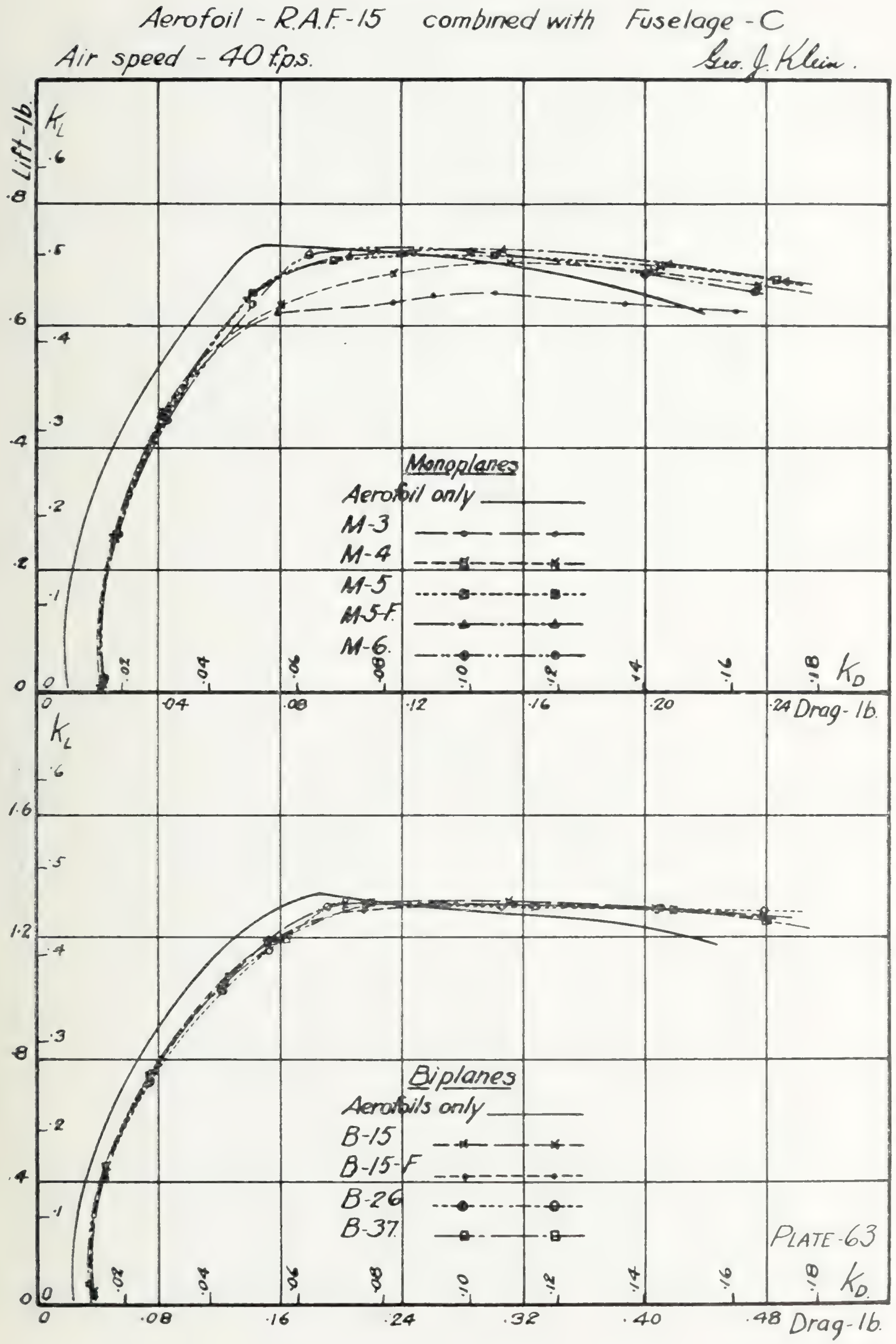

Aerofoil - R.A.F.-15 combined with Fuselage - C
Air speed - 40 f.p.s.
Geo. J. Klein.
Lift-lb.
K_L
Monoplanes
Aerofoil only
M-3
M-4
M-5
M-5-F
M-6
K_D
Drag-lb.
Biplanes
Aerofoils only
B-15
B-15-F
B-26
B-37
0
·02
·04
·06
·08
·10
·12
·14
·16
·18
·1
·2
·3
·4
·5
·6
·2
·4
·6
·8
1·2
1·6
·08
·16
·20
·24
·32
·40
·48

PLATE-63

Aerofoil - U.S.A-27 combined with Fuselage - C

Air speed - 40 f.p.s.

Geo J Klein.

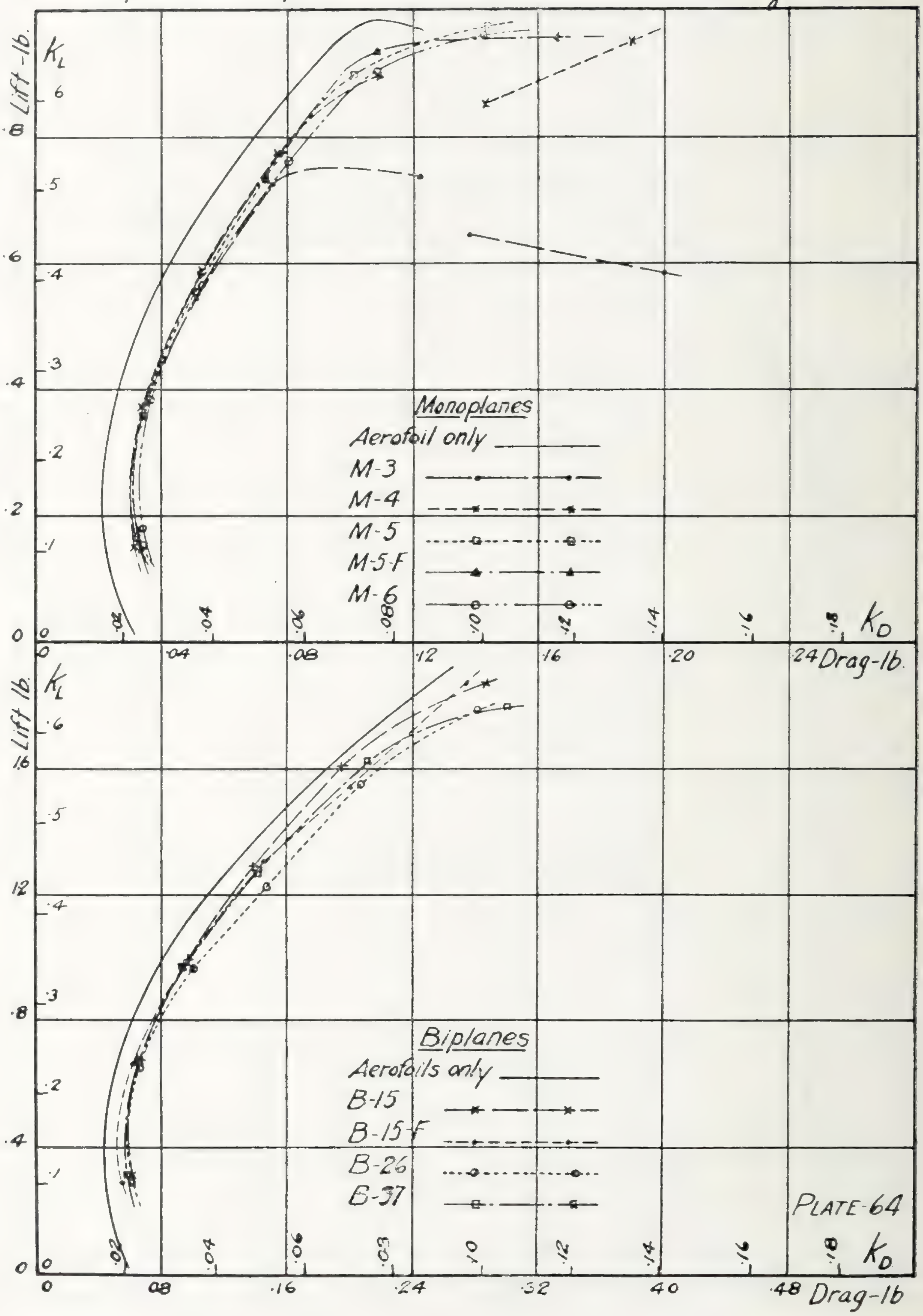
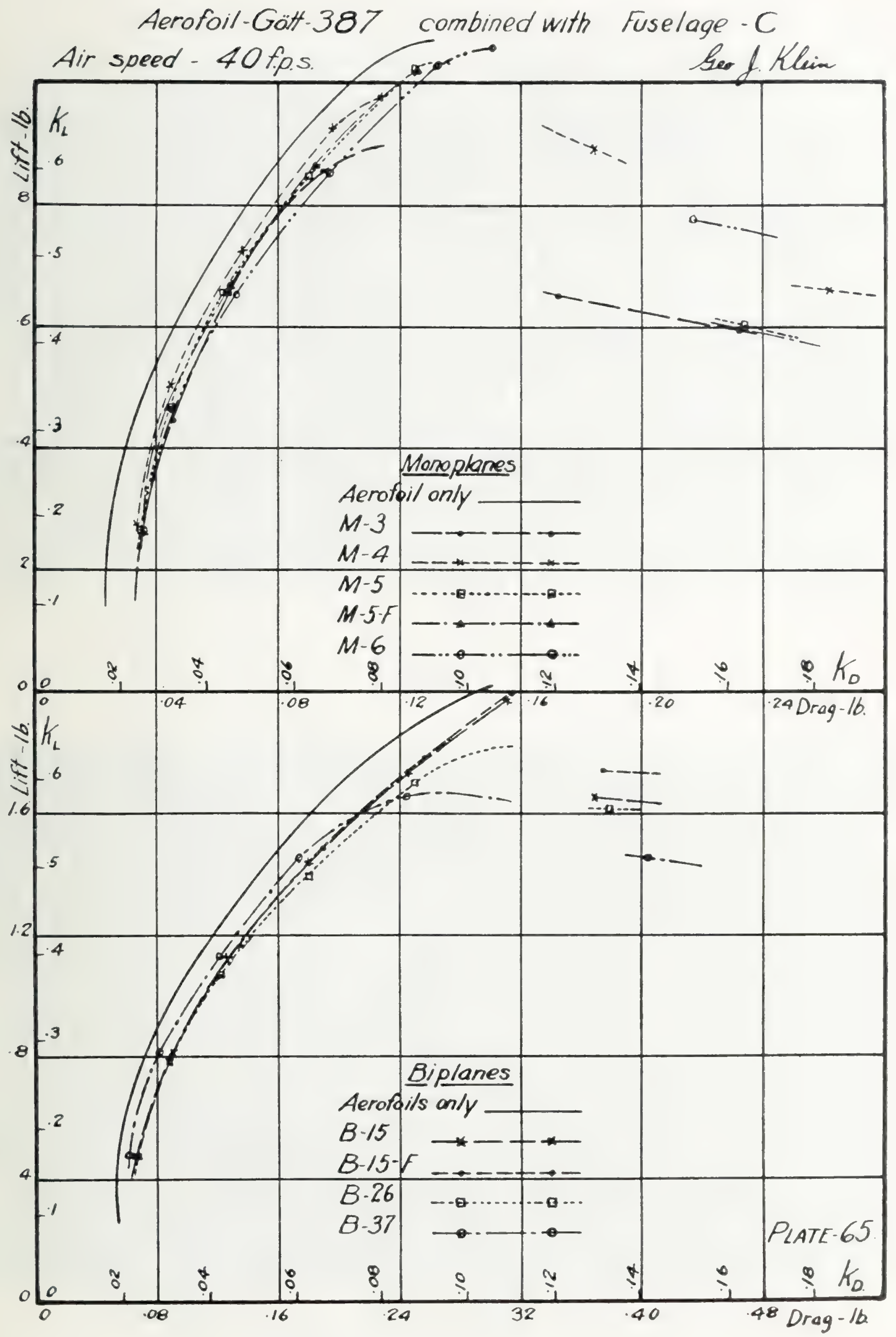


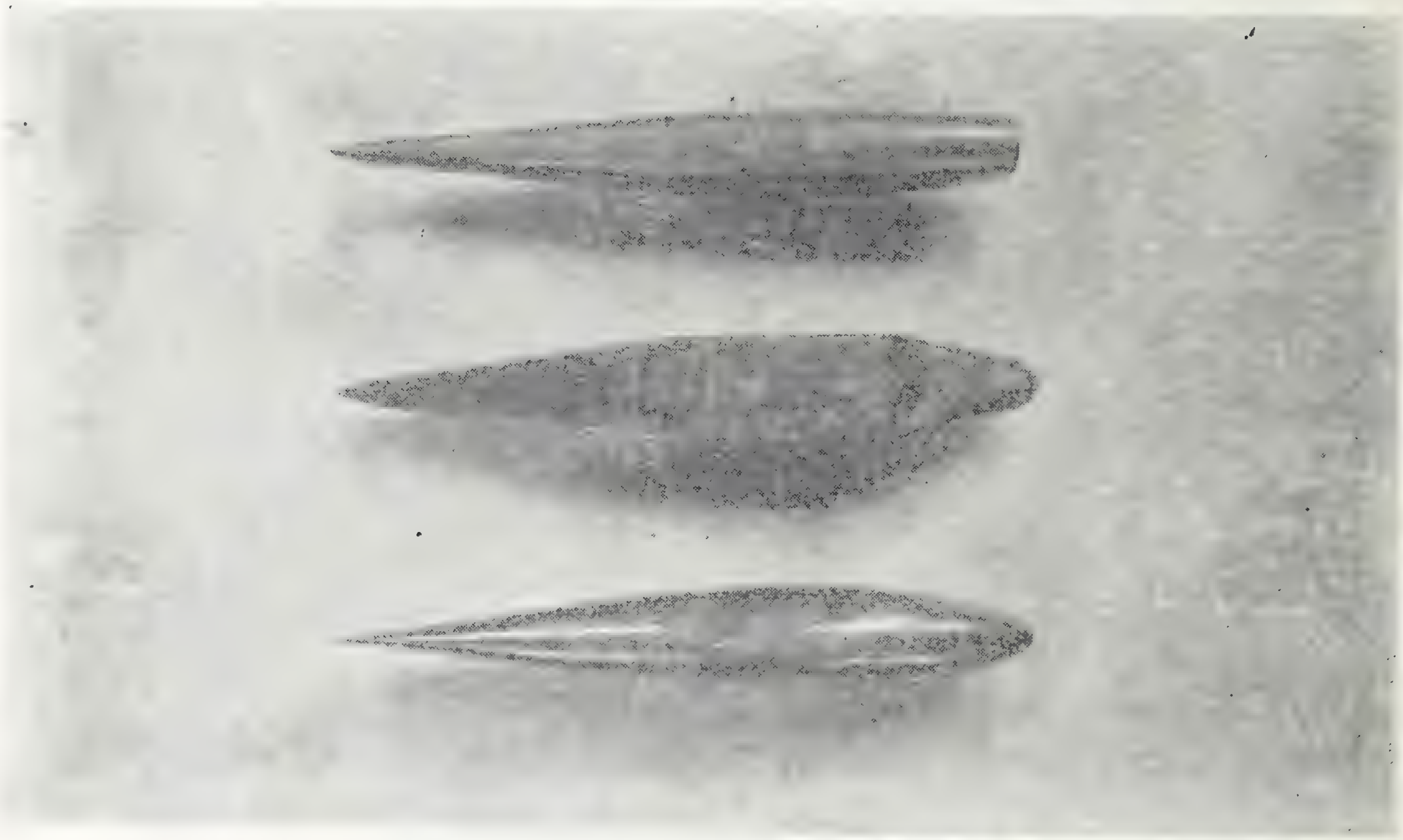
PLATE-64


Aerofoil-Gött-387 combined with Fuselage-C

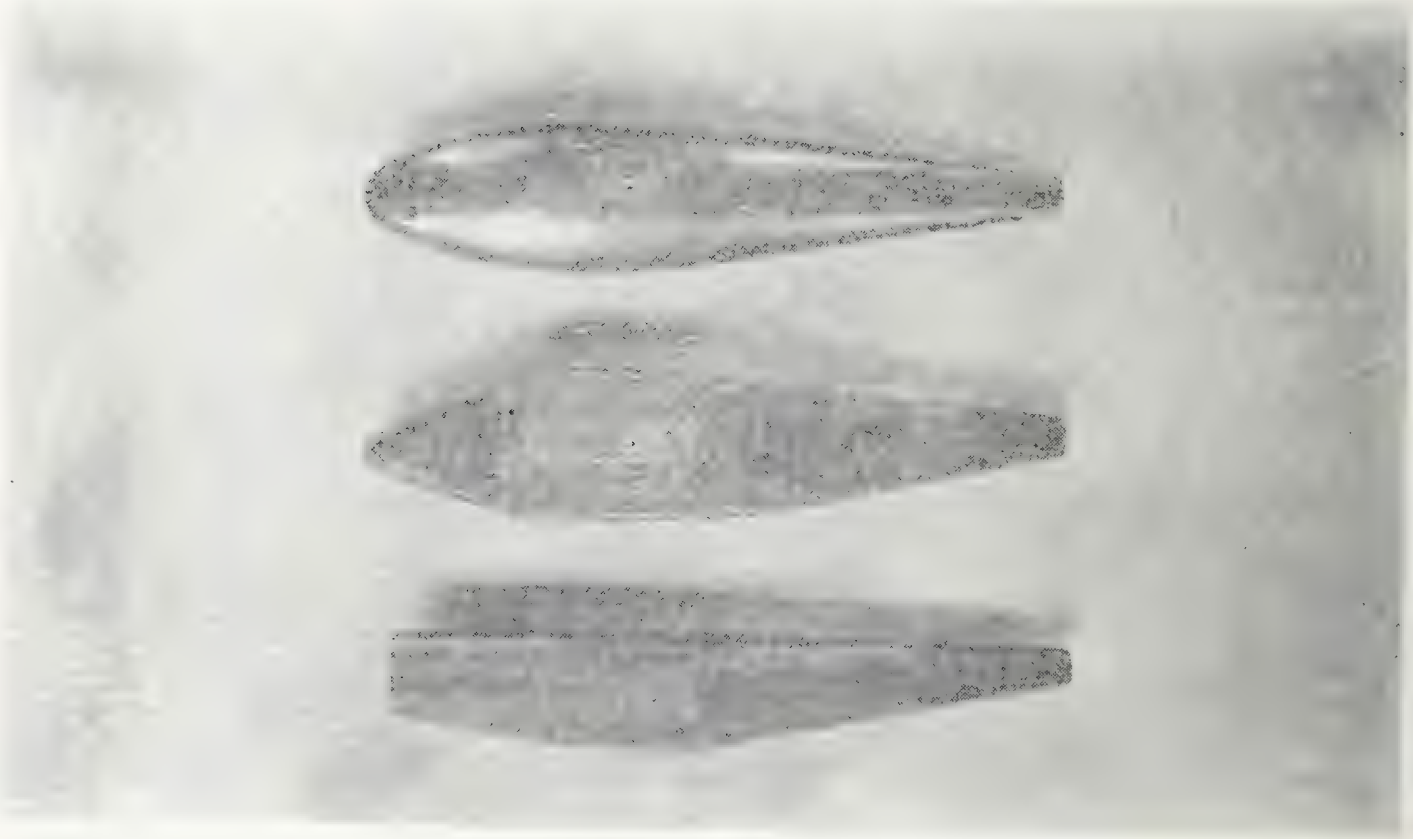
Air speed - 40 f.p.s.

Geo J. Klein

PLATE-65

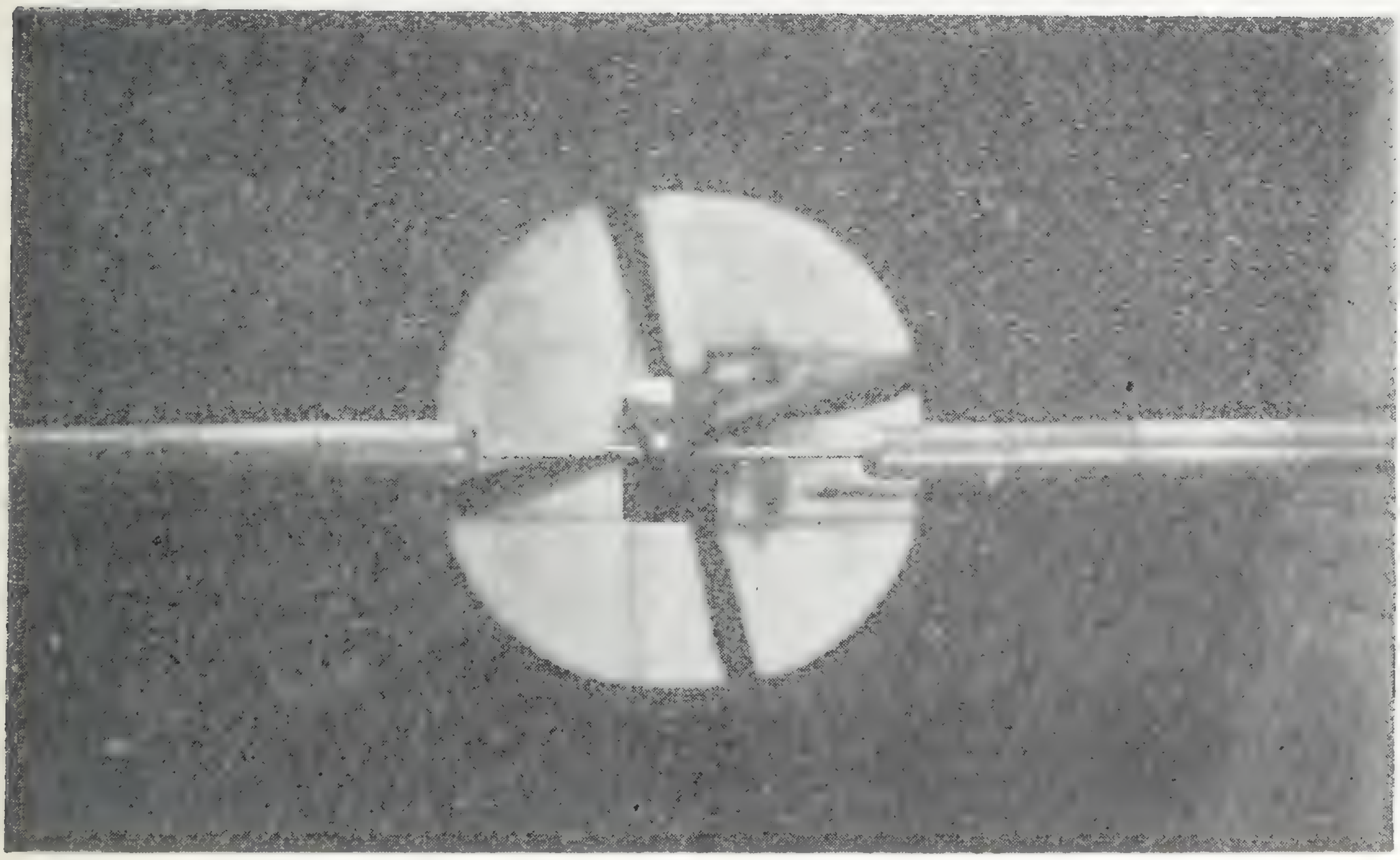


PHOTOGRAPH 1

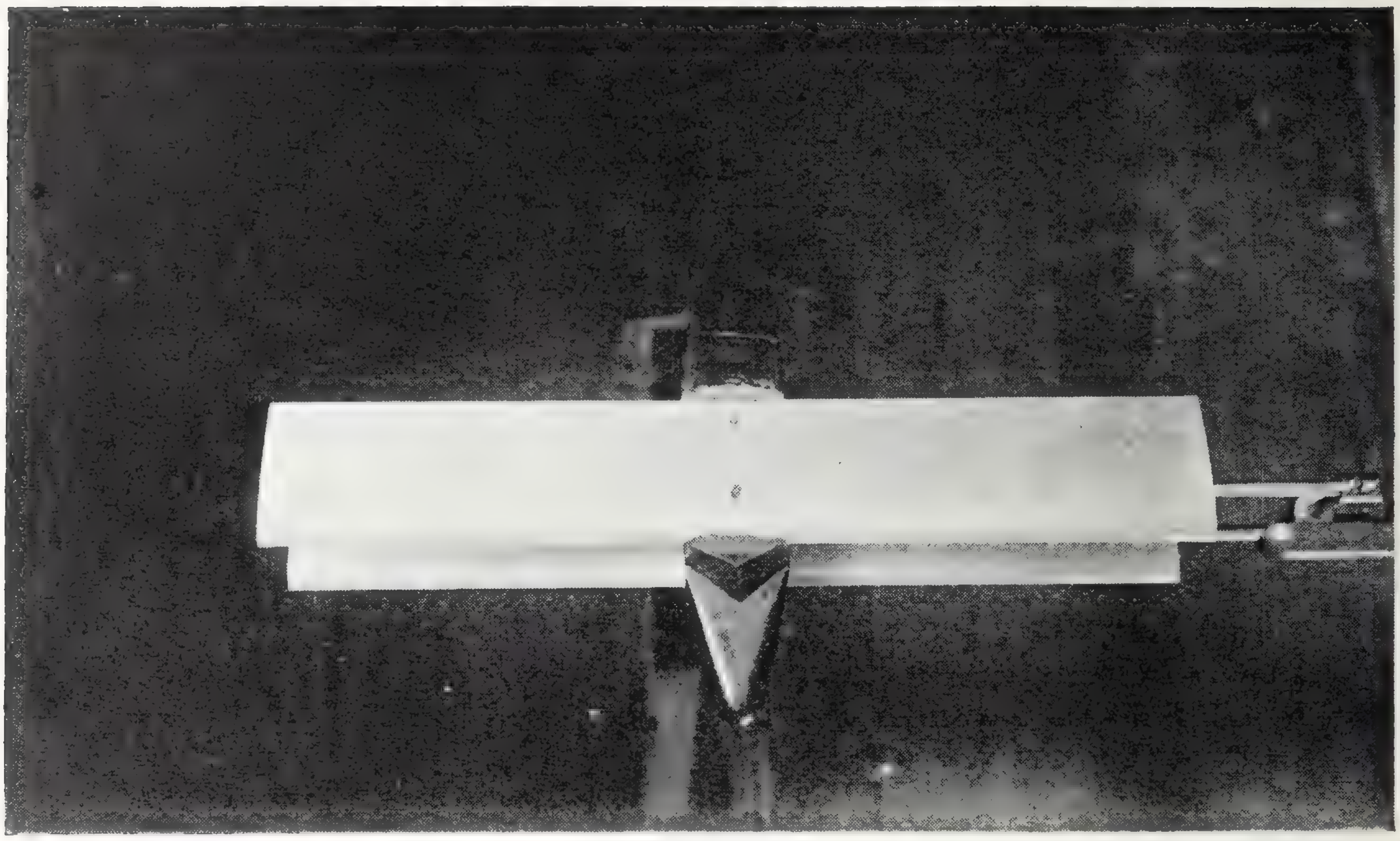


PHOTOGRAPH 2

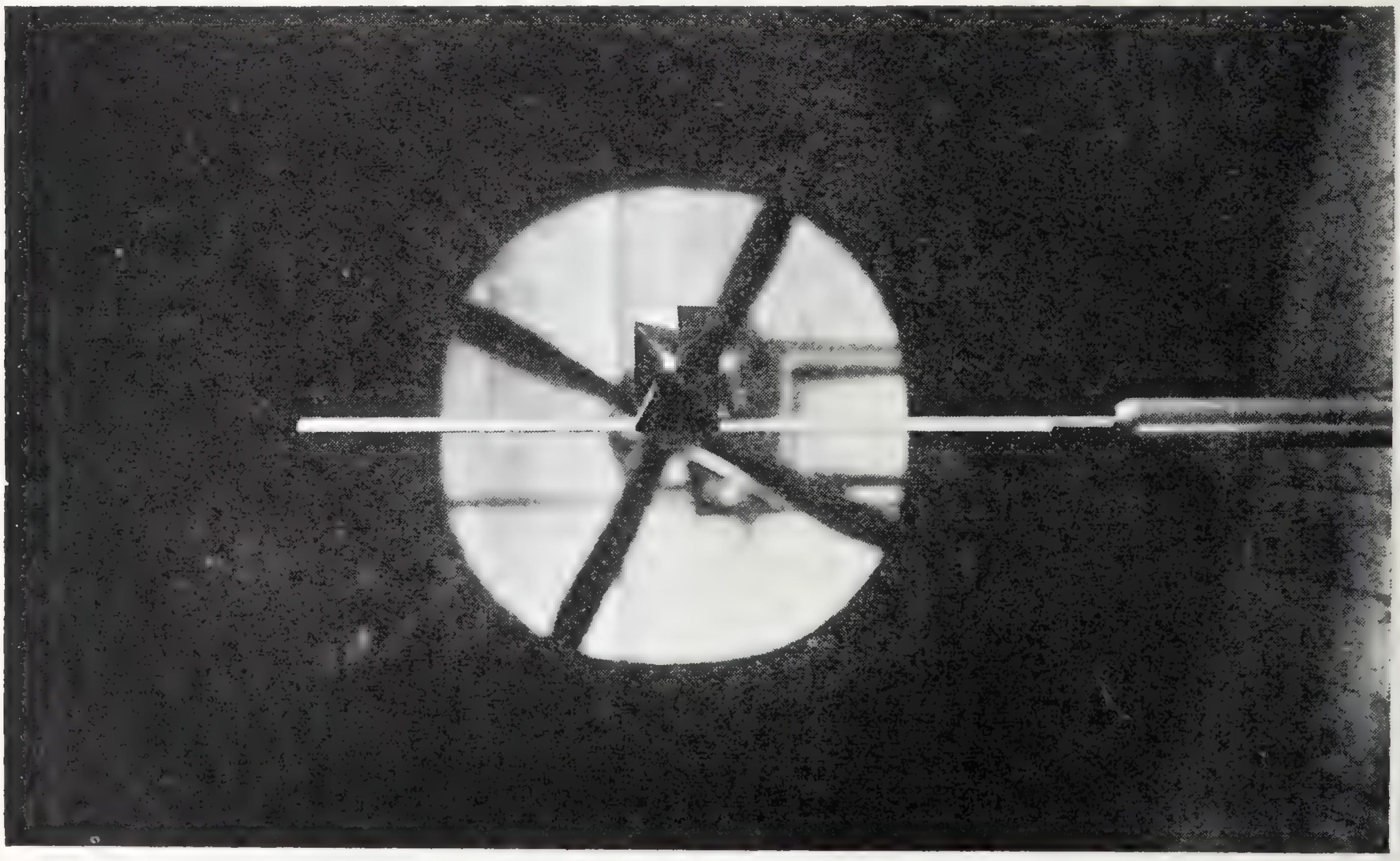
Photograph 4



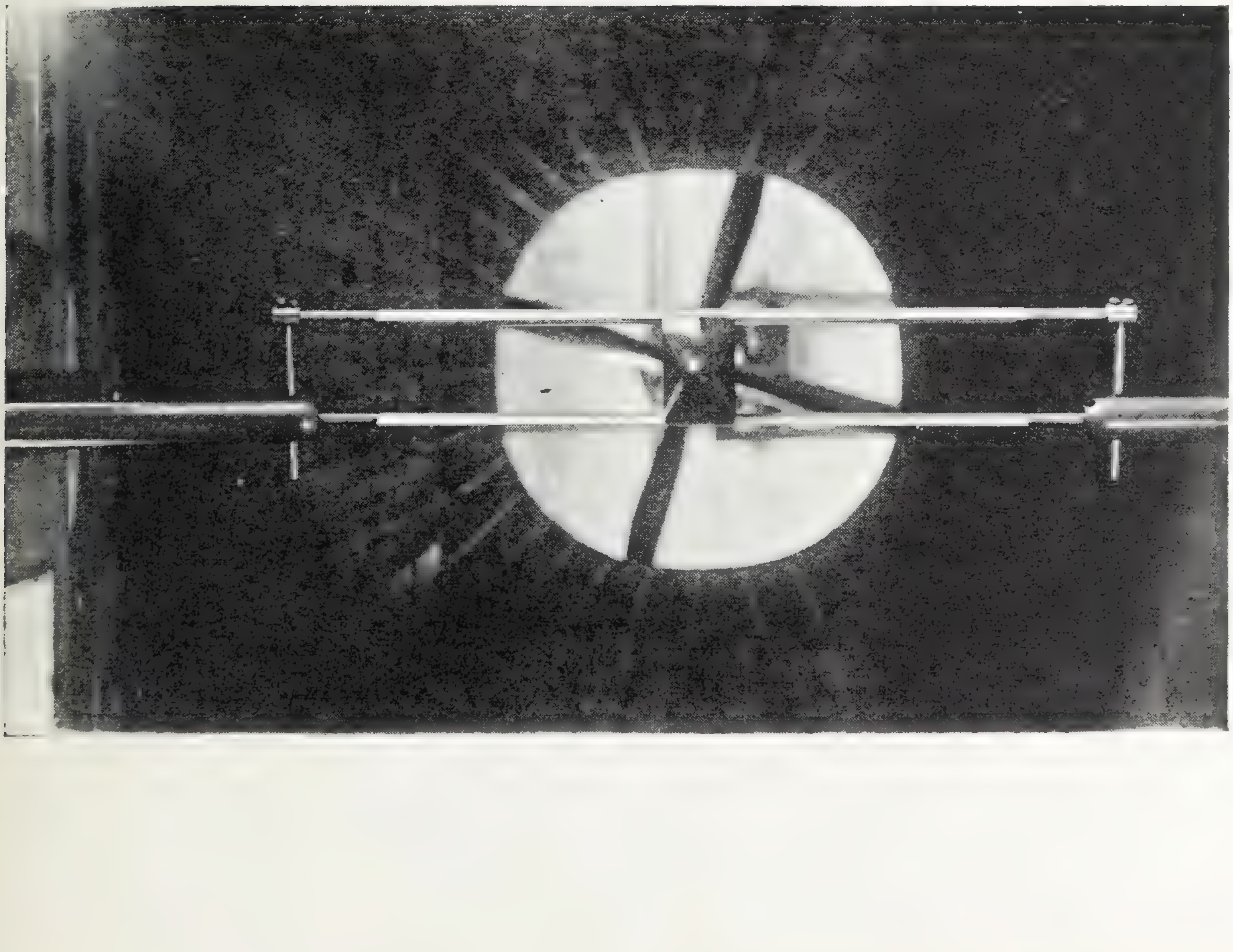
Photograph 3



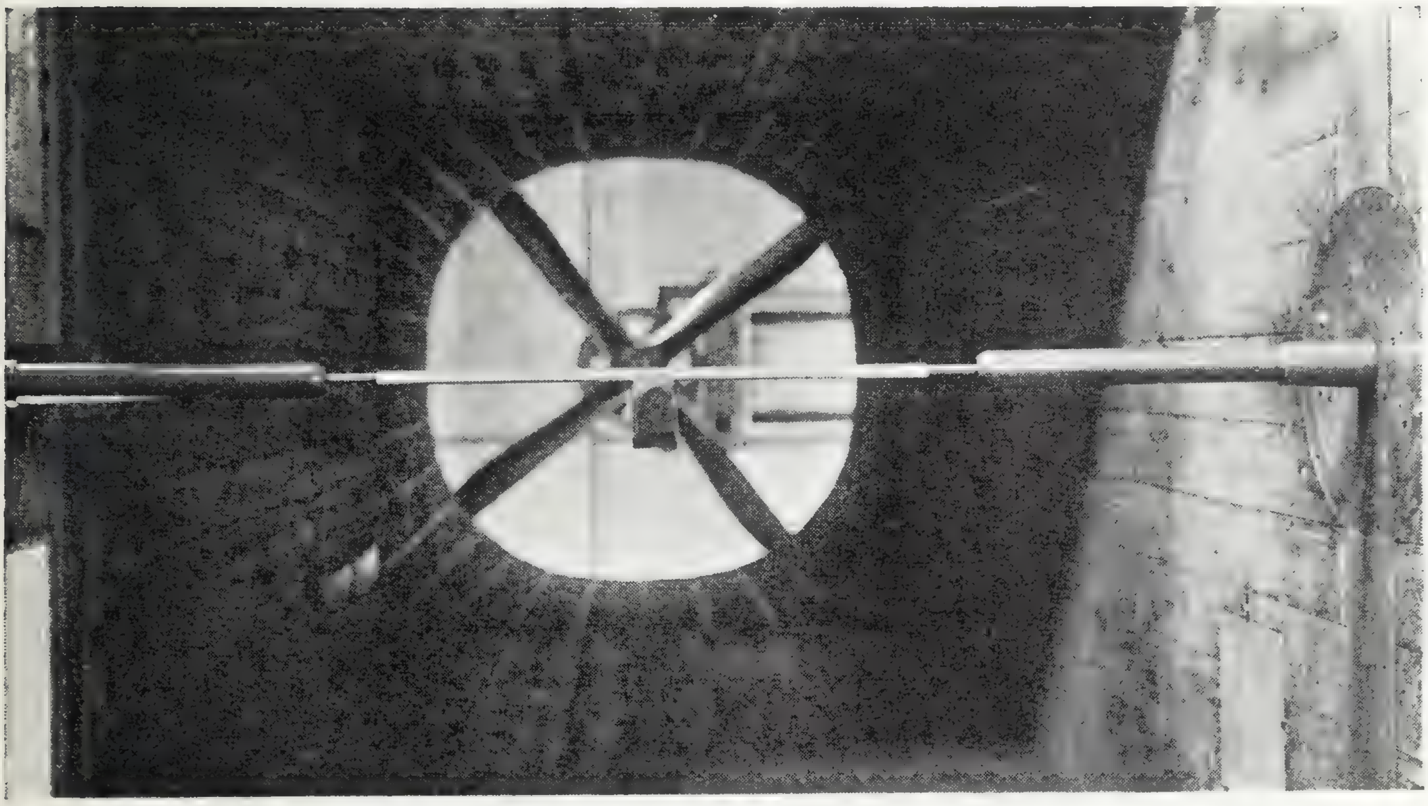
PHOTOGRAPH 6



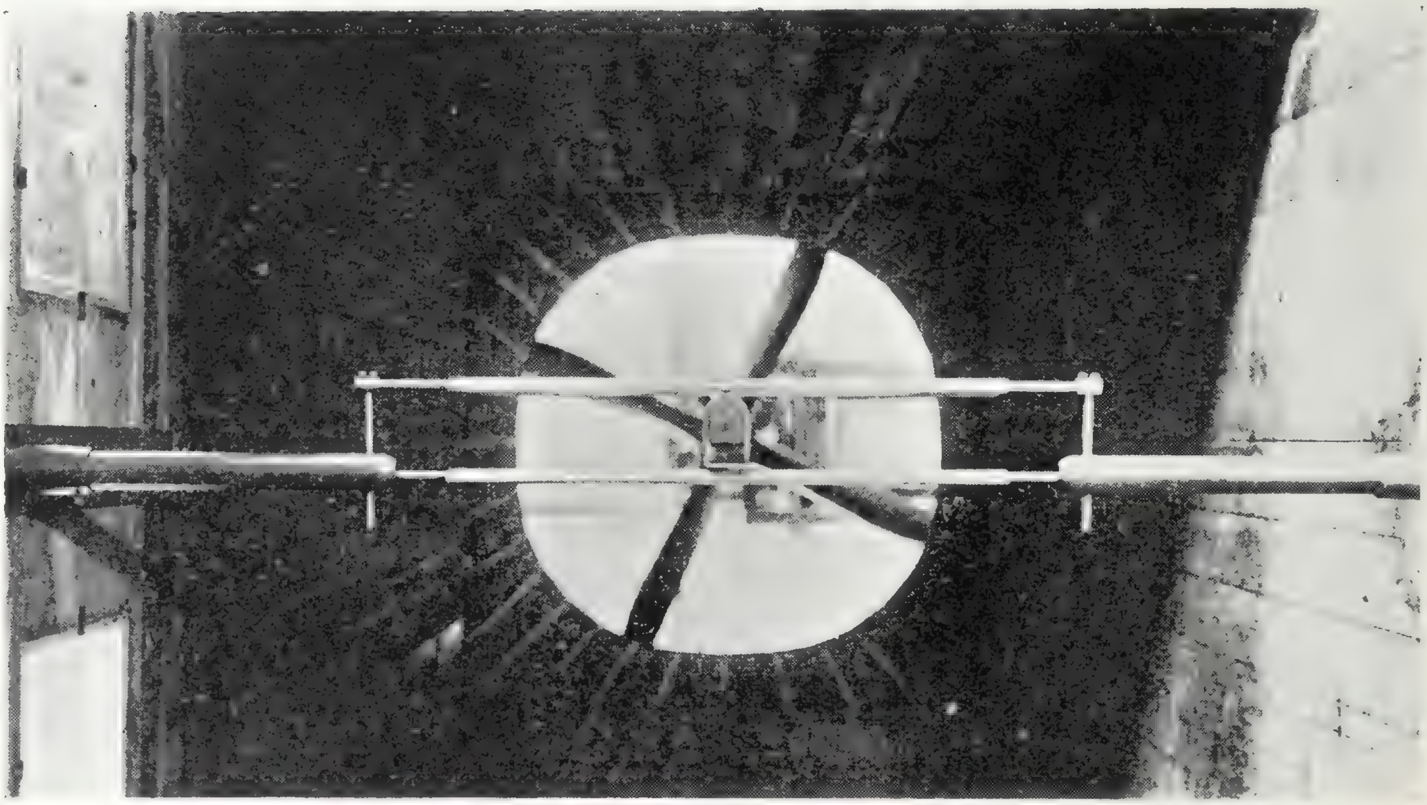
PHOTOGRAPH 5



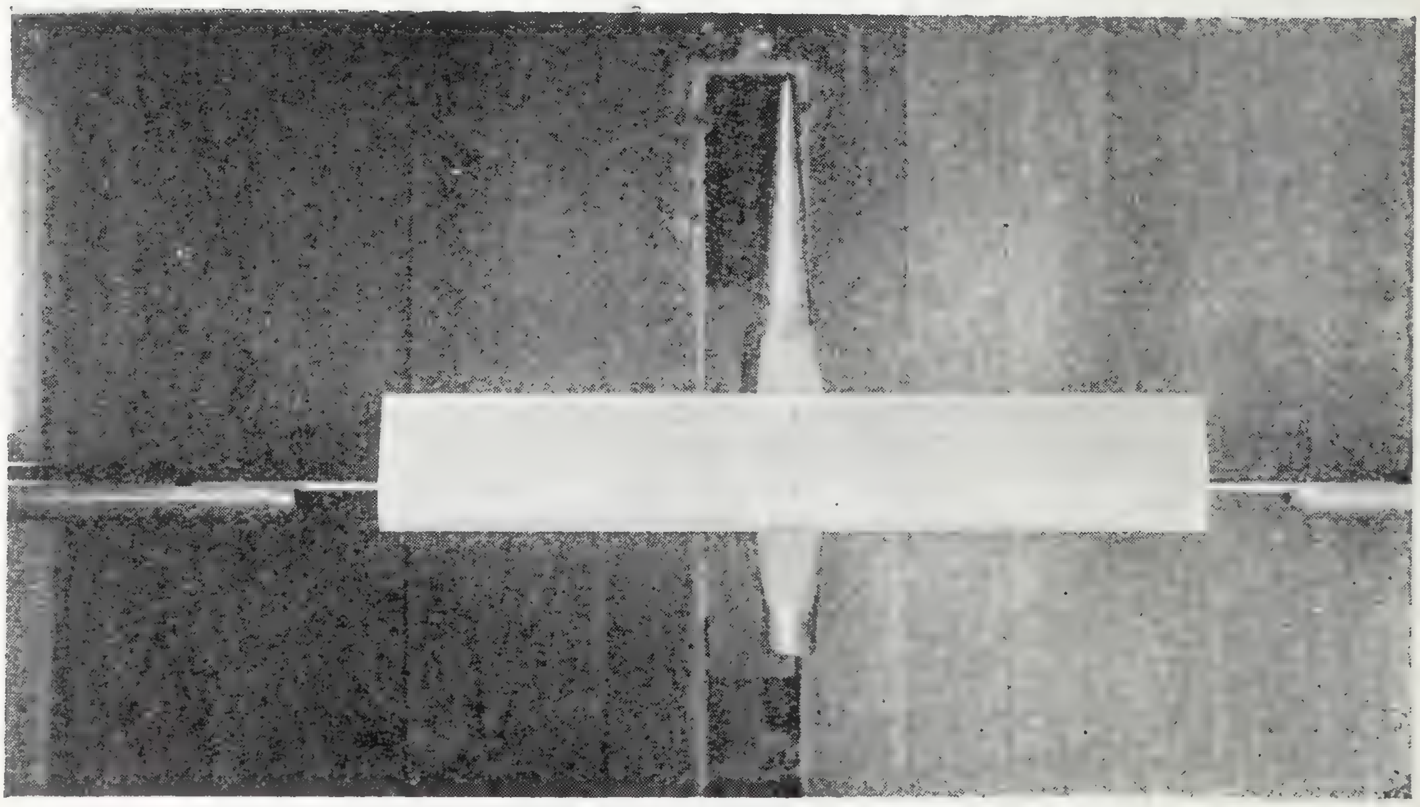
PHOTOGRAPH 7



PHOTOGRAPH 8



PHOTOGRAPH 10



PHOTOGRAPH 9

Aeronautical Research Paper No. 23.
October, 1928.

UNDERCARRIAGE RESISTANCE

By J. H. Parkin, M.E., F.R.Ae.S., W. H. Coates, B.A.Sc., and G. J. Klein, B.A.Sc.

Summary

Aircraft operating in Canada in winter, of necessity, are fitted with skis instead of wheels or floats. The effect on the performance of aircraft, due to changing from wheels to skis or from floats to skis, is, therefore, a matter of some importance to aircraft operators. The present investigation was accordingly undertaken to compare the performance of aircraft when fitted with wheel, ski and float undercarriages.

The investigation was made using a 1/16 scale model of a "general purpose" cabin biplane of more or less typical design, arranged to be fitted with wheels, skis or floats. Tests were also made with the lower wing removed as in the usual high wing cabin monoplane. Skis of two types were tested.

The results indicate that skis reduce the maximum speed of aircraft less than 5%, as compared with that when equipped with wheels. The reduction in speed is only slightly less than that due to floats. It is an advantage to fair the ski when it is built up of tubes or struts. With ski B tested in this research the reduction in maximum speed was about 50% greater when unfaired than when faired.

The resistance of the land undercarriage structure is about13-14%, and complete with wheels 19-20% of the drag of the remainder of the aeroplane. The complete marine undercarriage has a drag of more than 25% of that of the rest of the machine.

If the land undercarriage could be removed, or completely retracted, the maximum speed of the aeroplane would be increased nearly 20%.

Introduction

Aircraft in Canada are fitted with skis during the winter months for operating from snow and ice. The skis are commonly interchangeable with the wheels of the usual land undercarriage. Replacing the wheels with skis may affect performance of the aircraft either through (*a*) Increasing the weight, since the skis are generally heavier than the wheels.

(*b*) Increasing the parasite drag, the skis having a greater aerodynamic resistance than wheels.

Attention was directed to this question in 1927 through receipt of a letter from a large aircraft operating company in Canada inquiring:

(*a*) The lift, if any, and the drag on skis at various incidences.

(*b*) The most efficient incidence for skis.

(*c*) The effect of aspect ratio of skis on their aerodynamic efficiency.

As information on these points was lacking, and the subject was of particular interest to Canadian aviation, the investigation described in this paper was undertaken.

SCOPE OF INVESTIGATION

Initially, a rather comprehensive programme for the research was outlined, with a view to securing information applicable to the forms of landing gear and types of machine in general use.

The type of aircraft most generally employed for commercial purposes in Canada, at the time, was the cabin monoplane or biplane of about 200 h.p. and with a seating capacity of 4-5. The machines were of the "general purpose" type, so constructed as to permit use of a land undercarriage with wheels or skis, or a marine undercarriage with floats.

The tentative programme for the investigation was therefore as follows:—

(*a*) Determination of the aerodynamic characteristics of a cabin monoplane and of a cabin biplane when fitted with wheel, ski and float undercarriages.

(*b*) Comparison of the aerodynamic behaviour of skis of two or three types on the monoplane and the biplane.

(*c*) A detailed investigation of the effect of ski incidence, ski aspect ratio and the vertical position of the ski relative to wings and fuselage, on the aerodynamic performance of aircraft.

The results of sections (*a*) and (*b*) of the investigation were such that it was not considered justifiable to proceed with section (*c*).

MODELS

As a 1/16 scale model of a general purpose five-place cabin biplane was available at the laboratory, it was used in the tests.

The general specifications of the machine were:

Span, 35′ 0″. Length, 30′ 0″. Height, 12′ 0″. Weight, 2,200 lbs. Disposable load, 1,200 lbs. Power, 200 h.p. Wing section—Clark Y. Maximum speed, 100 m.p.h.

The general form of the machine will be apparent from the outline drawing on Plate 1 and the photographs. The model was of wood, with the exception of the aluminum tail plane and the brass struts. For the monoplane tests the lower wing of the biplane was removed.

The undercarriages were made up as nearly as possible to scale. The land undercarriage was of the split axle type with oleo legs. The wheels were of the usual type.

Models of three types of skis were made, but only two were tested, as the differences in aerodynamic chracteristics were found to be very slight.

The skis of which models were made were of the following types and were suitable for machines weighing from 2,500-3,500 pounds.

1. Ski A—a ski having a coarsely corrugated duralumin shoe and conical pedestal of sheet duralumin.

2. Ski B—a ski having a duralumin shoe with fine corrugations supported on a pedestal of steel tubing. The tubing is ordinarily covered with a fabric fairing.

3. Ski C—an all-wood ski having a flat grooved shoe and pedestal of wooden struts. The model of this ski was not completed and was not tested.

The models tested are shown in Plate 2 and the photographs.

APPARATUS AND TESTS

The measurements were made in the 4-foot closed RAE type wind tunnel of the University of Toronto. (Refer A.R.P. No. 16, 1927.)

The models were supported inverted, from roof balances, using two fine wires and a tail sting, as will be seen in the photographs.

The models were aligned, using protractor level, and the speed was indicated by means of a side plate and modified Chattock manometer.

The air speed was 40 ft. per sec. throughout the tests.

Measurements of lift and drag were made for the following:

1. Bare fuselage with empennage only.

For both monoplane and biplane arrangements:—

2. Fuselage with wings and empennage.
3. Fuselage with wings, empennage and bare undercarriage.
4. Model complete with undercarriage and wheels.
5. Model complete with undercarriage and skis—type A.
6. Model complete with undercarriage and skis—type B.
7. Model as in latter test but with skis faired with plasticene to represent fabric fairing.
8. Model complete with float undercarriage but without the floats.
9. Model complete with floats.

PRESENTATION OF RESULTS

The lift, drag and lift/drag are tabulated for each of the arrangements tested, as listed in the foregoing, in Table 1 for the monoplane and Table 2 for the biplane. Lifts and drags are in pounds.

The results are shown graphically in polar diagrams on Plates 3 to 8. Plates 3-5 are for the monoplane and Plates 6-8 for the biplane.

Plates 3 and 6 show polar curves for the fuselage and empennage, the model without undercarriage, the model with undercarriage but without wheels, and the model complete with wheels. Comparison of the curves on these plates thus shows the effects on the aerodynamic performance of the addition of the different components: wings, undercarriage and wheels.

On Plates 4 and 7 are plotted similar polar curves for the marine undercarriage, the curves showing the aerodynamic effects of the bare undercarriage and of the floats.

The polar curves for the wheel, ski and float undercarriages are plotted on Plates 5 and 8. The figures plotted for the ski undercarriage are those for the ski having the least drag, namely, the type B ski when faired. These curves enable the three types of undercarriage to be compared from an aërodynamic standpoint.

DISCUSSION OF RESULTS

The results are to be accepted only as comparative, showing the relative resistances of the undercarriages tested. The scale effects on the shaped struts, wires and other parts of the model undercarriages at the values of *vl* of the tests will be large, but of roughly the same magnitude for the different arrangements. Hence, the figures may be used to predict the effect of changing from wheels to skis, or wheels to floats, etc.

The relative drags of the different undercarriages and components are collected and tabulated in Table 3. It will be observed that the wheel undercarriage offers the least resistance. The undercarriage with skis has practically the same resistance as the float undercarriage. There is little aerodynamic difference between ski A and ski B when the latter is faired, but, when unfaired, ski B has appreciably greater resistance.

The marine undercarriage structure without floats has a lower resistance than the land undercarriage without wheels, presumably due to the absence of the oleo legs. The resistance of the monoplane float undercarriage is slightly less than that for the biplane, due to the former having fewer struts.

In the region of minimum drag, for the biplane, the bare land undercarriage adds 13-14% to the resistance of the remainder of the aeroplane and the complete undercarriage with wheels, 19-20%. With faired skis the additional resistance due to the undercarriage is about 25%, and with unfaired skis 29%; hence, it is advantageous to fair the skis. The marine undercarriage without floats adds 12%, and with floats about 26% to the resistance of the remainder of the machine. Thus, the resistance of the wheels is nearly 30% of that of the whole undercarriage, and of the skis rather less than 50% of that of the complete undercarriage, while for the marine undercarriage the float resistance is somewhat greater than 50% of that of the entire undercarriage.

The undercarriages contribute practically nothing to the lift of the aeroplane. Only at the maximum incidence tested, 18°, is there any lift effect, the floats and skis at this angle exerting a slight lift.

The presence of the lower wing of the biplane relatively close to the undercarriage has apparently no influence on the drag of the different undercarriages, as is shown by comparing the figures in columns 3 and 4, Table 3. Interference effects due to the lower wing are thus negligible.

The effects of the undercarriages on the performance of the aircraft have been calculated approximately and are tabulated in Table 4. These figures are based on the assumption that the maximum speed of 100 m.p.h. with the wheel undercarriage corresponds to the angle of minimum drag for the complete model, namely, 0°, and that the available power is constant at 200 h.p. This latter assumption is roughly true, since the usual curve of power available plotted on aircraft speed is nearly horizontal.

It will be seen from Table 4 that changing from wheels to reasonably faired skis reduces the maximum speed less than 5%, and if the skis are not faired about 7½%. Replacing the land undercarriage with the marine undercarriage causes a reduction in maximum speed of more than 5%. If the undercarriage could be entirely removed, or completely retracted, the maxmum speed would be increased nearly 20%. The full scale percentage effects will probably be rather less than as stated in the foregoing, due to the large scale effect on struts, etc., operative between the *vl* of the tests and the full scale *vl*. The comparative effects of the different undercarriages should be about the same on full scale, as shown in this report for the models.

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1. High Wing Cabin Monoplane—Aerodynamic characteristics of different undercarriages.
2. Cabin Biplane—Aerodynamic characteristics of different undercarriages.
3. Relative Drags of Undercarriages and Components.
4. Effect on Maximum Speed.

LIST OF PLATES

1. Cabin Biplane—Outline drawing.
2. Skis—Types A, B and C.
3. Cabin Monoplane—Graphs for land undercarriage and components.
4. “ “ —Graphs for marine undercarriage and components.
5. “ “ —Graphs for wheel, ski and float undercarriages.
6. Cabin Biplane—Graphs for land undercarriage and components.
7. “ “ —Graphs for marine undercarriage and components.
8. “ “ —Graphs for wheel, ski and float undercarriages.

LIST OF PHOTOGRAPHS

1. Cabin biplane with wheel undercarriage.
2. Cabin biplane with float undercarriage.
3. Cabin biplane with skis, Type A.
4. Cabin biplane with skis, Type B.
5. Cabin biplane with skis, Type B—front view.
6. Cabin biplane with skis, Type B—faired.

TABLE 1

UNDERCARRIAGE RESISTANCE

HIGH WING MONOPLANE

Wing Incidence (degrees)	Bare Fuselage	Fuselage with Wing	——Fuselage with Wheel Undercarriage——					Float Undercarriage	
			Without Wheels	With Wheels	With Ski A	With Ski B	With Ski B Faired	Without Floats	With Floats
				LIFT (pounds)					
−8	−0.154	−0.392	−0.371	−0.375	−0.388	−0.382	−0.388	−0.385	−0.460
−4	−0.056	+0.135	+0.155	+0.151	+0.139	+0.140	+0.148	+0.147	+0.091
0	+0.042	0.618	0.628	0.621	0.618	0.618	0.627	0.623	0.591
+2	0.111	0.856	0.884	0.859	0.866	0.856	0.881	0.862	0.842
4	0.164	1.112	1.122	1.102	1.108	1.086	1.131	1.112	1.102
6	0.226	1.341	1.352	1.365	1.353	1.339	1.376	1.361	1.360
8	0.297	1.584	1.584	1.596	1.596	1.563	1.617	1.586	1.583
10	0.369	1.830	1.833	1.819	1.826	1.789	1.849	1.839	1.812
12	0.432	2.006	2.017	2.044	2.025	1.972	2.023	2.033	2.010
14	0.483	2.121	2.165	2.143	2.164	2.098	2.161	2.177	2.184
16	0.502	2.106	2.119	2.098	2.147	2.109	2.151	2.138	2.269
18	0.492	1.995	2.032	2.016	2.087	2.069	1.992	2.060	2.179

TABLE NO. 1—*Continued*

Wing Incidence (degrees)	Bare Fuselage	Fuselage with Wing	—Fuselage with Wheel Undercarriage—					Float Undercarriage	
			Without Wheels	With Wheels	With Ski A	With Ski B	With Ski B Faired	Without Floats	With Floats
				DRAG (pounds)					
−8	0.0690	0.1290	0.1491	0.1576	0.1728	0.1751	0.1663	0.1452	0.1809
−4	0.0522	0.0861	0.1063	0.1163	0.1281	0.1303	0.1227	0.1024	0.1267
0	0.0510	0.0840	0.1040	0.1128	0.1233	0.1272	0.1204	0.0994	0.1197
+2	0.0563	0.0956	0.1156	0.1233	0.1343	0.1369	0.1323	0.1109	0.1289
4	0.0635	0.1137	0.1346	0.1409	0.1529	0.1521	0.1506	0.1286	0.1474
6	0.0761	0.1381	0.1593	0.1681	0.1781	0.1780	0.1783	0.1539	0.1735
8	0.0966	0.1681	0.1917	0.1976	0.2111	0.2085	0.2099	0.1884	0.2051
10	0.1198	0.2128	0.2309	0.2399	0.2536	0.2493	0.2496	0.2306	0.2521
12	0.1504	0.2606	0.2855	0.2928	0.3089	0.3002	0.3031	0.2863	0.3086
14	0.1771	0.3278	0.3544	0.3617	0.3789	0.3712	0.3778	0.3585	0.3814
16	0.2018	0.4928	0.5293	0.5369	0.5531	0.5571	0.5326	0.5368	0.4749
18	0.2201	0.6530	0.6961	0.6964	0.7048	0.7222	0.6690	0.6949	0.6787
				LIFT/DRAG					
−8	−2.23	−3.04	−2.49	−2.38	−2.25	−2.18	−2.33	−2.65	−2.54
−4	−1.07	+1.57	+1.46	+1.30	+1.08	+1.07	+1.21	+1.44	+0.72
0	+0.82	7.35	6.04	5.50	5.00	4.86	5.20	6.27	4.94
+2	1.97	8.95	7.65	6.95	6.45	6.25	6.65	7.78	6.57
4	2.58	9.80	8.35	7.81	7.25	7.13	7.51	8.70	7.48
6	2.97	9.70	8.50	8.11	7.60	7.52	7.71	8.85	7.84
8	3.07	9.42	8.27	8.07	7.55	7.50	7.70	8.42	7.72
10	3.08	8.60	7.95	7.57	7.20	7.17	7.41	7.97	7.19
12	2.87	7.69	7.06	6.98	6.55	6.57	6.68	7.10	6.51
14	2.73	6.48	6.10	5.93	5.72	5.65	5.72	6.07	5.73
16	2.49	4.28	4.00	3.91	3.88	3.78	4.03	3.98	4.78
18	2.23	3.06	2.92	2.89	2.96	2.865	2.98	2.97	3.21

TABLE 2

UNDERCARRIAGE RESISTANCE

BIPLANE

Wing Incidence (degrees)	Bare Fuselage	Fuselage with Wing	Fuselage with Wheel Undercarriage					Float Undercarriage	
			Without Wheels	With Wheels	With Ski A	With Ski B	With Ski B Faired	Without Floats	With Floats
				LIFT (pounds)					
−8	−0.154	−0.522	−0.523	−0.523	−0.519	−0.513	−0.530	−0.532	−0.601
−4	−0.056	+0.239	+0.263	+0.264	+0.259	+0.270	+0.254	+0.253	+0.209
0	+0.042	0.983	0.985	0.992	0.991	1.003	0.992	0.996	0.966
+2	0.111	1.349	1.385	1.375	1.349	1.367	1.369	1.376	1.346
4	0.164	1.745	1.750	1.739	1.718	1.753	1.736	1.751	1.715
6	0.226	2.093	2.122	2.117	2.083	2.137	2.088	2.136	2.092
8	0.297	2.432	2.371	2.424	2.474	2.468	2.456	2.473	2.443
10	0.369	2.820	2.813	2.758	2.854	2.821	2.849	2.859	2.830
12	0.432	3.146	3.135	3.063	3.180	3.134	3.165	3.166	3.170
14	0.483	3.391	3.423	3.418	3.424	3.448	3.396	3.446	3.452
16	0.502	3.589	3.598	3.583	3.615	3.635	3.596	3.606	3.647
18	0.492	3.495	3.479	3.328	3.565	3.570	3.520	3.529	3.599

TABLE NO. 2—*Continued*

Wing Incidence (degrees)	Bare Fuselage	Fuselage with Wing	——Fuselage with Wheel Undercarriage——					Float Undercarriage	
			Without Wheels	With Wheels	With Ski A	With Ski B	With Ski B Faired	Without Floats	With Floats
				DRAG (pounds)					
−8	0.0690	0.1880	0.2069	0.2191	0.2306	0.2345	0.2253	0.2080	0.2393
−4	0.0522	0.1318	0.1556	0.1646	0.1743	0.1810	0.1699	0.1540	0.1781
0	0.0510	0.1398	0.1590	0.1669	0.1752	0.1804	0.1752	0.1564	0.1765
+2	0.0563	0.1611	0.1808	0.1858	0.1932	0.2000	0.1936	0.1768	0.1929
4	0.0635	0.1916	0.2094	0.2145	0.2238	0.2283	0.2218	0.2067	0.2249
6	0.0761	0.2304	0.2492	0.2549	0.2592	0.2683	0.2627	0.2462	0.2665
8	0.0966	0.2776	0.2942	0.2988	0.3153	0.3118	0.3128	0.2965	0.3155
10	0.1198	0.3423	0.3562	0.3578	0.3826	0.3805	0.3707	0 3605	0 3860
12	0.1504	0.4106	0.4332	0.4331	0.4548	0.4515	0.4480	0.4319	0.4647
14	0.1771	0.4945	0.5144	0.5124	0.5374	0.5412	0.5313	0.5191	0.5480
16	0.2018	0.6079	0.6241	0.6280	0.6505	0.6549	0.6438	0.6289	0.6675
18	0.2201	0.8503	0.8598	0.8674	0.8962	0.8863	0.8880	0.8642	0.9120
				LIFT/DRAG					
−8	−2.23	−2.78	−2.52	−2.39	−2.25	−2.19	−2.36	−2.56	−2.51
−4	−1.07	+1.81	+1.69	+1.60	+1.48	+1.49	+1.50	+1.64	+1.17
0	+0.82	7.03	6.20	5.95	5.65	5.56	5.65	6.37	5.47
+2	1.97	8.37	7.66	7.40	6.98	6.83	7.07	7.80	6.98
4	2.58	9.11	8.35	8.10	7.68	7.67	7.83	8.48	7.63
6	2.97	9.09	8.52	8.30	8.05	7.95	7.95	8.67	7.85
8	3.07	8.77	8.40	8.11	7.85	7.92	7.85	8.35	7.75
10	3.08	8.23	7.90	7.70	7.45	7.42	7.68	7.93	7.33
12	2.87	7.66	7.24	7.08	7.00	6.95	7.06	7.33	6.83
14	2.73	7.06	6.66	6.67	6.37	6.37	6.40	6.64	6.30
16	2.49	5.90	5.76	5.70	5.55	5.55	5.60	5.73	5.46
18	2.23	4.10	4.05	3.84	3.98	4.03	3.96	4.10	3.94

TABLE 3

RELATIVE DRAGS OF UNDERCARRIAGES AND COMPONENTS

(In the region of minimum drag)

	Drag in pounds		Increase in drag over that with no undercarriage		Per cent increase in drag compared with that for model without undercarriage	
	Mono.	Biplane	Mono.	Biplane	Mono.*	Biplane
Aircraft with:						
No undercarriage........	.0840	.1398	0	0	0	0
Bare land undercarriage..	.1040	.1590	.0200	.0192	23.8	13.7
Undercarriage with wheels	.1128	.1669	.0288	.0271	34.1	19.3
Undercarriage with ski A.	.1233	.1752	.0393	.0354	46.8	25.3
“ “ “ B.	.1272	.1804	.0432	.0406	51.3	29.0
“ “ “ B. faired..............	.1204	.1752	.0364	.0354	43.3	25.3
Bare marine undercarriage	.0994	.1564	.0154	.0166	18.2	11.9
Undercarriage with floats.	.1197	.1765	.0357	.0367	42.3	26.2

TABLE 4

EFFECT ON MAXIMUM SPEED

	Maximum speed Miles per hour		Per cent change in maximum speed compared with that for wheel undercarriage	
	Mono.	Biplane	Monoplane*	Biplane
Aircraft with:				
No undercarriage.................	134.2	119.4	+34.2	+19.4
Bare land undercarriage...........	108.4	105.0	+ 8.4	+ 5.0
Undercarriage with wheels.........	100.00	100.0	0	0
“ “ ski A..........	91.4	95.3	− 8.6	− 4.7
“ “ ski B..........	88.6	92.5	−11.4	− 7.5
“ “ ski B faired....	93.6	95.3	− 6.4	− 4.7
Bare marine undercarriage.........	113.4	106.7	+13.4	+ 6.7
Undercarriage with floats..........	94.3	94.5	− 5.7	− 5.5

*These figures are somewhat misleading as the monoplane drag is much less than the biplane drag, due to the wing area being about one-half that of the biplane. Actually, of course, the monoplane wing area would be about twice that used in the model. The wing areas of the monoplane and biplane then would be nearly the same and the drags more nearly equal.

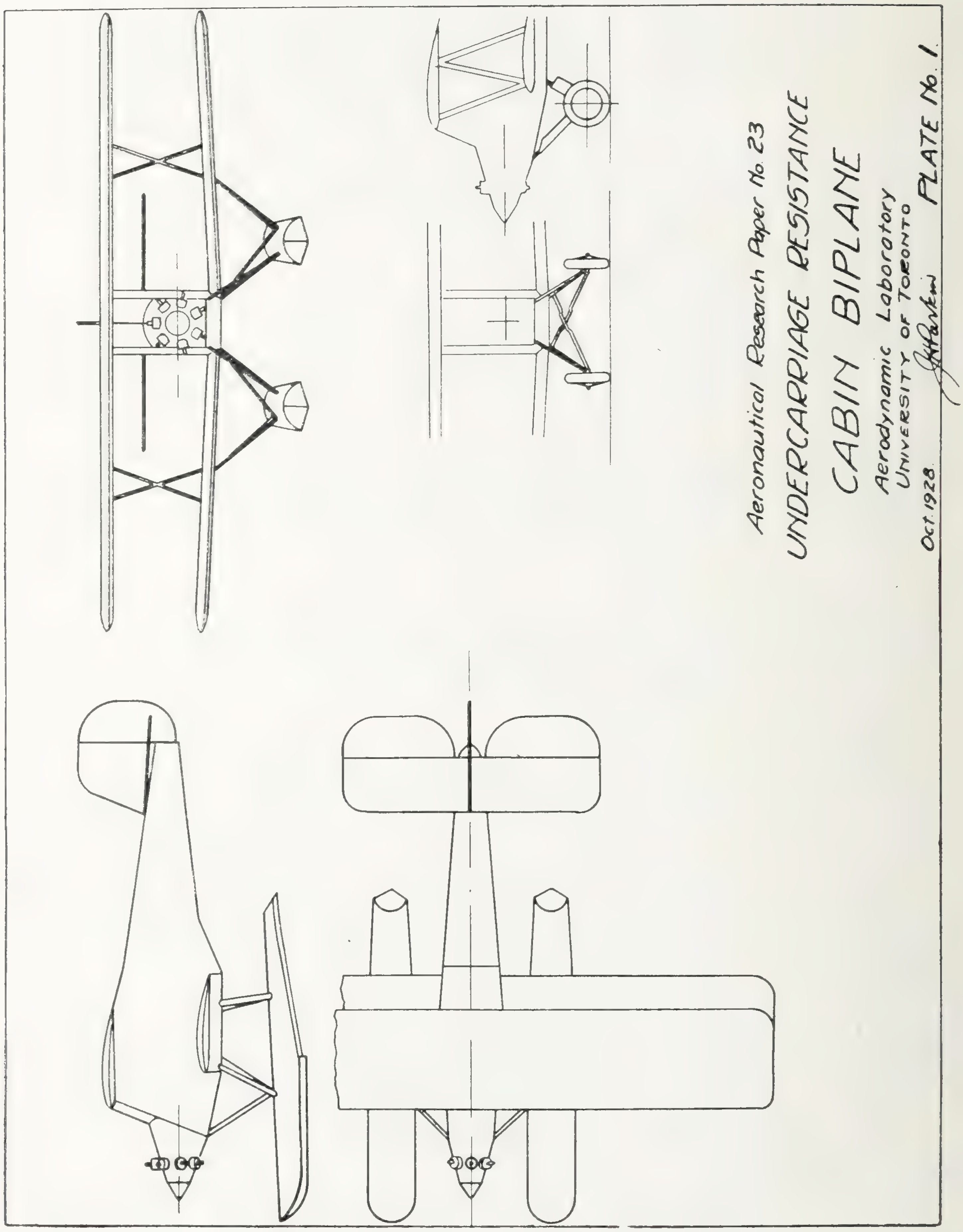

Aeronautical Research Paper No. 23
UNDERCARRIAGE RESISTANCE
CABIN BIPLANE
Aerodynamic Laboratory
UNIVERSITY OF TORONTO
Oct. 1928.
JHParkin
PLATE No. 1.

PLATE 1

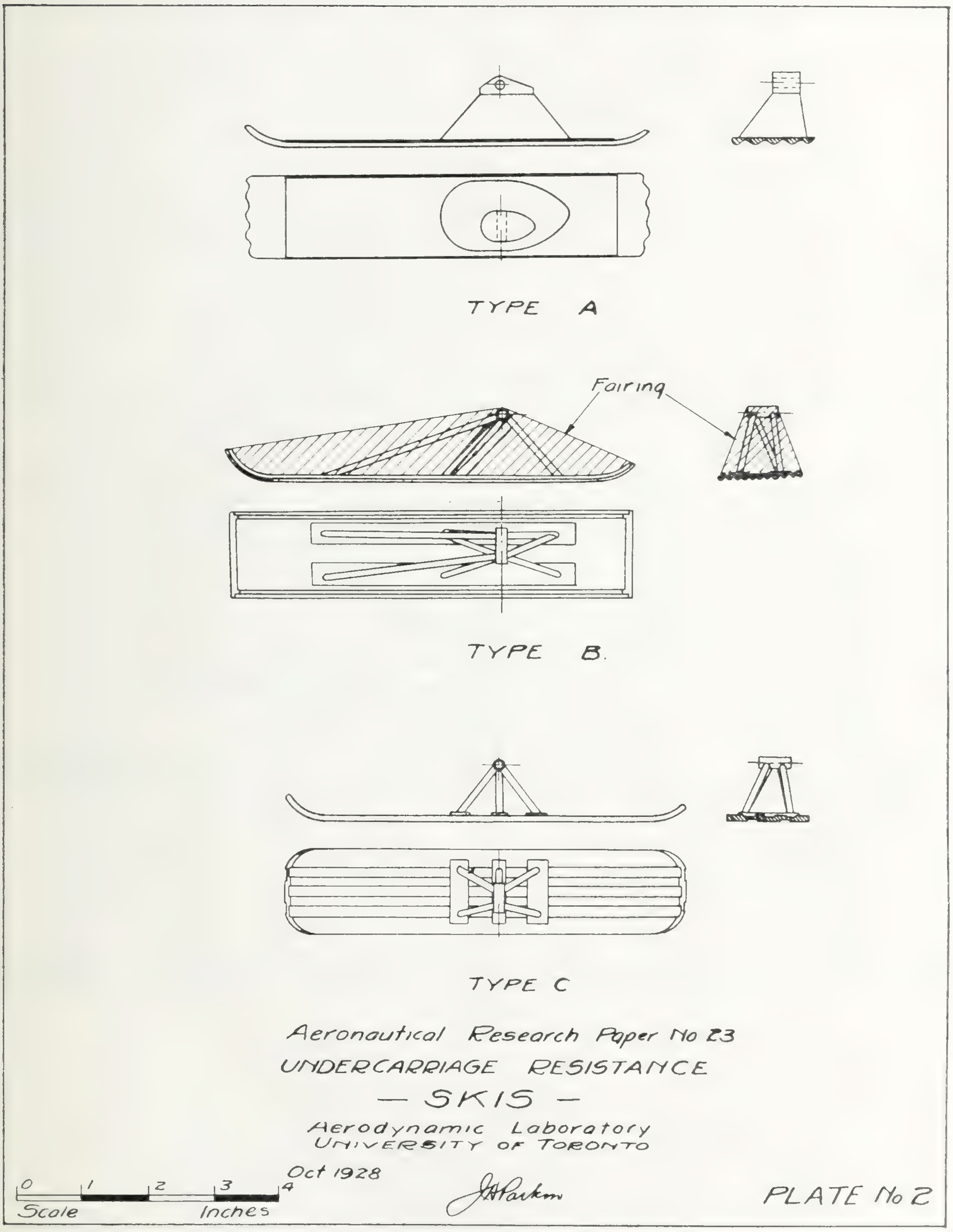

TYPE A
Fairing
TYPE B.
TYPE C
Aeronautical Research Paper No 23
UNDERCARRIAGE RESISTANCE
— SKIS —
Aerodynamic Laboratory
UNIVERSITY OF TORONTO
Oct 1928
0 1 2 3 4
Scale Inches
PLATE No 2

PLATE 2

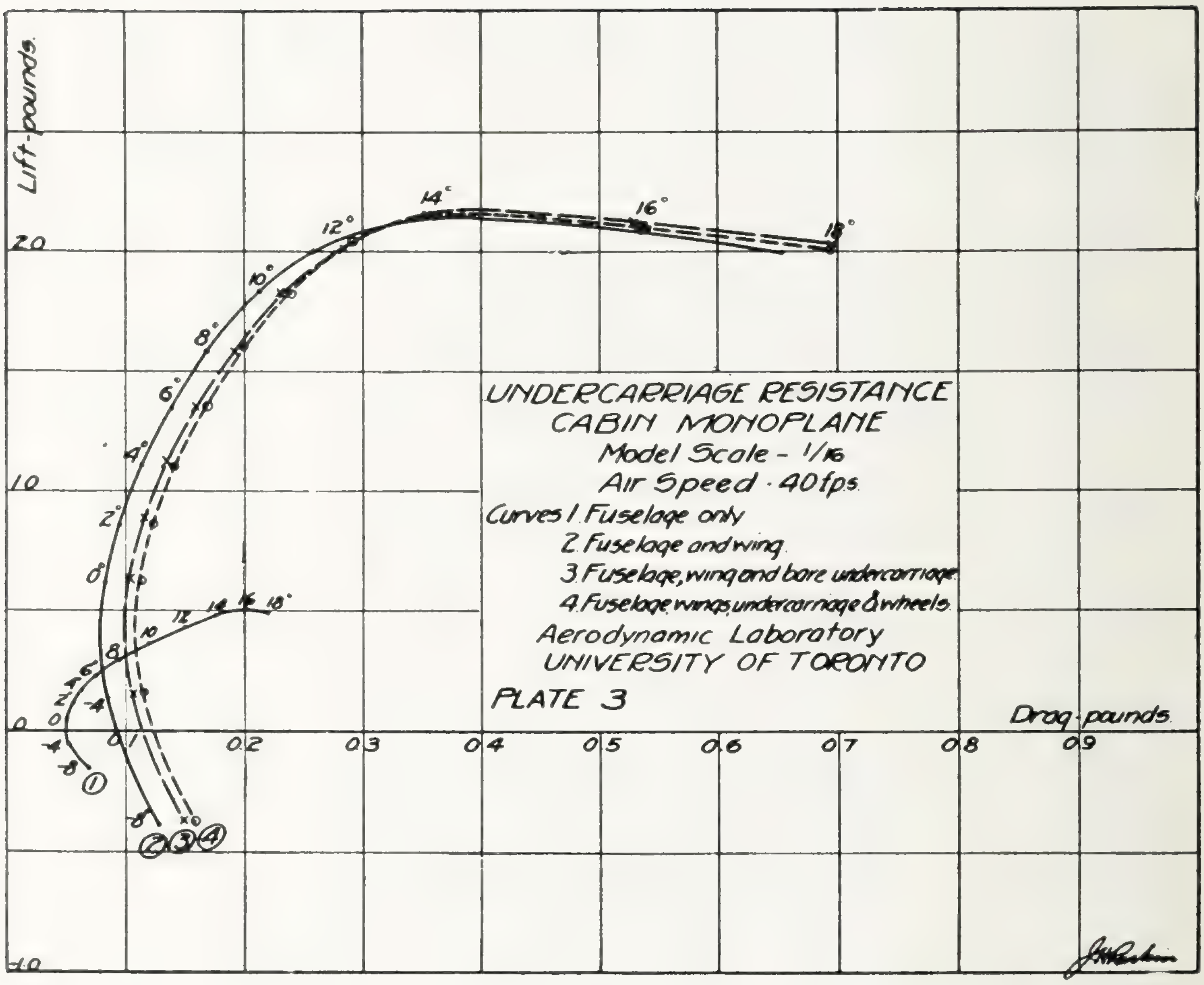

UNDERCARRIAGE RESISTANCE
CABIN MONOPLANE
Model Scale - 1/16
Air Speed · 40 fps.
Curves 1. Fuselage only
2. Fuselage and wing
3. Fuselage, wing and bare undercarriage
4. Fuselage wings, undercarriage & wheels
Aerodynamic Laboratory
UNIVERSITY OF TORONTO
PLATE 3
Lift-pounds
Drag-pounds
2.0
1.0
0
-1.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
14°
16°
18°
12°
10°
8°
6°
4°
2°
0°
-4
-8
10
12
14
16
18°
8
6
2
0
-4
-8
①
②
③
④

PLATE 3

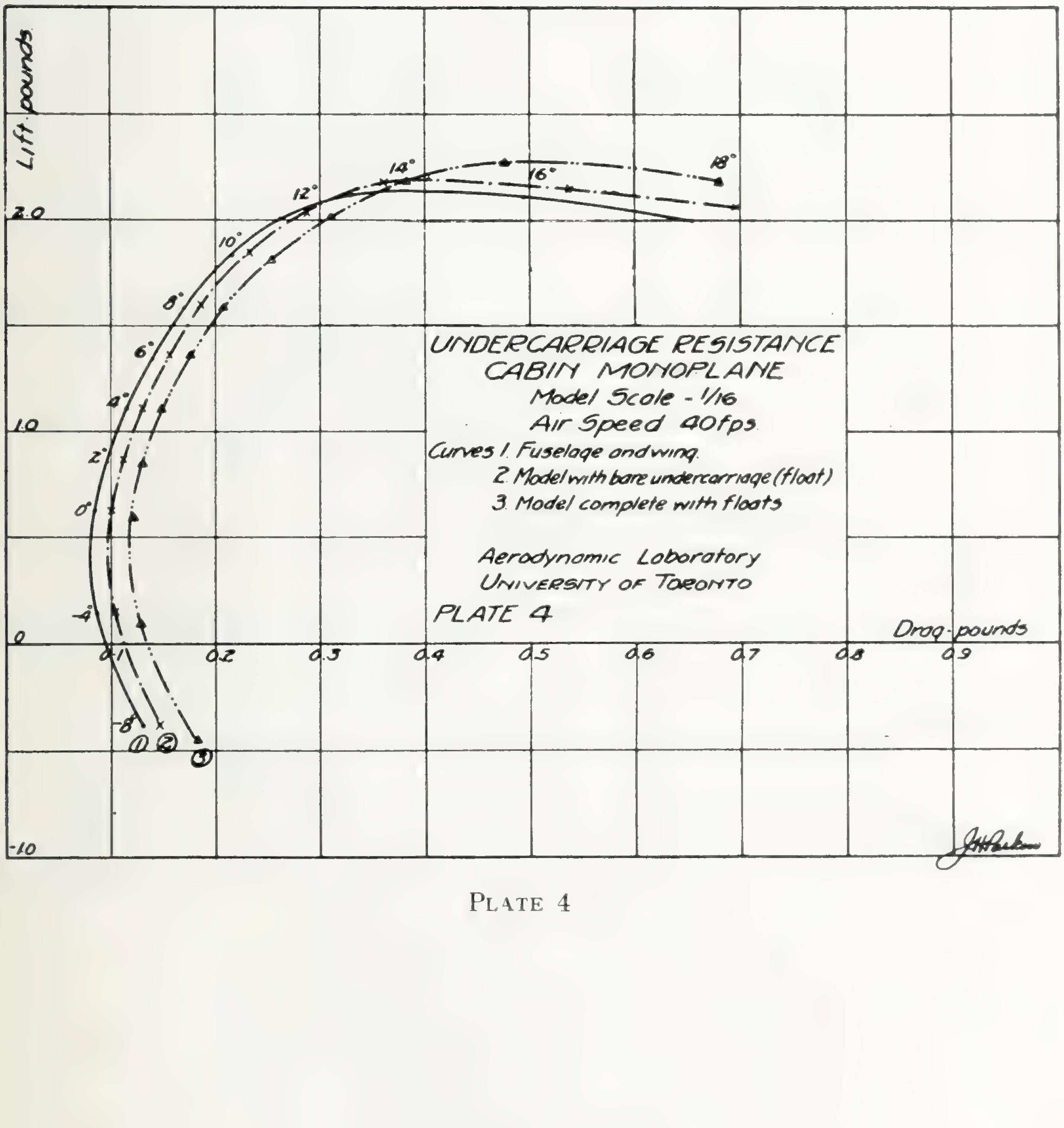
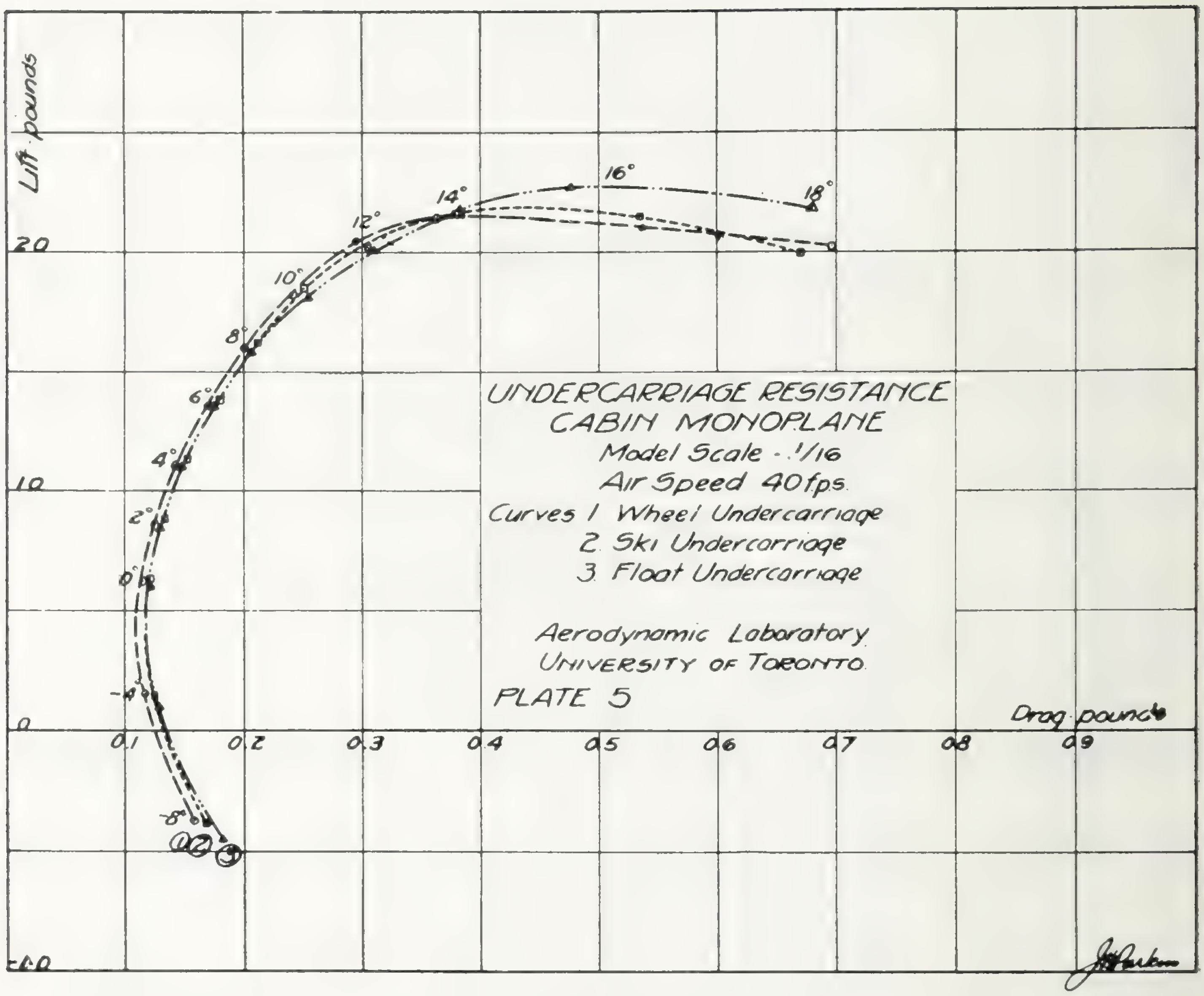

Lift-pounds
2.0
1.0
0
-1.0
Drag-pounds
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
-8°
-4°
0°
2°
4°
6°
8°
10°
12°
14°
16°
18°
①
②
③
UNDERCARRIAGE RESISTANCE
CABIN MONOPLANE
Model Scale - 1/16
Air Speed 40fps
Curves 1. Fuselage and wing.
2. Model with bare undercarriage (float)
3. Model complete with floats
Aerodynamic Laboratory
UNIVERSITY OF TORONTO
PLATE 4

PLATE 4


Lift pounds
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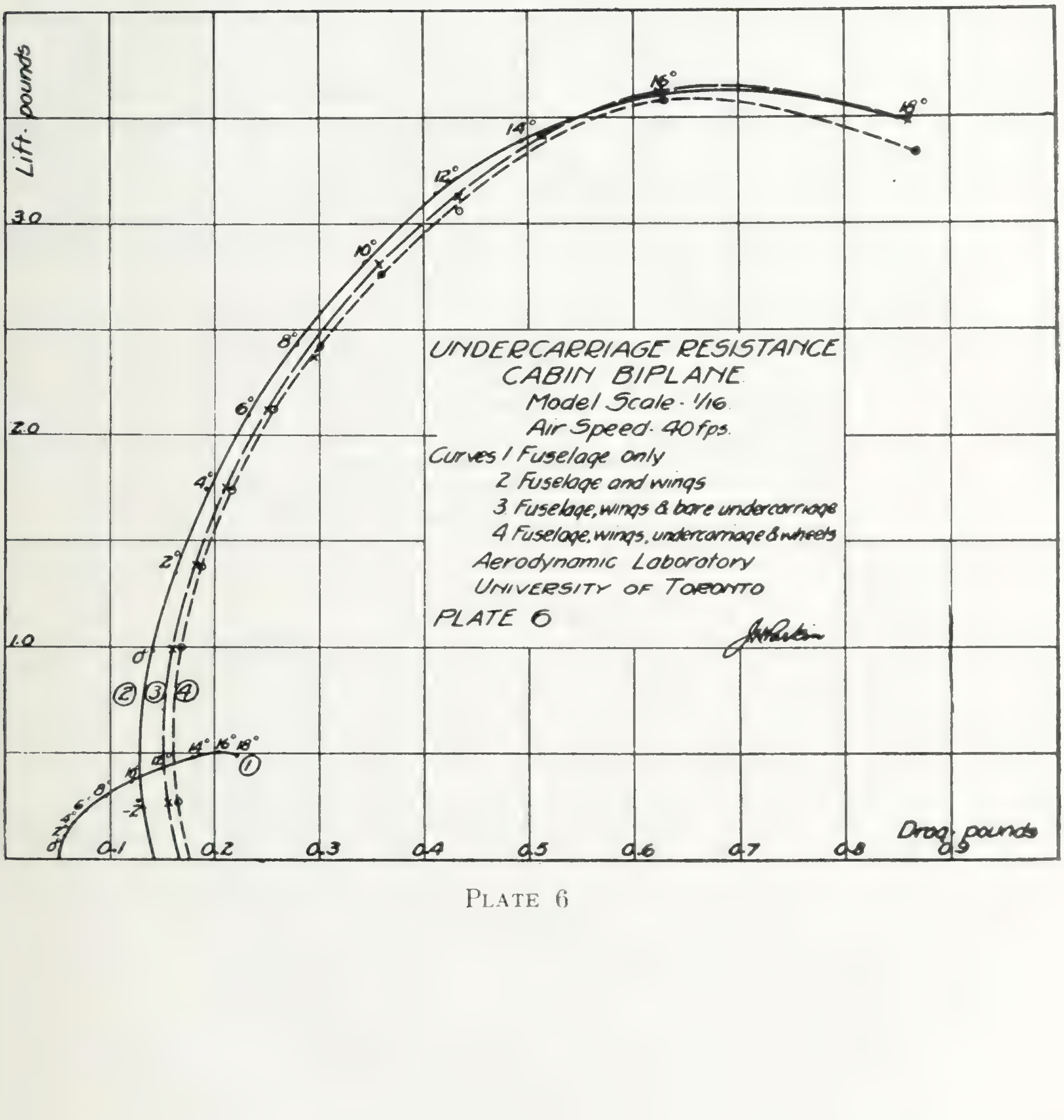
10
0
-10
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Drag pounds
-8°
-4°
0°
2°
4°
6°
8°
10°
12°
14°
16°
18°
1
2
3
UNDERCARRIAGE RESISTANCE
CABIN MONOPLANE
Model Scale - 1/16
Air Speed 40 fps.
Curves 1 Wheel Undercarriage
2. Ski Undercarriage
3. Float Undercarriage
Aerodynamic Laboratory
UNIVERSITY OF TORONTO
PLATE 5

PLATE 5


Lift · pounds
3.0
2.0
1.0
Drag · pounds
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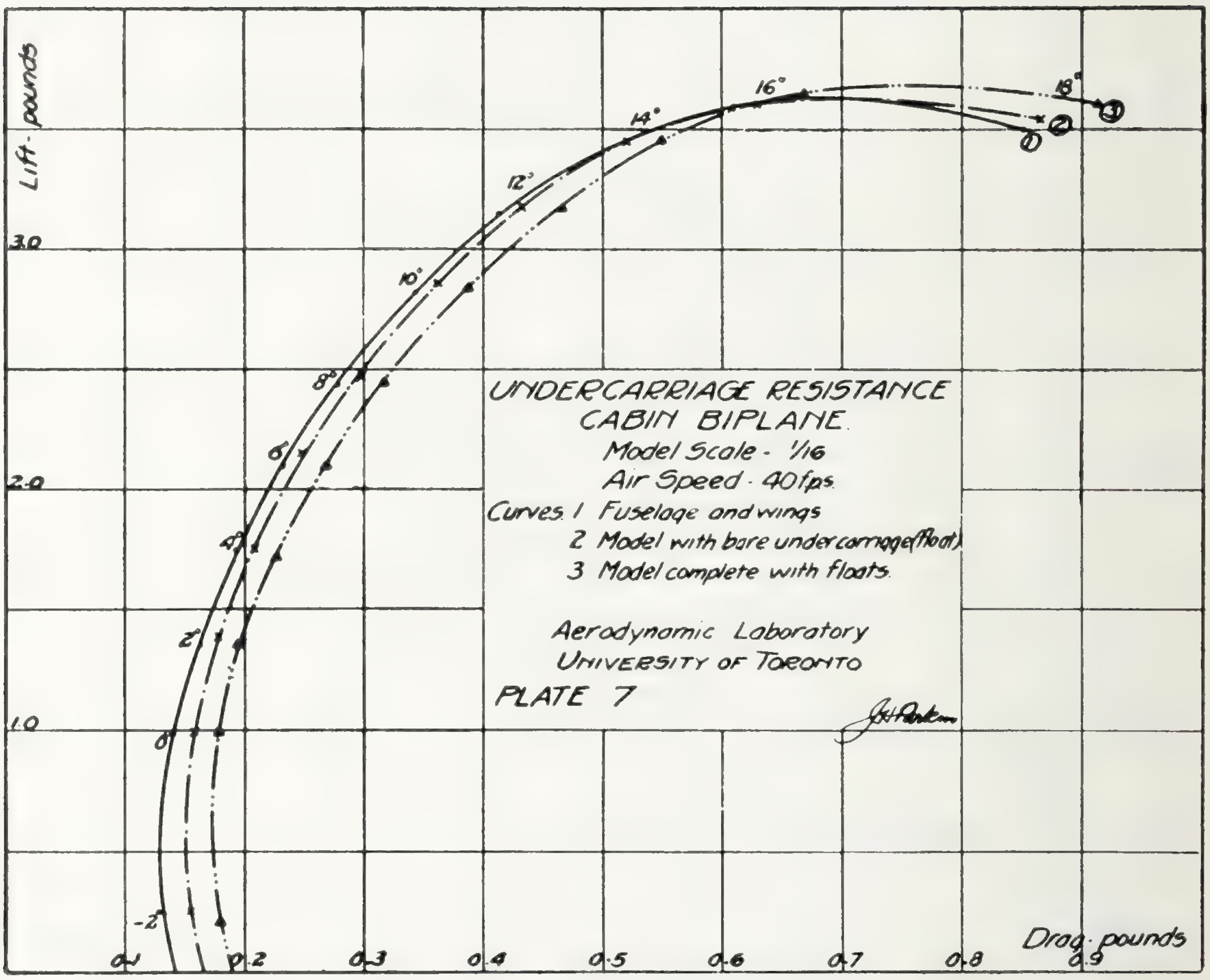
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
UNDERCARRIAGE RESISTANCE
CABIN BIPLANE
Model Scale · 1/16
Air Speed · 40 fps.
Curves 1 Fuselage only
2 Fuselage and wings
3 Fuselage, wings & bare undercarriage
4 Fuselage, wings, undercarriage & wheels
Aerodynamic Laboratory
UNIVERSITY OF TORONTO
PLATE 6
①
②
③
④
-2
0°
2°
4°
6°
8°
10°
12°
14°
16°
18°

PLATE 6


Lift-pounds
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2.0
1.0
Drag-pounds
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0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
-2°
0°
2°
4°
6°
8°
10°
12°
14°
16°
18°
①
②
③
UNDERCARRIAGE RESISTANCE
CABIN BIPLANE.
Model Scale - 1/16
Air Speed - 40fps.
Curves. 1 Fuselage and wings
2 Model with bare undercarriage(float)
3 Model complete with floats.
Aerodynamic Laboratory
UNIVERSITY OF TORONTO
PLATE 7

PLATE 7

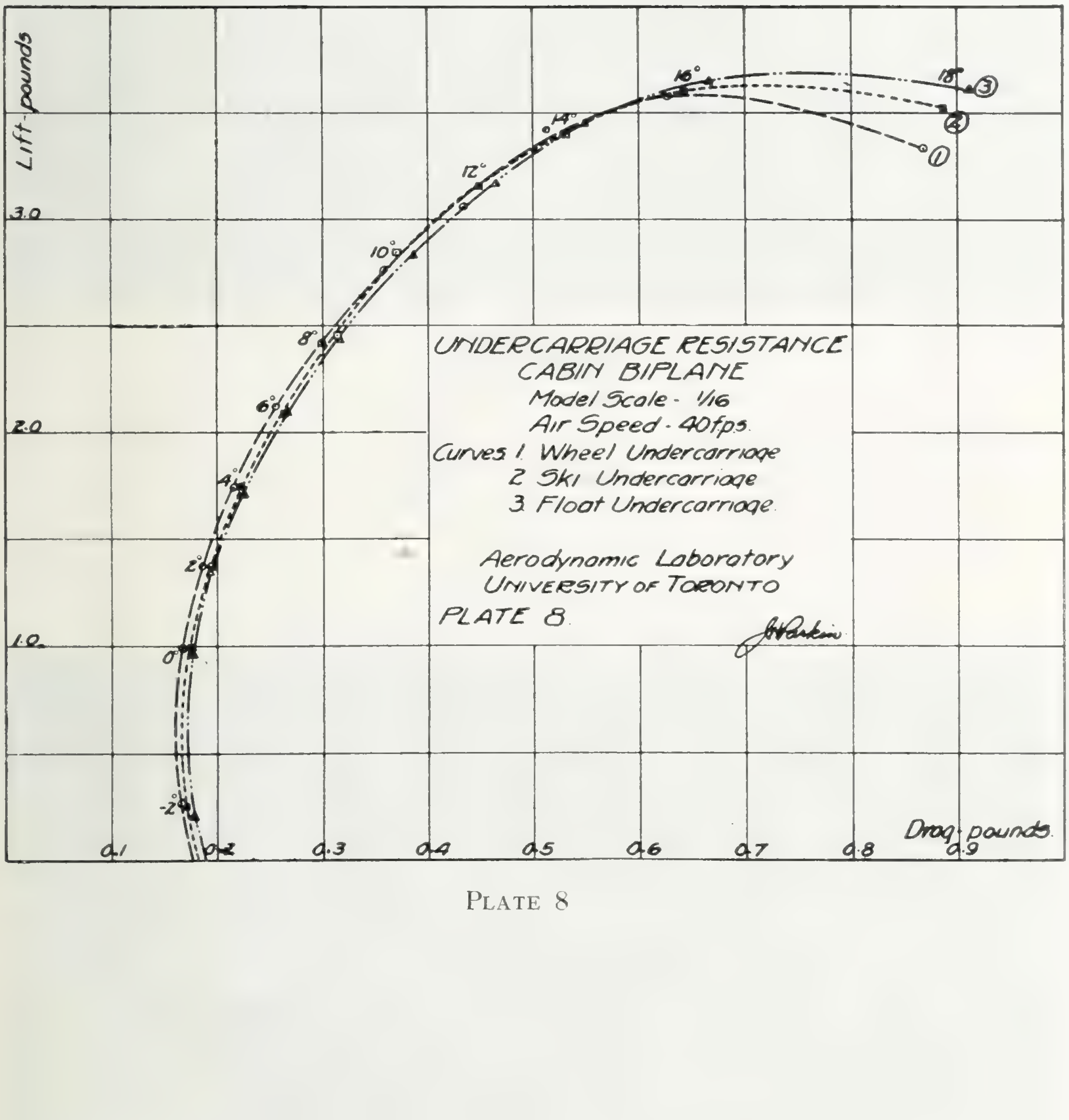
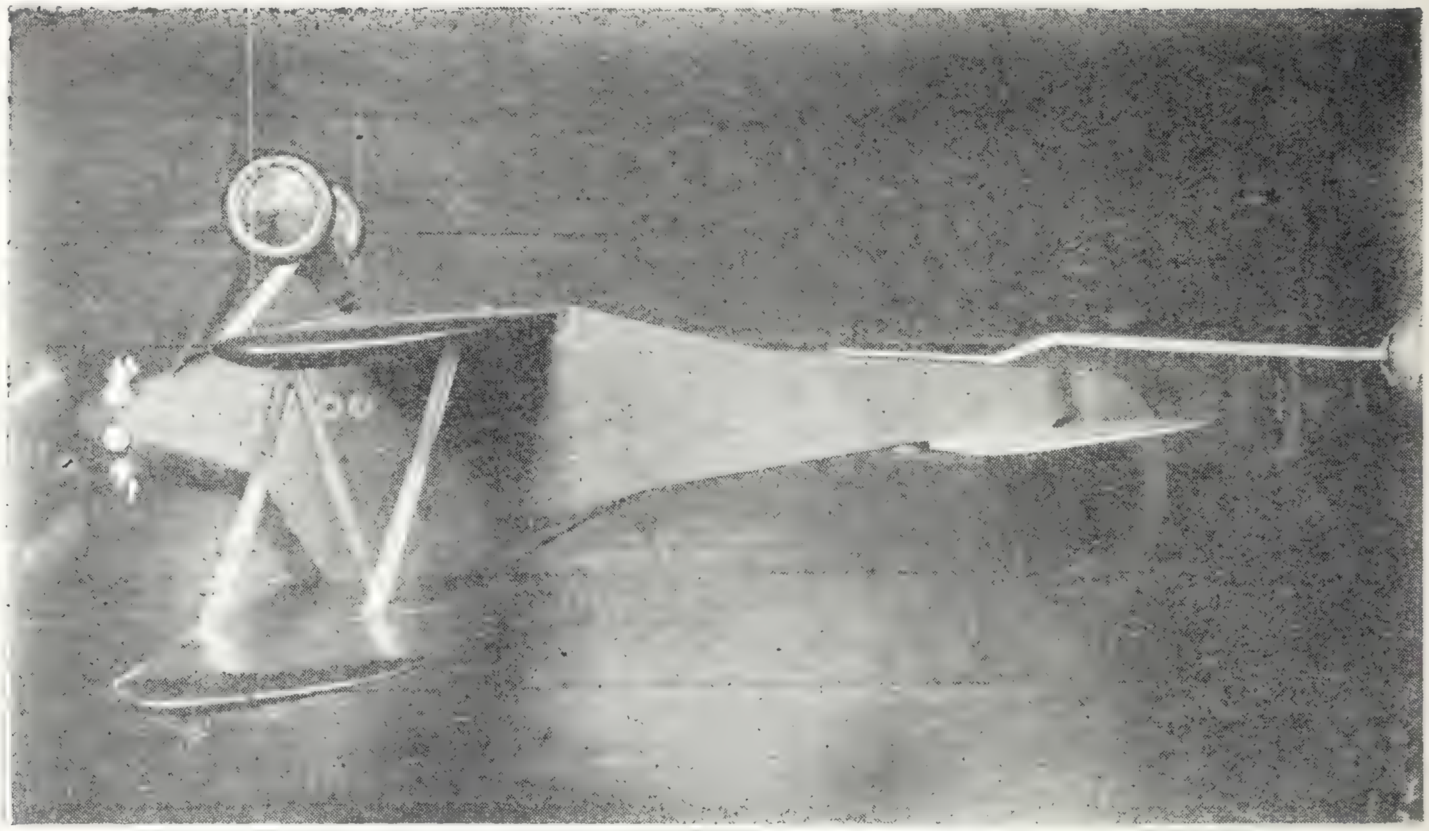
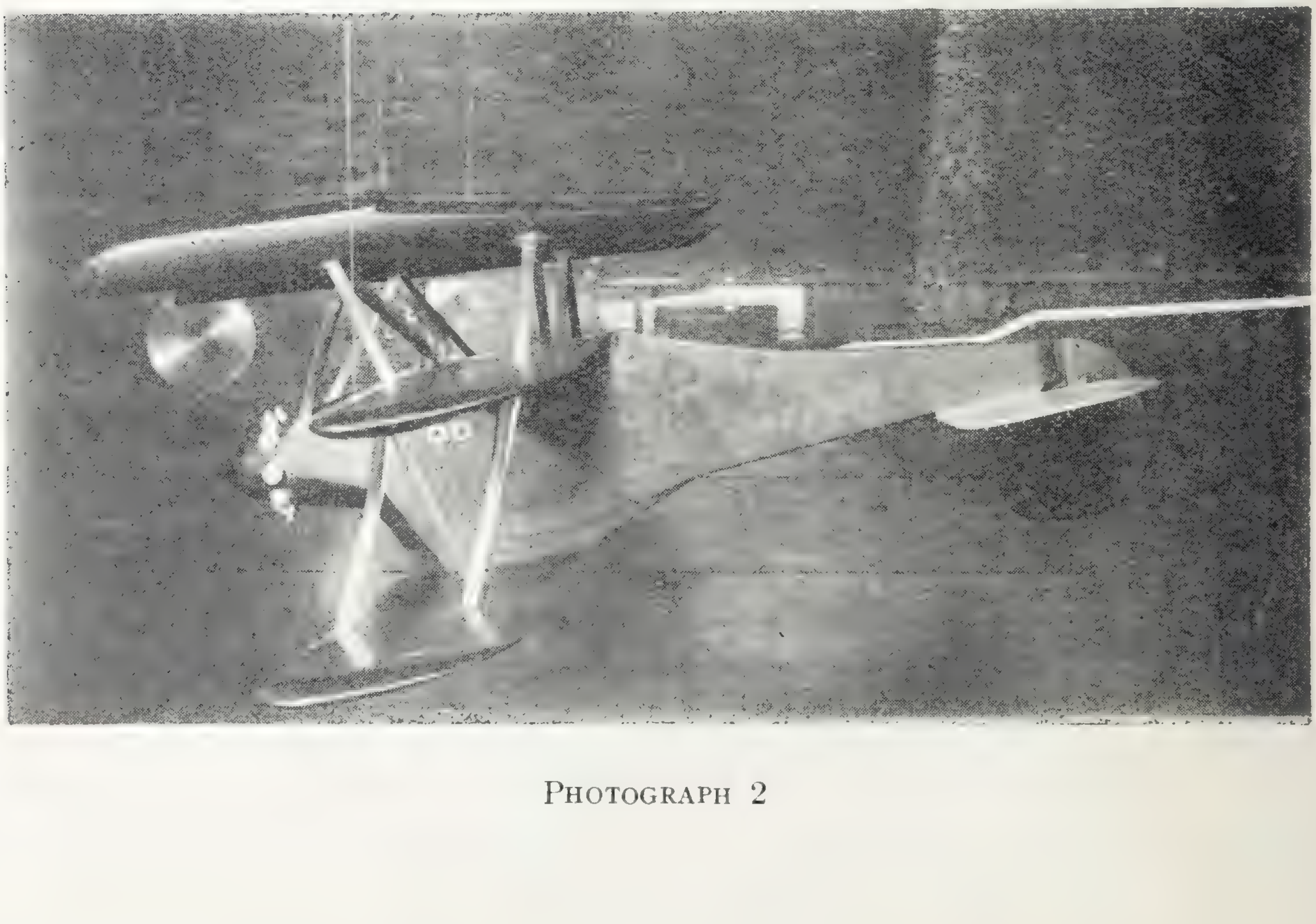

Lift-pounds
3.0
2.0
1.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Drag-pounds
-2°
0°
2°
4°
6°
8°
10°
12°
14°
16°
18°
①
②
③
UNDERCARRIAGE RESISTANCE
CABIN BIPLANE
Model Scale - 1/16
Air Speed - 40 fps.
Curves 1. Wheel Undercarriage
2. Ski Undercarriage
3. Float Undercarriage
Aerodynamic Laboratory
UNIVERSITY OF TORONTO
PLATE 8

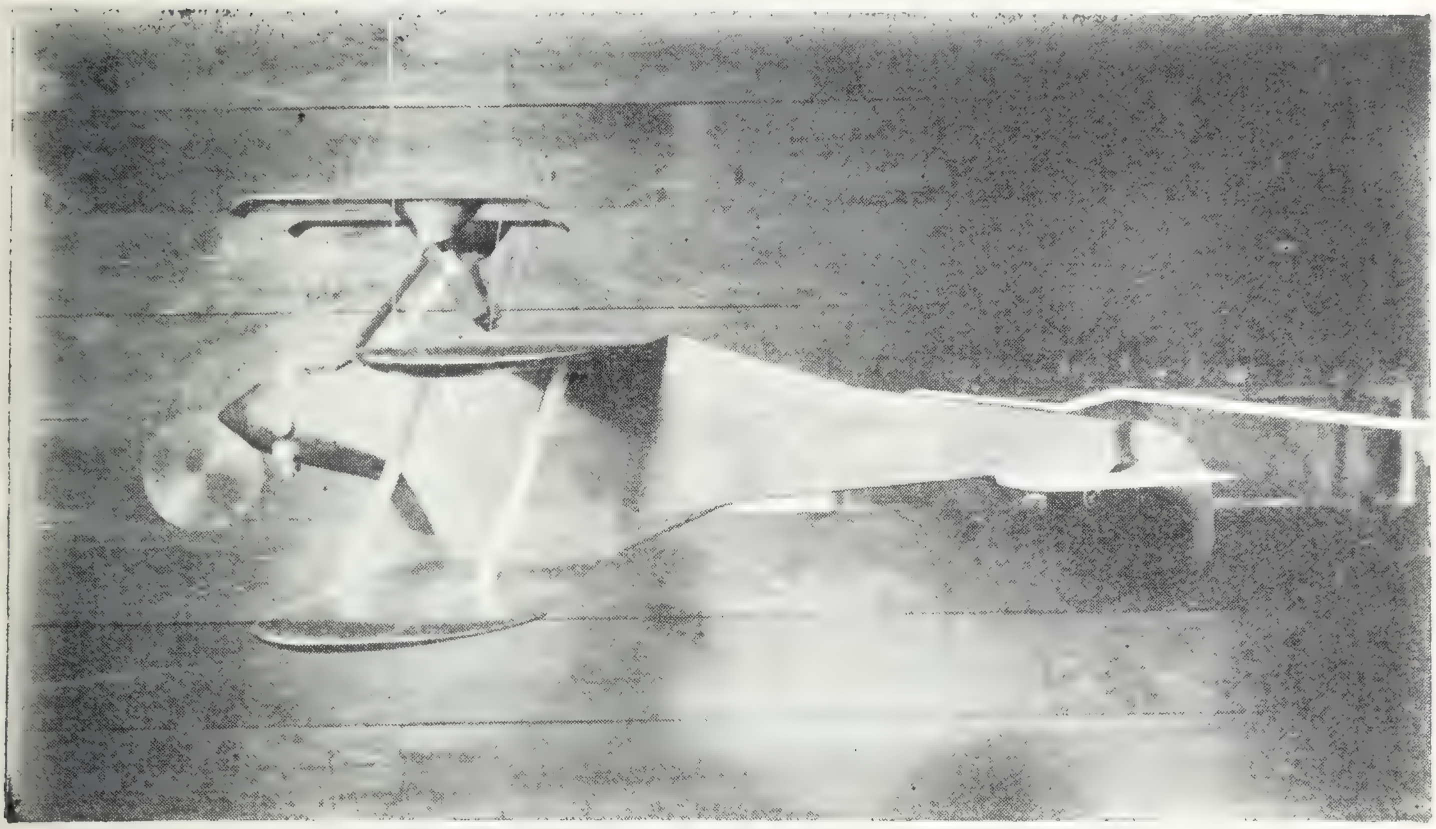
PLATE 8



PHOTOGRAPH 1



PHOTOGRAPH 2



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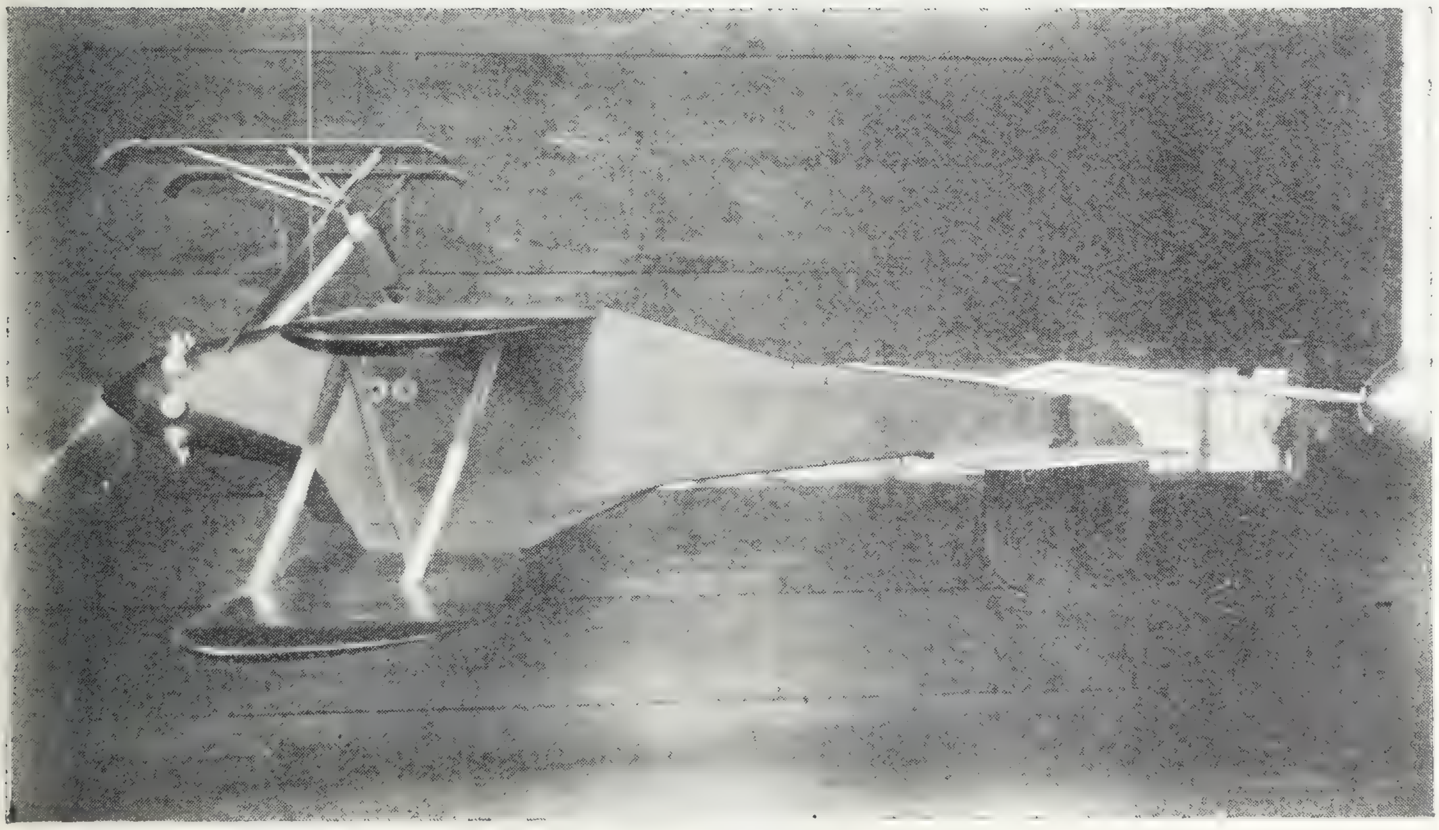

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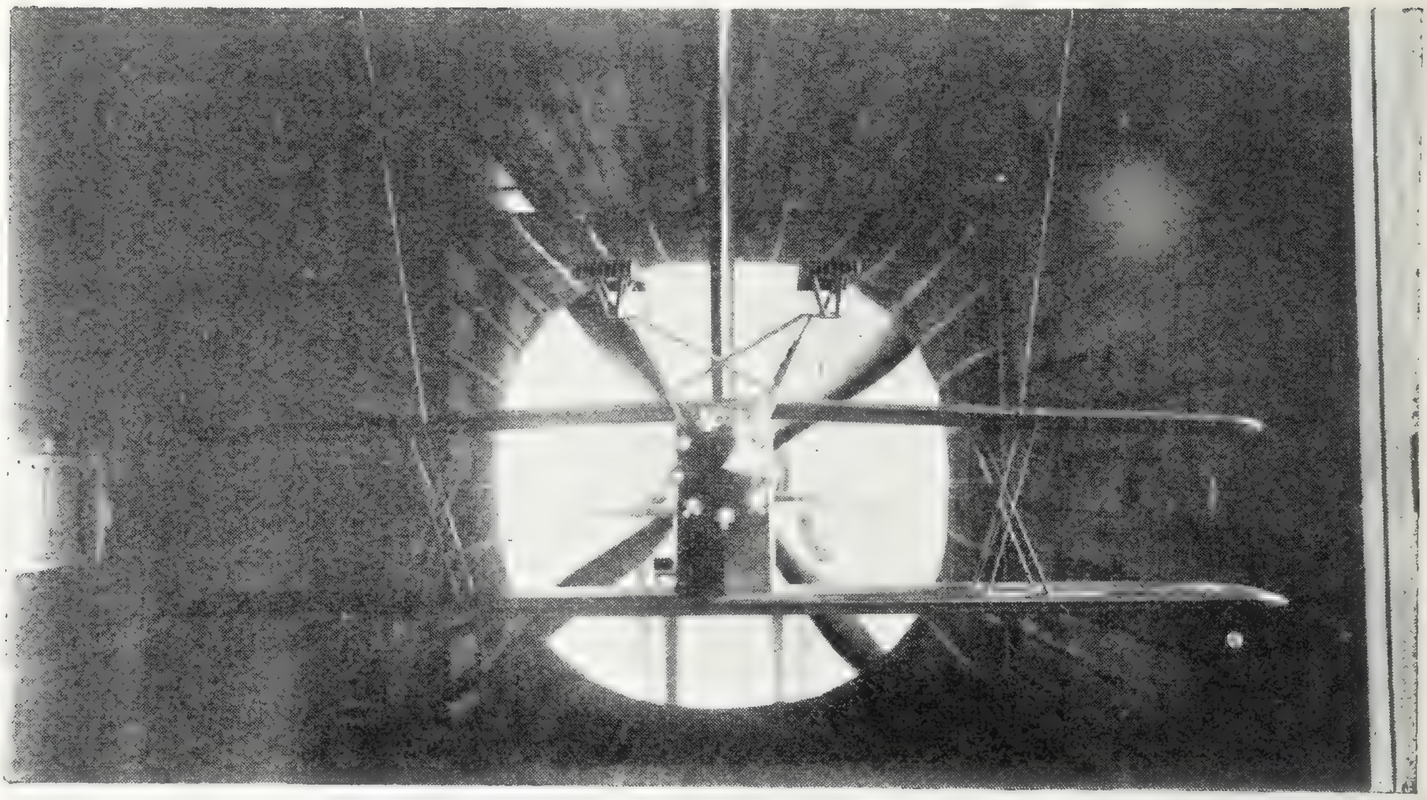

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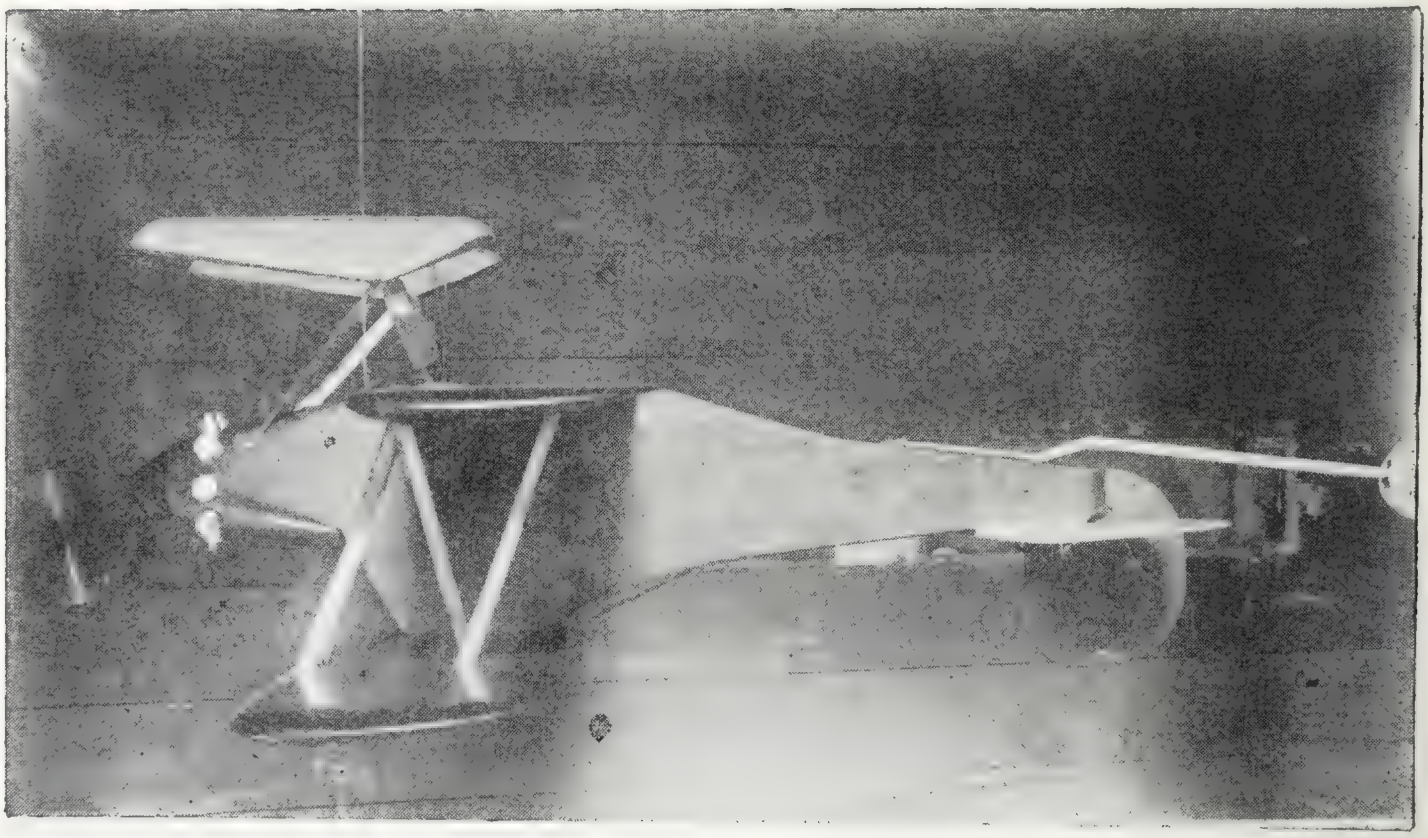

PHOTOGRAPH 6

THE INFLUENCE OF VARIOUS FACTORS ON THE POWER AND ECONOMY OF A GASOLINE ENGINE

By E. A. Allcut
Associate Professor of Mechanical Engineering, University of Toronto

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General

The tests recorded in this paper were performed on a six-cylinder Durant Continental engine (No. 15-L-10045), equipped with Tillotson carburettor, and both hand and automatic spark advance. The bore and stroke were 2-7/8 and 4-3/4 inches, respectively, the firing order 153624, and compression ratio 5.35. The cranks were 120 degrees apart, Nos. 1 and 6, 2 and 5, 3 and 4 being together. The speed range investigated was 850 to 2,800 revolutions per minute, and in each case the throttle was fully open and the ignition advance adjusted to give maximum power.

The engine was mounted on a frame, as shown in Figs. A and B, and was connected through the gear box to a Froude water dynamometer, by means of which the power developed was calculated for each test. The speed was taken with a speed counter, the exhaust temperatures with a calibrated Chromel Copel thermocouple, and the exhaust pressures with a mercury gauge. The fuel consumption was observed by weight at intervals during each test, so that the time taken for the consumption of each pound of gasoline was accurately known, and no test was started until the conditions became constant. The cooling water was weighed and inlet and outlet temperatures were measured in each case.

The tests resolve themselves into four series:

(1) Tests made with same fuel but different mixture strengths.
(2) Tests made with different fuels but same carburettor setting.
(3) Tests made with different cooling water temperatures.
(4) Tests made with patent lubricants mixed with fuels.

Variable Mixture Tests

In these tests the first series (A 1-5) was made with the main jet needle open 2-1/2 turns, so that the analysis of the exhaust gases showed a CO_2 content of approximately 8% (Table 1), and a CO content of

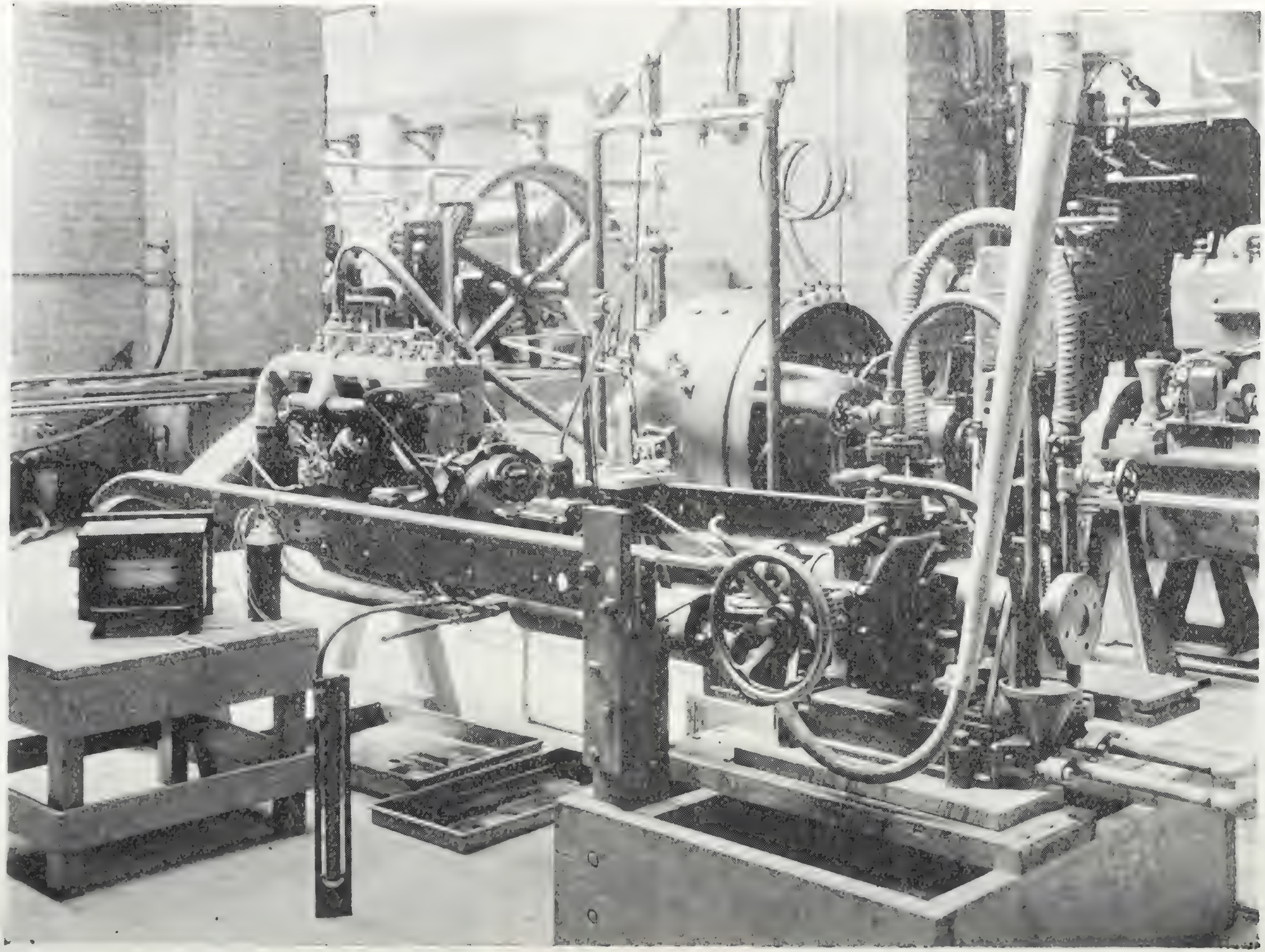

Fig. A.—Arrangement of Engine for Test.

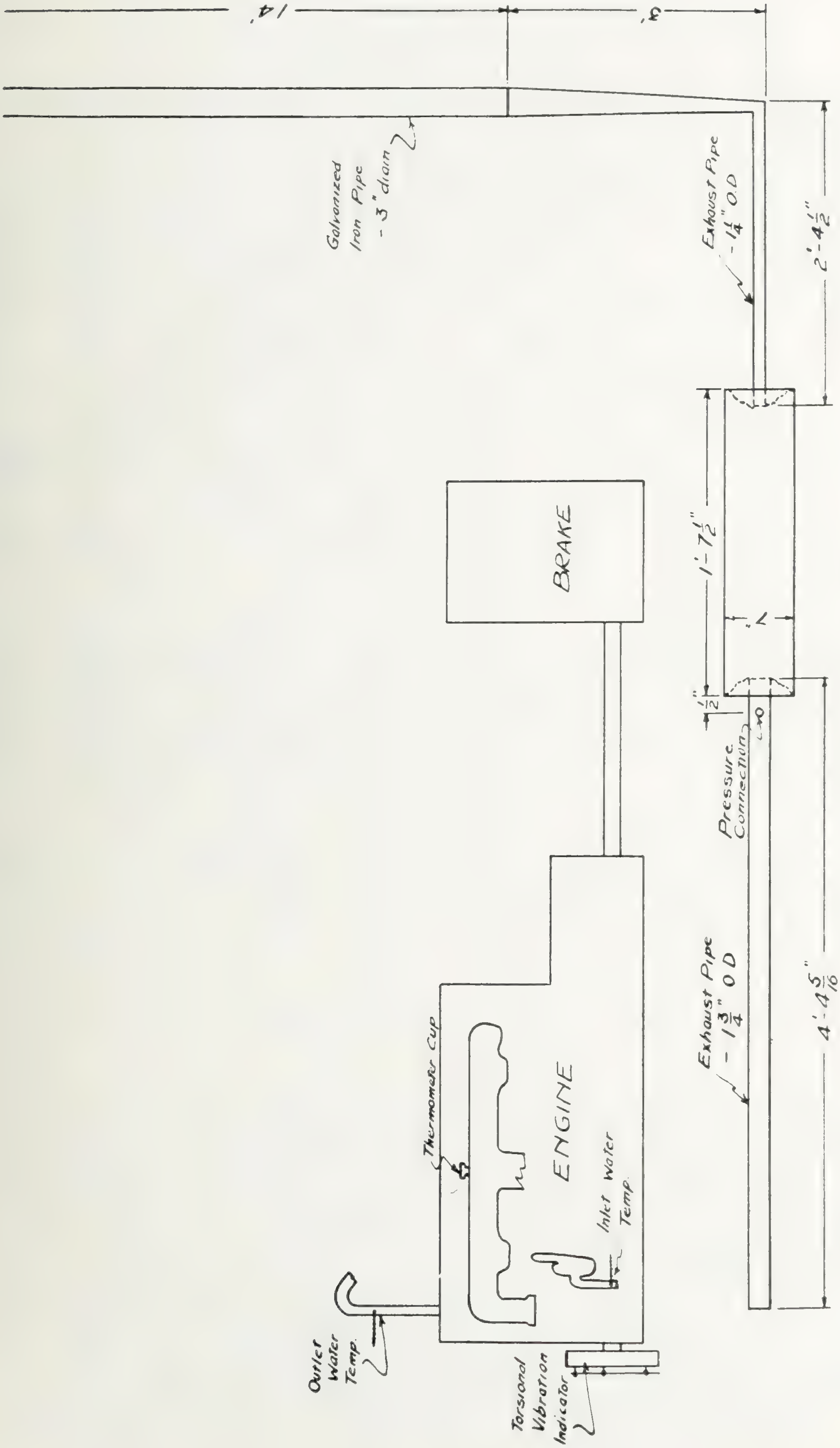

Fig. B.—Diagram of Test Layout.

TABLE No. 1

DURANT—*Continental Motor*

Test No	A 1	A 2	A 3	A 4	A 5
Date	June 11	June 11	June 11	June 11	June 11
Air Temp. F	77	74	72.2	71.5	68
Exhaust Temp. F	792	900	1040	1107	1240
Fuel per hour, lbs	15.05	20.08	27.18	32.88	36.83
Cooling water, per hour, lbs	632	913	1105	1273	1472
" " Inlet temp., F	50	49	49	49	50.7
" " Outlet temp., F	172	160.1	165.4	167	162.9
" " Temp. rise F	122	111.1	116.4	118	112.2
" " B.T.U. per hr	77100	101500	128700	150200	165100
Brake load net lbs	60.5	54.6	54.0	49.5	39.8
Brake M.E.P. lbs. per sq. inch	92.3	83.6	82.4	75.6	60.8
" " torque lbs. ft	113.8	102.7	101.5	93.1	74.8
Speed r.p.m	850	1244	1760	2244	2830
Brake horse-power	18.35	24.3	33.9	39.7	40.2
Gasoline—S.G. ("A")	.738				
Cal. value	19000 B.T.U./lb.				
Fuel per B.H.P. hour, lbs	.82	.827	.801	.829	.917
Thermal effy per cent	16.35	16.20	16.75	16.18	14.62
Cooling loss B.T.U. per B.H.P. hour	4200	4175	3795	3785	4110
Cooling loss per cent	26.95	26.55	24.90	24.05	23.60
Exhaust and radiation per cent	56.70	57.25	58.35	59.77	61.78
Ignition advance	14°	17°	20°	23°	28°
Throttle	Full open				
Oil temp. in crank case, °F	....	....	....	....	....
Barometer—Ins	29.83	29.83	29.83	29.83	29.85
Exhaust pressure—Ins. mercury	1.2	2.1	3.7	5.5	6.9
Humidity, per cent	52	62	58	57	63
Exhaust analysis: CO_2	7.83	8.2	8.63	7.9	8.0
" " O	.10	.4	.20	.4	.2
" " CO	8.63	7.7	7.63	8.8	8.6
" " N	83.44	83.7	83.54	82.9	83.2

approximately 8%. This corresponds to a fuel supply about 15% in excess of that required for perfect combustion. The second series A 8-12) gave a CO_2 content of about 10.5%, and CO approximately 3% (Table 2). The third series (A 13-17) was made with what was practically a correct mixture, the exhaust analysis being about 13% CO_2 and 0.4% free oxygen. (Table 3.)

The curves showing the results obtained at different speeds are given in Figs. 1-3. In some cases (as in the power curve) the various lines are so close together that only one curve can be drawn.

The results, so far as power and economy are concerned, are given in Fig. 4. For the purpose of extending the mixture range, a set of points

TABLE No. 2

DURANT—*Continental Motor*

Test No	A 12	A 11	A 8	A 9	A 10
Date	June 13	June 13	June 13	June 13	June 13
Air Temp. F	77.5	75.7	71	73.5	74.5
Exhaust Temp. F	830	947	1050	1125	1170
Fuel per hour, lbs	12.73	16.68	23.43	28.35	31.38
Cooling water per hour lbs	758	977	1138	1330	1497
" " Inlet temp., F	50	49	50.5	49.5	49
" " Outlet temp., F	161	160.1	169	169.5	167
" " Temp. rise F	111	111.1	118.5	120	118
" " B.T.U. per hr	84200	108400	134800	159600	176700
Brake load, lbs	60.5	56.3	54.4	49.6	40.3
Brake M.E.P. lbs. per sq. inch	92.5	86.1	83.1	75.6	61.6
" " torque, lbs. ft	113.8	105.9	102.2	93.2	75.8
Speed r.p.m	860	1240	1760	2250	2819
Brake horse-power	18.6	24.95	34.2	39.8	40.6
Gasoline—S.G. ("A")	———	———	.738———	———	
Cal. value		———	19000 B.T.U. lb.———		
Fuel per B.H.P. hour, lbs	.684	.668	.685	.712	.772
Thermal effy per cent	19.55	20.05	19.45	18.8	17.35
Cooling loss B.T.U. per B.H.P. hour	4530	4350	3940	4010	4350
Cooling loss per cent	34.85	34.25	30.05	29.6	29.60
Exhaust and radiation per cent	45.60	45.70	50.50	51.6	53.05
Ignition advance	18°	21°	25°	28°	32°
Throttle	———	———	Full open	———	———
Oil temp. in crank case, °F	....	....	....	....	....
Barometer—Ins. mercury	29.61	29.61	29.62	29.62	29.62
Exhaust pressure—Ins. mercury	1.2	1.9	3.4	5.2	6.4
Humidity, per cent	70	65	76	75	70
Exhaust analysis—CO_2	10.2	11.2	10.5	10.5	10.2
" " O	.5	0.5	0.4	0.3	0.3
" " CO	3.0	2.4	3.2	3.2	3.9
" " N	86.3	85.9	85.9	86.0	85.6

obtained with 15% excess air on a similar engine (No. 15-L 5057) is included, and it is evident that for all speeds, the most economical figures are obtained with about 2% of free oxygen in the exhaust gas. This corresponds to about 9% excess air in the mixture.

Many tests have been made, and several papers have been published dealing with the effect of mixture strength on power and economy. The experiments of Ricardo[1] indicate that with gasoline the minimum fuel consumption is obtained with 10-15% of excess air in the mixture, and maximum power with about 20% excess fuel.

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[1] "The Internal Combustion Engine," Ricardo, Vol. II, p. 46.

TABLE No. 3

DURANT—*Continental Motor*

Test No	A 17	A 16	A 13	A 15	A 14	A 18
Date	June 15	June 15	June 15	June 15	June 15	July 6
Air Temp. F	75	74.5	69	73	71	81
Exhaust Temp. F	856	985	1150	1260	1302	1190
Fuel per hour, lbs	11.29	14.64	20.43	23.28	26.08	17.75
Cooling water per hour, lbs	757	955	1303	1330	1430	1329
" " Inlet temp., F	49	50	50	50	50	56
" " Outlet temp., F	166	167.5	163	179	184	160
" " Temp. rise, F	117	117.5	113	129	134	104
" " B.T.U. per hr	88500	112200	147300	171700	191800	138100
Brake load, lbs.	61.0	57.5	55.2	49.5	40.5	50.8
Brake M.E.P., lbs. per sq. in	93.2	87.8	84.3	75.6	61.9	77.6
" " torque, lbs. ft	114.8	108.1	103.9	93.1	76.1	95.5
Speed r.p.m	855	1225	1760	2247	2807	1734
Brake horse-power	18.62	25.15	34.7	39.7	40.6	31.45
Gasoline, S.G. ("A")		————	.738————			
Cal. value		————	19000 B.T.U./lb.————			
Fuel per B.H.P. hour, lbs	.606	.582	.589	.586	.642	.564
Thermal effy, per cent	22.1	23.0	22.75	22.9	20.9	23.75
Cooling loss B.T.U. per B.H.P. hour	4750	4460	4250	4320	4720	4390
Cooling loss per cent	41.2	40.3	37.9	38.8	38.7	40.9
Exhaust and radiation per cent	36.7	36.7	39.35	38.3	40.4	35.35
Ignition advance	16°	20°	22°	24°	29°	17°
Throttle		————	————Full open————	————		
Oil temp. in crank case, °F	..	....	....	....	....	210
Barometer—Ins. mercury	29.97	29.97	29.96	29.96	29.96	....
Exhaust pressure—Ins. mercury	1.1	1.7	3.3	4.9	6.2	3.3
Humidity, per cent	62	60	66	65	65	58
Exhaust analysis: CO_2	12.2	13.1	13.3	13.6	13.5	13.75
" " O	0.6	0.4	0.4	0.3	0.3	0.65
" " CO	0	0	0	0	0	0.00
" " N	87.2	86.5	86.3	86.1	86.2	85.60

Berry[2] shows similar curves with maximum economy at about 5% excess air, and maximum power at about 16% excess fuel. A paper read by H. M. Jacklin[3] includes a series of curves with different spark plug combinations, and the following figures are taken from the curves with one spark plug:—

Spark Advance	Max. B.H.P.	Mixture Ratio Air/Fuel	Max. I.H.P.	Mixture Ratio Air/Fuel
10°	1.14	14.5	1.92	14.0
20°	1.26	13.5	2.02	13.0
30°	1.21	10.5	1.97	12.5

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[2] "More Car Miles per Gallon of Fuel," S.A.E. Jrl., Aug., 1922.

[3] "Improving Engine Performance," S.A.E. Jrl., March, 1928.

Taking the correct mixture ratio for complete combustion as 15:1, it is evident that all of these maxima are obtained with rich mixtures. This paper also indicates a decrease of mechanical efficiency as the mixture increases in richness.

A summary of the situation, given by S. W. Sparrow[4], points to the same conclusions, that the maximum economy is obtained with the leanest mixture upon which the engine will run regularly, and the maximum power with a mixture 10 to 20% richer than that required for complete combustion. The reason for the former is not far to seek. Excess air is required for maximum efficiency, because the air and fuel are never completely mixed in the cylinder, and therefore there must be sufficient oxygen present at all points to burn the fuel available, otherwise, some incomplete combustion will result. With this conclusion the present series of results is in agreement.

The case of the power curve, however, is different. In the tests cited above, the power curve rises rapidly to a maximum on the rich side, and then slowly decreases as the quantity of fuel is increased. The present tests (Fig. 4) indicate no appreciable difference in power over the range of mixtures investigated. It is reasonably certain that weaker mixtures than those given in Fig. 4 would show progressive decreases in power, but at each of the speeds indicated the power curves were practically flat, thus differing from those obtained in the experiments described above. In several of these cases, the engine experimented on was a single cylinder variable compression engine, and the question arises whether the results obtained in such engines are generally applicable to multi-cylinder engines. This series of tests, made on a six-cylinder engine manufactured under commercial conditions, indicates that there is some doubt on that point.

The fuel consumption curves are fairly flat, reaching a minimum in each case between 1200 and 2000 r.p.m.

TABLE No. 4.

Engine speed, r.p.m.	850	1250	1750	2250	2800
Car speed—miles per hour	16.8	24.8	34.8	44.7	55.6

In all cases the curves representing the percentage of CO_2 in the exhaust gases were very flat. It is notable that the tests made at 850 r.p.m. in all cases gave a lower percentage of CO_2 and more CO and O_2 than those made at higher speeds.

Warner[5] states that with 7% of CO_2 in the exhaust gases, combustion is about 57.5% complete, and with 14% CO_2 combustion is 97% com-

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[4] "Elements of Fuel Economy," S.A.E. Jrl., June, 1921.

[5] 'Instruments for Automotive Research," S.A.E. Jrl., June, 1925.

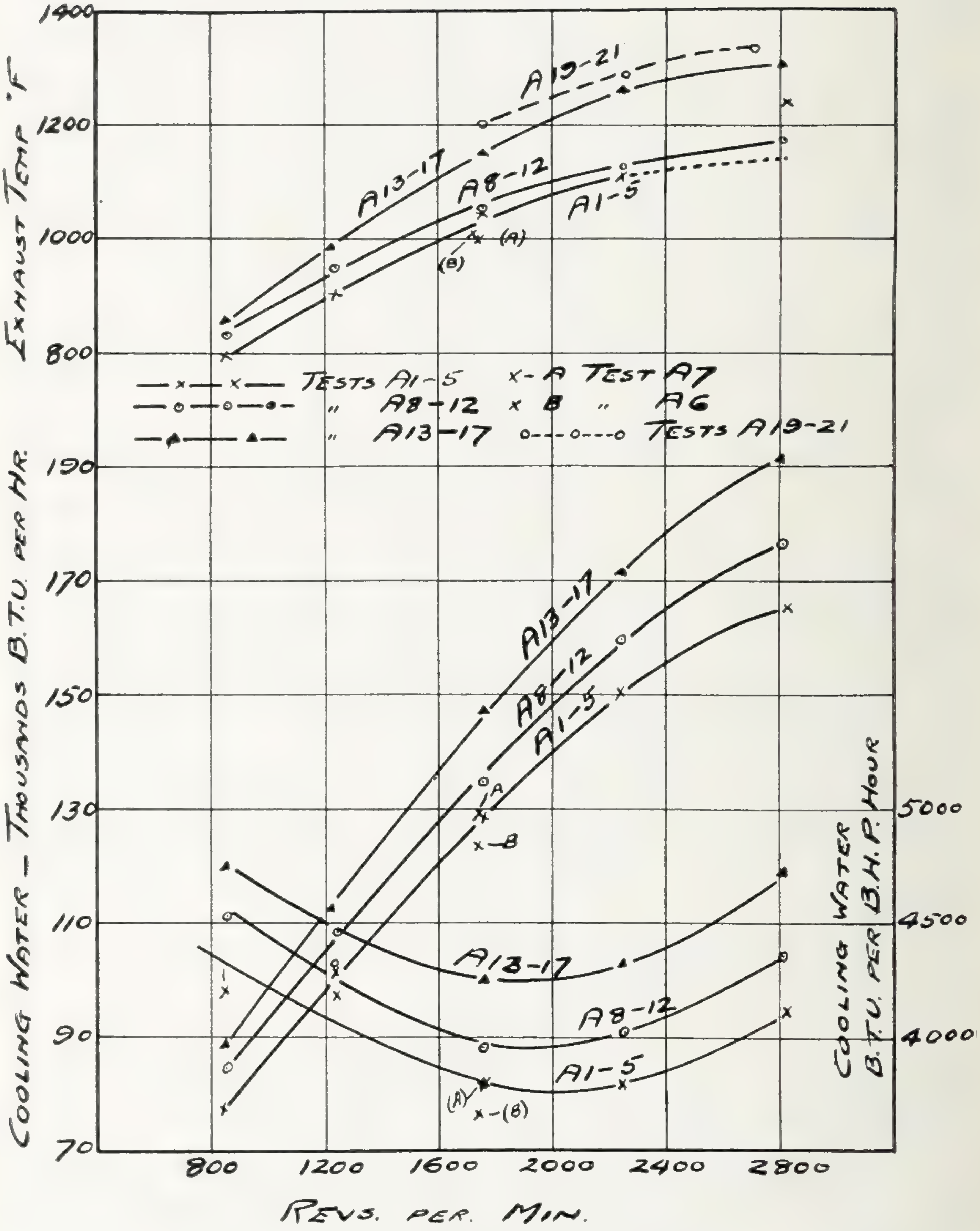

Fig. 1.—Different Mixture Strengths.

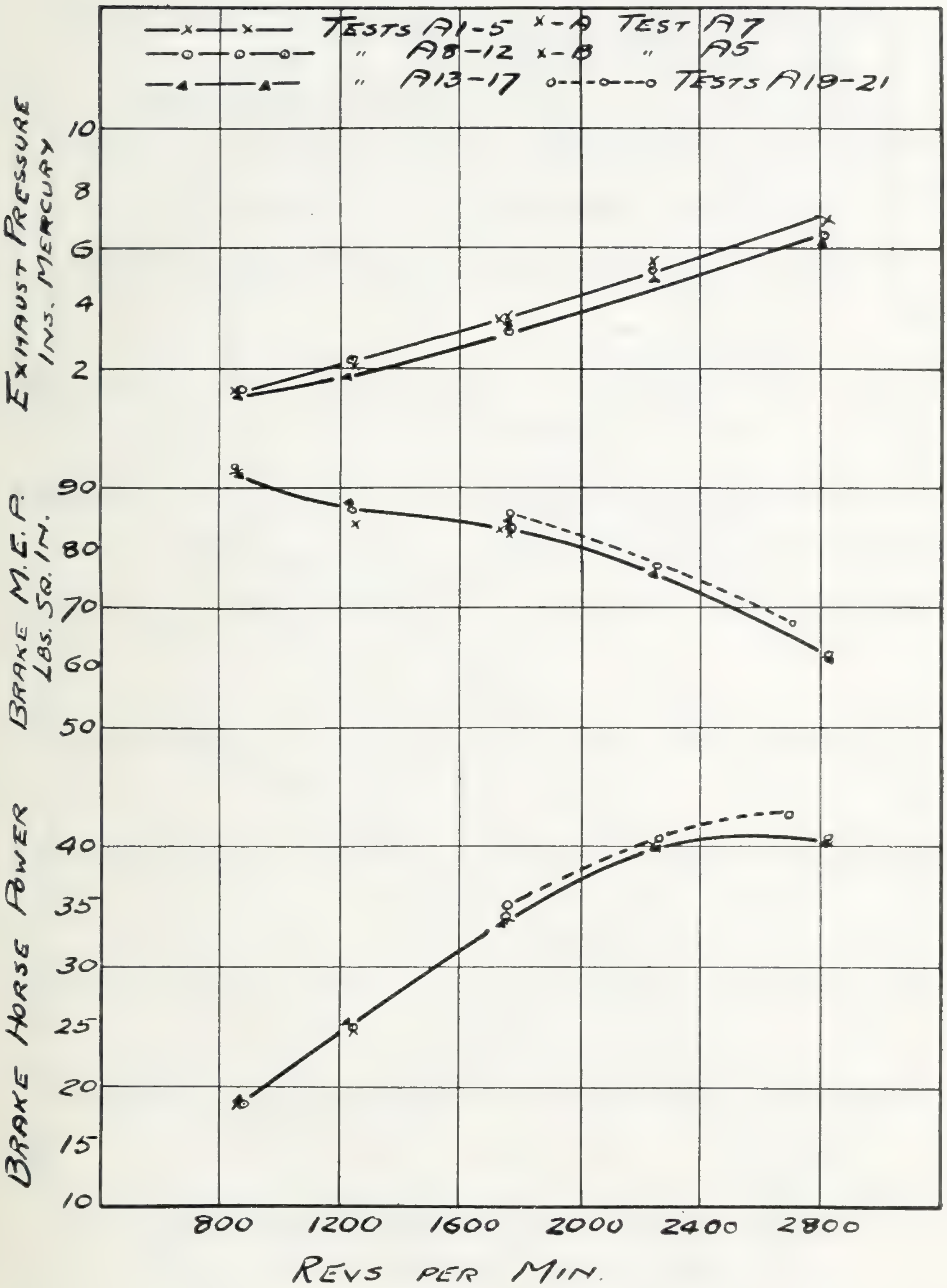

Fig. 2.—Different Mixture Strengths.

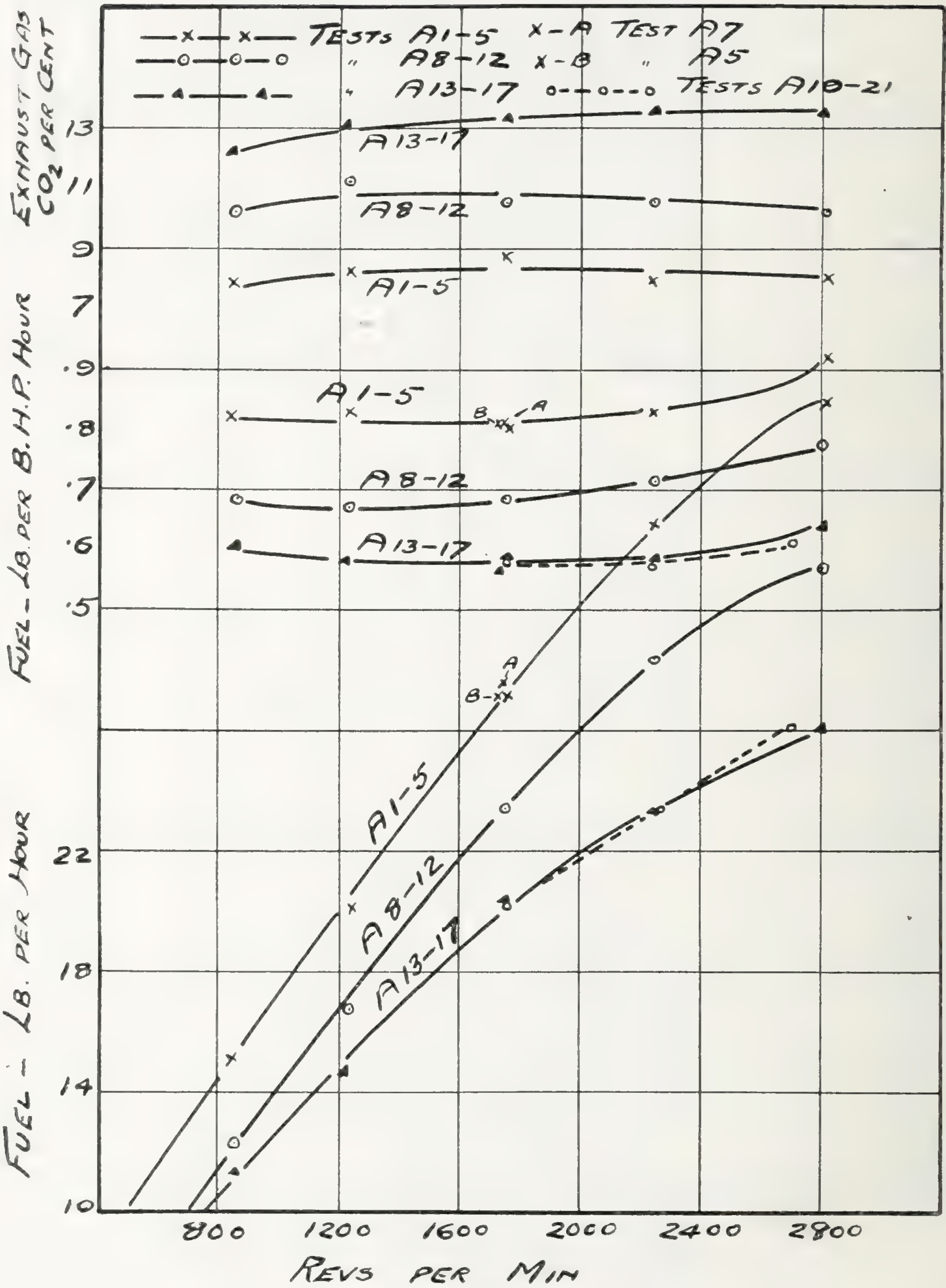

Fig. 3.—Different Mixture Strengths.

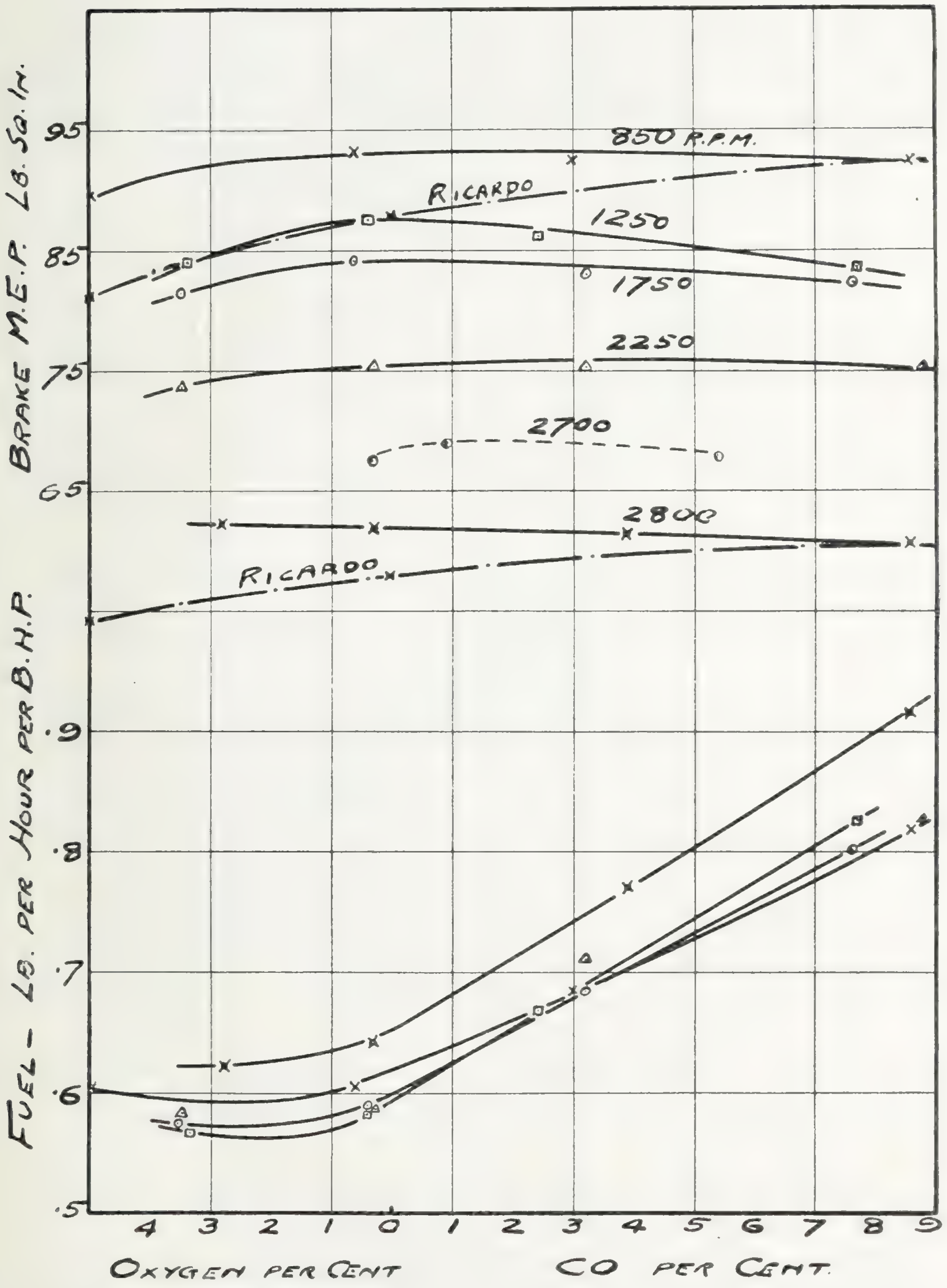

Fig. 4.—Different Mixture Strengths.

plete. Between the two (according to Fieldner & Jones) there is a straight line relationship. As in the present series, 13-14% CO_2 was obtained in many tests, it is evident that the combustion efficiency was high.

The cooling water losses decreased as the speed increased, and increased as the mixture strength was decreased. That is to say, the weak mixtures which developed the highest exhaust temperatures also rejected to the cooling water approximately 40% of the heat generated, as against 25% in the case of the rich mixtures.

In all cases, the thermal efficiencies were calculated on the higher calorific values of the fuels as obtained in the Parr calorimeter. In these tests, gasoline "A" was used throughout, and brake thermal efficiencies up to 24% were obtained. In a previous series made with engine No. 15-L 5057, efficiencies of over 24% were obtained on several occasions.

A further series of tests was made on this engine to determine the effect of the muffler and air cleaner on the power and economy obtained. The air cleaner was removed and the muffler and tail pipe were replaced by 21 feet of 1-1/2 to 1-3/4-inch exhaust piping. The carburettor setting was approximately that used in tests A 13-17, and the mixture was practically the same as that indicated by the exhaust analyses for tests A 19-21 (Table 13). At 1760 r.p.m. the power was increased from 34.7 to 35.0 B.H.P. At 2260 r.p.m. the increase was from 39.7 to 40.6, and at maximum speed the power was raised from 40.6 to 42.6 B.H.P. The economy was unaltered, but the exhaust temperatures were higher than those in the previous series. The results obtained are indicated by dotted lines in Figs. 1-4.

Advantage was taken of this opportunity to check the results obtained with different mixture strengths, and therefore tests A 22 and 23 were performed under exactly the same conditions as test A 21, but with the main jet of the carburettor opened 1/4 turn between each pair of tests.

Again, it is evident that the power is practically unaltered as the mixture is made richer than the chemically correct mixture.

DIFFERENT FUELS

During the past ten years considerable attention has been paid to the nature of the fuel used in gasoline engines, and the various characteristics that conduce to satisfactory performance on the road.

The desirable features are:

(*a*) Easy starting;

(*b*) Good acceleration;

(*c*) High anti-detonation, or anti-knocking value.

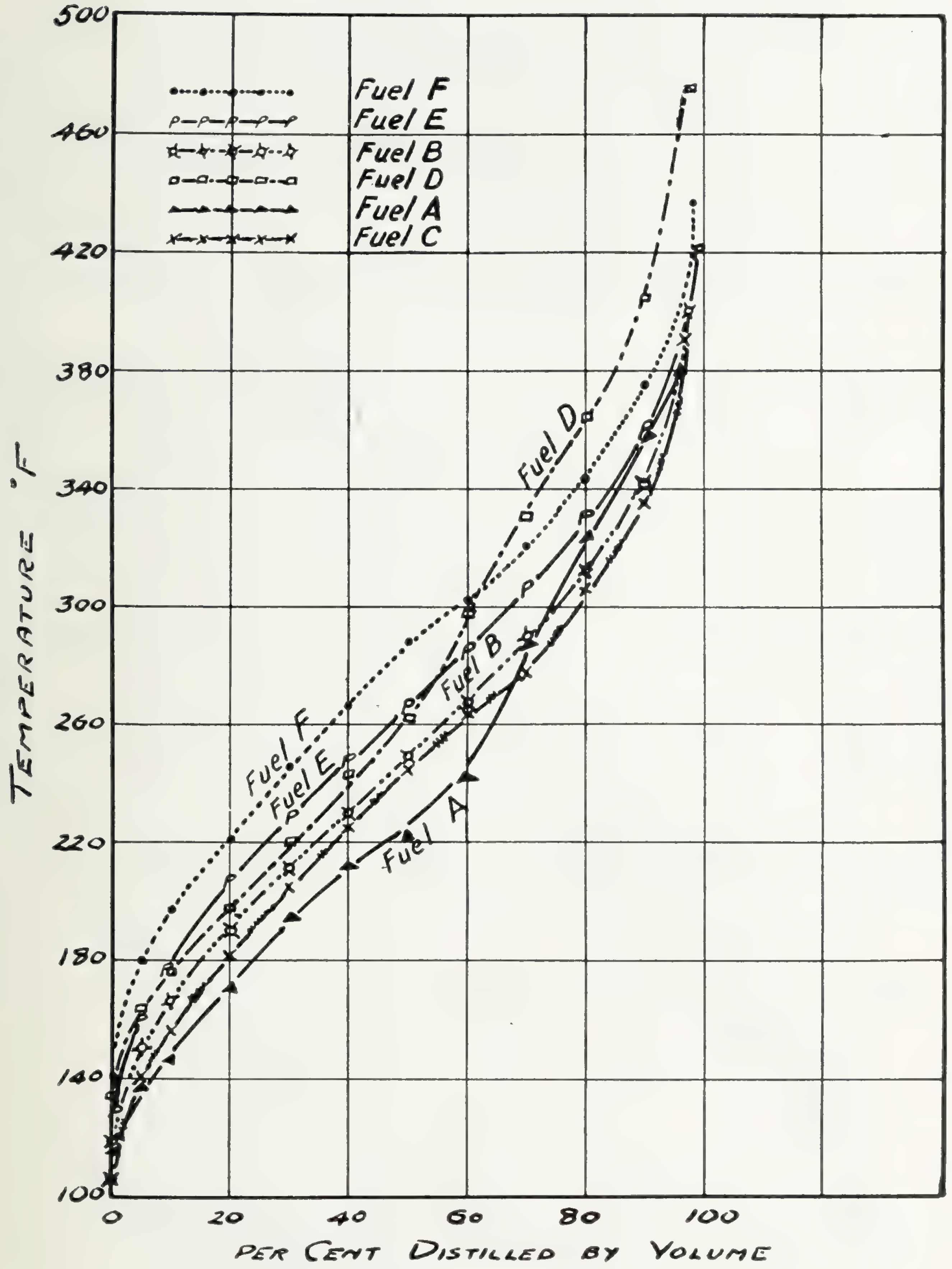

Fig. C.—Fractional Distillation for 1926-7 from Dominion Fuel Report.

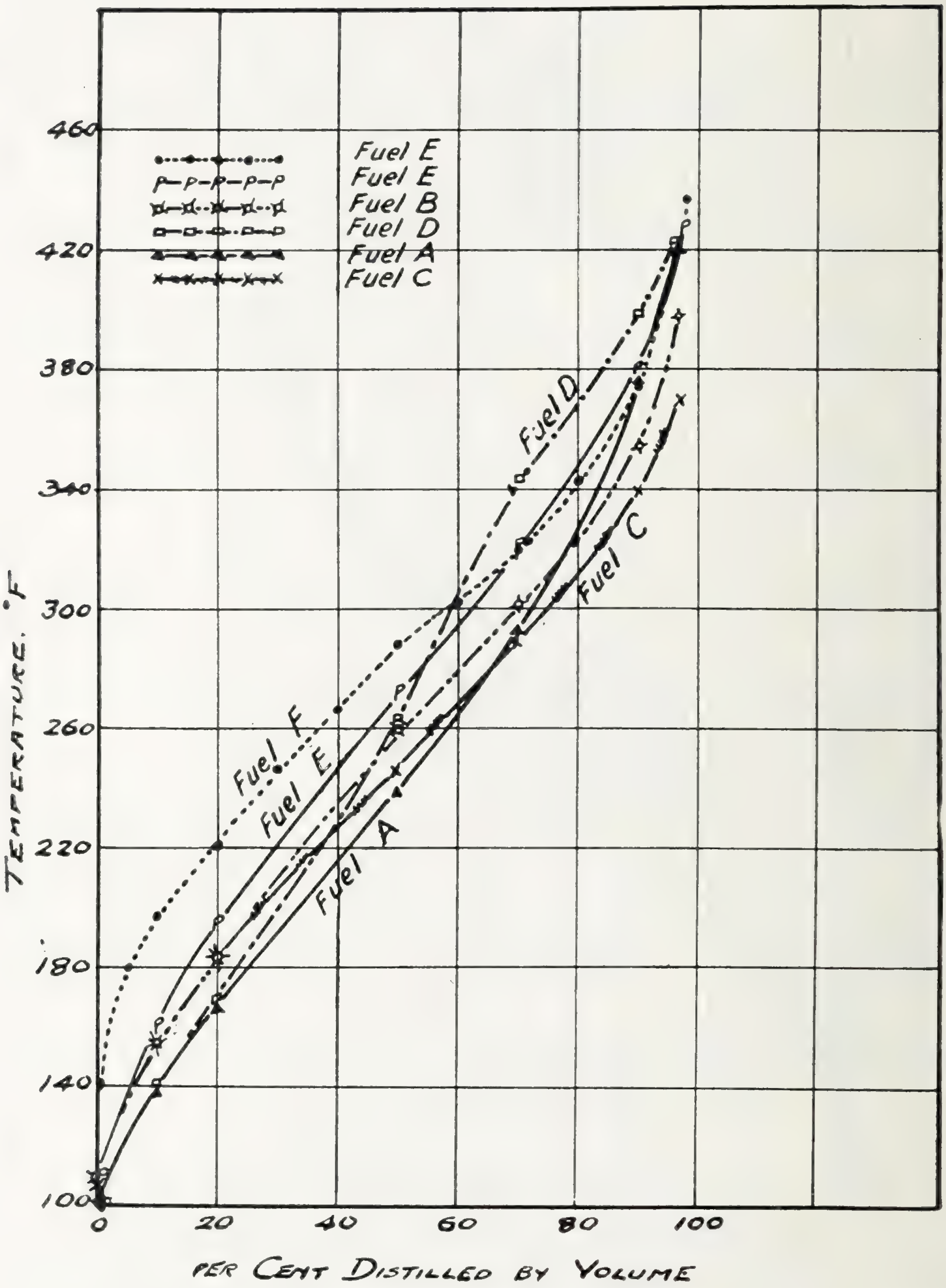

Fig. D.—Fractional Distillation of Samples Taken During Tests.

Easy starting from cold requires a comparatively large percentage of constituents with low boiling points. In this connection it was pointed out by Professor G. G. Brown[6] that fuels giving easy starting and having low boiling points are liable to vaporize in the carburettor under road conditions, and also that the mixture is liable to be superheated in the intake manifold in some cases. This causes a loss of power and economy, particularly at part throttle. Figs. C and D give the distillation curves for the fuels under consideration. Fig. C was taken from the Dominion Survey,[7] and Fig. D contains the actual distillation curves obtained from samples of the fuels used in the tests.

Detonation is a complex subject and has been the subject of numerous papers. Brown[8] has investigated the relationship between volatility and detonating properties of fuels, and Ricardo has established the importance, from a detonation standpoint, of the relative proportions of paraffins, aromatics and naphthenes in the fuel. Numerous papers have appeared, and the S.A.E. has held symposiums on volatility and detonation,[9] so that the subject has been well considered. However, there is no finality of opinion, either on the nature of detonation, or on the method of measuring it, so that these matters will not be considered here.

Anti-detonating fuels, however, are in common use, and various claims of increased power, economy and carbon removal are made for them. The advantage of ethyl and similar gasolines in increasing the possible compression ratio (and therefore the power and economy of engines) may be conceded, but the question arises whether any benefit is derivable from their use in engines having compression ratios less than 5-1/2 to 1, and well-designed combustion spaces where detonation would not normally occur. The present engine, having a compression ratio of 5.35:1, was well suited to such an investigation, and the fuels were chosen with this idea in view. Fuels "A" and "E" were chosen as representative of the ordinary low-priced gasolines, and "B," "C" and "D" were anti-detonating gasolines costing 3c. per gallon more than "A" and "E."

Their characteristics were as follows:—

A—A blend of straight run and cracked gasoline from certain California crudes.

B—A gasoline blended with lead tetra ethyl to a forty per cent. benzol equivalent.

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[6] "Present Tendencies in Motor Fuel Quality," S.A.E. Jrl., May, 1928.

[7] "Gasoline Survey for 1927," Rosewarne & Offord, Dept. of Mines, Ottawa, April, 1928.

[8] "The Relation of Motor Fuel Characteristics to Engine Performance," University of Michigan Bulletin, May, 1927.

[9] cf. S.A.E. Jrl., April, 1928.

C—A gasoline covered by U.S. Navy aviation gasoline, specification *domestic grade.*

D—A gasoline of straight run and vapour phase cracking blend.

E—A gasoline according to U.S. Government motor gasoline specification.

In these tests the engine was run with the same carburettor setting throughout. The first series was run on gasoline "A," and the carburettor setting (1-1/2 turns) corresponding to tests A 13-17, was chosen as a standard, because the exhaust analysis showed that under these conditions the mixture was very nearly chemically correct. Table 3 indicates that in this series there was no CO in the exhaust gases and the free oxygen content was about 0.4%.

TABLE No. 5.

DURANT—*Continental Motor*

Test No	B 5	B 4	B 1	B 2	B 3
Date	June 25	June 25	June 25	June 25	June 25
Air Temp. F	80	77.5	75	76	77
Exhaust Temp., F	850	978	1138	1220	1250
Fuel per hour, lbs	11.73	15.72	20.68	24.35	28.06
Cooling waterper hour, lbs	787	1060	1436	1469	1755
" " Inlet temp., F	56	55	55	54.5	55
" " Outlet temp,, F	164	162.5	157	167.5	160
" " Temp. rise, F	108	107.5	102	113	105
" " B.T.U. per hour	85000	114000	146400	166000	184300
Brake load net lbs	59.2	55.8	52.8	48.7	38.9
Brake M.E.P. lbs., per sq. inch	90.5	85.4	80.8	74.4	59.5
" " torque, lbs. ft	111.3	105	99.3	91.7	73.1
Speed, r.p.m	851	1267	1780	2226	2837
Brake horse-power	18.0	25.22	33.6	38.7	39.4
Gasoline—S.G.			.744		
Cal. value			19635 B.T.U./lb.		
Fuel per B.H.P. hour, lbs.	.652	.622	.615	.629	.713
Thermal eff'y., per cent	19.9	20.82	21.1	20.6	18.2
Cooling loss B.T.U. per B.H.P. hour	4720	4510	4360	4290	4680
Cooling loss per cent	36.9	36.90	36.1	34.7	33.4
Exhaust and radiation, per cent	43.2	42.28	42.8	44.7	48.4
Ignition advance	....	....	....	....	....
Throttle			Full open		
Oil temp. in crank case, °F	....	....	....	....	....
Barometer—Ins. mercury	29.40	29.40	29.40	29.40	29.40
Exhaust pressure—Ins. mercury	1.1	1.8	3.3	4.8	6.2
Humidity, per cent	69	72	73	71	71
Exhaust analysis: CO_2	11.5	12.25	12.1	12.7	12.0
" " O	0.25	0.15	0.2	0.1	0.2
" " CO	3.45	2.60	2.4	2.30	3.1
" " N	84.80	85.00	85.3	85.9	84.7

TABLE No. 6.

DURANT—*Continental Motor*

Test No	B 12	B 11	B 8	B 10	B 9
Date	July 6	July 6	July 6	July 6	July 6
Air Temp. F	89	78.5	75	79	76
Exhaust Temp. F	837	1011	1183	1282	1327
Fuel per hour, lbs	10.44	14.03	18.95	22.42	25.62
Cooling water per hour, lbs	750	997	1264	1534	1629
" " Inlet temp., F	56	56	57	55	56
" " Outlet temp., F	167	168.5	170	167	173
" " Temp. rise, F	111	112.5	113	112	117
" " B.T.U. per hour	83200	112100	142900	171900	190600
Brake load, net lbs	57.0	54.3	52.4	47.6	38.4
Brake M.E.P., lbs. per sq. in.	87.1	82.9	80.1	72.7	58.6
" " torque, lbs. ft	107.1	102.1	98.5	89.5	72.1
Speed, r.p.m	844	1268	1752	2244	2842
Brake horse-power	17.18	24.55	32.8	38.2	39.0
Gasoline—S.G.			.744		
Cal. value			19635 B.T.U./lb.		
Fuel per B.H.P. hour, lbs	.608	.572	.578	.587	.658
Thermal eff'y., per cent	21.35	22.7	22.45	22.1	19.7
Cooling loss B.T.U. per B.H.P. hour	4840	4570	4360	4500	4885
Cooling loss per cent	40.6	40.7	38.40	39.0	37.8
Exhaust and radiation per cent	38.05	36.6	39.15	38.9	42.5
Ignition advance	7°	18°	17°	19°	21°
Throttle			Full open		
Oil temp. in crank case, °F	175	....	214	260	276
Barometer—Ins. mercury	29.8	29.8	29.76	29.76	29.76
Exhaust pressure—Ins. mercury	1.0	1.8	3.3	4.8	6.3
Humidity, per cent	58	61	65	57	63
Exhaust analysis: CO_2	12.30	13.45	13.95	14.05	14.1
" " O	1.30	0.45	.30	0.20	0.1
" " CO	0.35	0.00	0.00	0.00	0.0
" " N	86.05	86.10	85.75	85.75	85.8

Tests B 1-5 were made under the same conditions with "B" gasoline, but it was found that the exhaust gases contained about 2.5% of CO (Table 5). A further series of tests was then made with the carburettor jet closed 1/8 turn. The results obtained are given in Table 6 (tests B 8-12), and under these conditions the fuel consumption at the most economical speed was cut down from .615 to .578 lbs. per B.H.P. hour.

Allowing for the slight difference in speed for tests B-1 and B-8, respectively, the power of the engine again was not affected by the change of mixture. The lower and higher speeds in this case showed a slight loss of power with the leaner mixture.

TABLE No. 7.

DURANT—*Continental Motor*

Test No	C 7	C 4	C 1	C 3	C 2
Date	June 27	June 27	June 27	June 27	June 27
Air Temp. F	81	76.25	72	76.5	75.5
Exhaust Temp. F	851	992	1165	1243	1313
Fuel per hour, lbs	11.0	14.07	20.08	23.3	25.45
Cooling water per hour, lbs	816	1092	1361	1558	1690
" " Inlet Temp., F	54.5	53	55.5	53	53
" " Outlet Temp., F	161.5	157	162.5	163	163.5
" " Temp. rise, F	107	104	107.0	110	110.5
" " B.T.U. per hour	87300	113700	145700	171300	186800
Brake load, net lbs	58.7	54.8	52.8	48.05	39.6
Brake M.E.P., lbs. per sq. inch	89.8	83.8	80.7	73.5	60.5
" " torque, lbs. ft	110.4	103	99.2	90.3	74.5
Speed, r.p.m	851	1236	1770	2255	2797
Brake horse-power	17.85	24.2	33.4	38.7	39.6
Gasoline—S.G.			.717		
Cal. value			19485 B.T.U./lb.		
Fuel per B.H.P. hour, lbs.	.616	.581	.601	.602	.643
Thermal eff'y., per cent	21.2	22.5	21.8	21.7	20.35
Cooling loss B.T.U. per B.H.P. hour	4890	4695	4360	4430	4720
Cooling loss per cent	40.7	41.4	37.2	37.8	37.75
Exhaust and radiation per cent	38.1	36.1	41.0	40.5	41.90
Ignition advance	15°	20°	17°	24°	19°
Throttle			Full open		
Oil temp. in crank case, °F	....	195	....	....	....
Barometer—Ins. mercury	29.68	29.68	29.66	29.68	29.66
Exhaust pressure—Ins. mercury	1.0	1.7	3.3	4.9	6.2
Humidity, per cent	58	61	70	64	63
Exhaust analysis: CO_2	11.38	12.95	13.28	13.1	13.6
" " O	.10	0.35	0.15	0.05	0.0
" " CO	1.75	0.20	0.35	0.25	0.25
" " N	86.77	86.50	86.22	86.60	86.15

Tests C (Table 7), D (Table 8), and E (Table 9) were all made under the same conditions as tests A 13-17 and B 1-5. The results are plotted in Figs. 5, 6 and 7.

Fig. 5 indicates that the minimum consumption in pounds per B.H.P. hour was obtained over the entire speed range with "A" gasoline. The hourly consumption of "C" fuel was less with the same carburettor setting, but the power obtained was also less. Fig. 9 gives the fuel consumption curves on a volumetric basis, as this is the condition under which each fuel is sold; and, taking into account the retail prices as on July 14th, 1928, Table 10 shows in cents per B.H.P. hour the cost of running at 1750 r.p.m. (35 miles per hour) and full load. This, of course,

TABLE No. 8.

DURANT—*Continental Motor*

Test No	D 5	D 4	D 1	D 3	D 2
Date	June 29	June 29	June 29	June 29	June 29
Air Temp. F	70	70.5	71.5	71.5	72
Exhaust Temp. F	850	995	1150	1247	1297
Fuel per hour, lbs	11.31	15.09	19.9	24.0	27.25
Cooling water per hour, lbs	837	1029	1251	1561	1754
" " Inlet temp., F	51	50.5	51	50	50
" " Outlet temp., F	153	160	165	158	156
" " Temp. rise, F	102	109.5	114	108	106
" " B.T.U. per hour	85400	112800	142700	168800	186000
Brake load, net lbs	60.0	56.1	53.2	48.8	38.8
Brake M.E.P., lbs. per sq. inch	91.8	85.6	81.4	74.6	59.4
" " torque, lbs. ft	112.9	105.3	100.0	91.8	73.0
Speed r.p.m	853	1266	1745	2231	2846
Brake horse-power	18.28	25.35	33.2	38.9	39.45
Gasoline—S.G.	.754				
Cal. value	19655 B.T.U./lb.				
Fuel per B.H.P. hour, lbs	.618	.595	.599	.617	.690
Thermal eff'y., per cent	20.95	21.8	21.6	21.0	18.8
Cooling loss B.T.U. per B.H.P. hour	4670	4445	4295	4340	4710
Cooling loss per cent	38.50	38.0	36.4	35.8	34.8
Exhaust and radiation per cent	40.55	40.2	42.0	43.2	46.4
Ignition advance	....	....	....	....	....
Throttle	Full open				
Oil temp. in crank case, °F	178	211	200	263	267
Barometer—Ins. mercury	29.40	29.40	29.45	29.45	29.45
Exhaust pressure—Ins. mercury	1.1	1.8	3.2	4.9	6.2
Humidity, per cent	80	81	78	80	78
Exhaust analysis: CO_2	11.9	12.8	13.55	13.5	13.18
" " O	0.35	0.1	0.15	0.0	0.0
" " CO	1.60	0.85	0.30	0.45	0.4
" " N	86.15	86.25	86.00	86.05	86.42

does not take into account variations in actual running conditions, but it is the only basis of comparison available. The minimum cost of running varies from 1.99 cents per B.H.P. hour on "A" gasoline to 2.433 cents on "C" fuel. The latter figure is due to the low specific gravity of this fuel, which is .717, as compared with .74 for most of the other gasolines tested. The cooling water losses do not vary very greatly for the different fuels. In each series the water outlet temperature was adjusted to 160-170° F., so that variations from this cause should not affect the results.

The general conclusion to be drawn from these figures is identical

TABLE No. 9.

DURANT—*Continental Motor*

Test No	E 5	E 4	E 1	E 3	E 2
Date	July 4	July 4	July 4	July 4	July 4
Air Temp. F	85	84.5	77	83	82
Exhaust Temp. F	860	938	1143	1219	1263
Fuel per hour, lbs.	11.15	14.7	20.0	23.38	26.5
Cooling water per hour, lbs	799	1126	1271	1552	1663
" " Inlet temp., F.	57	56.5	57	55	57
" " Outlet temp., F.	163	157.5	170	165	167
" " Temp. rise, F	106	101	113	109	110
" " B.T.U. per hour	84700	113600	143700	169000	182900
Brake load, net lbs	58.0	55.0	52.8	47.9	39.0
Brake M.E.P., lbs. per sq. inch	88.6	84.0	80.7	73.2	59.6
" " torque, lbs. ft	109	103.4	99.3	90.1	73.3
Speed, r.p.m	855	1245	1777	2220	2807
Brake horse-power	17.7	24.45	33.5	38.0	39.1
Gasoline—S.G	.737				
Cal. value	19830 B.T.U./lb.				
Fuel per B.H.P. hour, lbs.	.630	.601	.597	.615	.678
Thermal eff'y., per cent.	20.4	21.4	21.55	20.9	18.95
Cooling loss B.T.U. per B.H.P. hour	4780	4640	4290	4450	4680
Cooling loss, per cent.	38.3	39.0	36.30	36.5	34.80
Exhaust and radiation, per cent.	41.3	39.6	42.15	42.6	46.25
Ignition advance	....	....	....	....	....
Throttle	Full open				
Oil temp. in crank case, °F	175	208	202	266	269
Barometer—Ins. mercury	29.65	29.59	29.59	29.59	29.59
Exhaust pressure—Ins. mercury	1.1	1.8	3.3	4.8	6.0
Humidity, per cent	63	61	80	65	65
Exhaust analysis: CO_2	12.00	12.75	13.2	13.25	12.7
" " O	.45	0.30	0.2	0.20	0.1
" " CO	1.90	0.90	0.7	0.65	1.2
" " N	85.65	86.05	85.9	85.90	86.0

with that of Professor Brown,[10] who states that there is no economic advantage in buying anti-knock fuels at a 3-cent premium (about 11% on the selling price), if an engine having a compression ratio of 5-1/2 to 1 or less can be operated satisfactorily on an ordinary gasoline.

COOLING WATER TEMPERATURES

At the end of each series of tests described above, further experiments were made with the maximum and minimum quantities of cooling water available, so that the influence of cooling water temperature on the power

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[10] "Present Tendencies in Motor Fuel Quality," S.A.E. Jrl., May, 1928.

TABLE No. 10.

COMPARATIVE COSTS OF POWER FROM DIFFERENT FUELS AT 1750 R.P.M.

Test No.	A 13	A 18	B 7	B 8	C 1	D 1	E 1	F 1	F 2	F 3	F 4
Gasoline	A	A	B	B	C	D	E	E	E + Mixtrol	C	C + — Mix-Ur-Oil
Retail price, c. per Imp. gal.	26c.	26c.	30c.	30c.	29c.	29c.	26c.	26c.	26c.	29c.	29c.
Imp. gal. per B.H.P. hour	.0798	.0764	.0785	.0777	.0838	.0794	.081	.0822	.0799	.082	.0826
Gasoline cost, c. per B.H.P. hr.	2.075	1.985	2.36	2.33	2.430	2.305	2.105	2.135	2.078	2.38	2.395
Date	June 15	July 6	June 25	July 6	June 27	June 29	July 4	July 9	July 9	July 9	July 9
Specific grav. gasoline	.738	.737	.744	.744	.717	.754	.737	.737	....	.717	....
B.T.U., Imp. gal	140000	140000	146000	146000	139500	148000	146000	146000	....	139500	....
Additional price per Imp. gal.	....	....	....	....	....	....	....	....	2c.	....	*.362c.
Inclusive c. per B.H.P. hour	....	....	....	....	....	....	....	....	2.238	....	2.423

*Wholesale price.

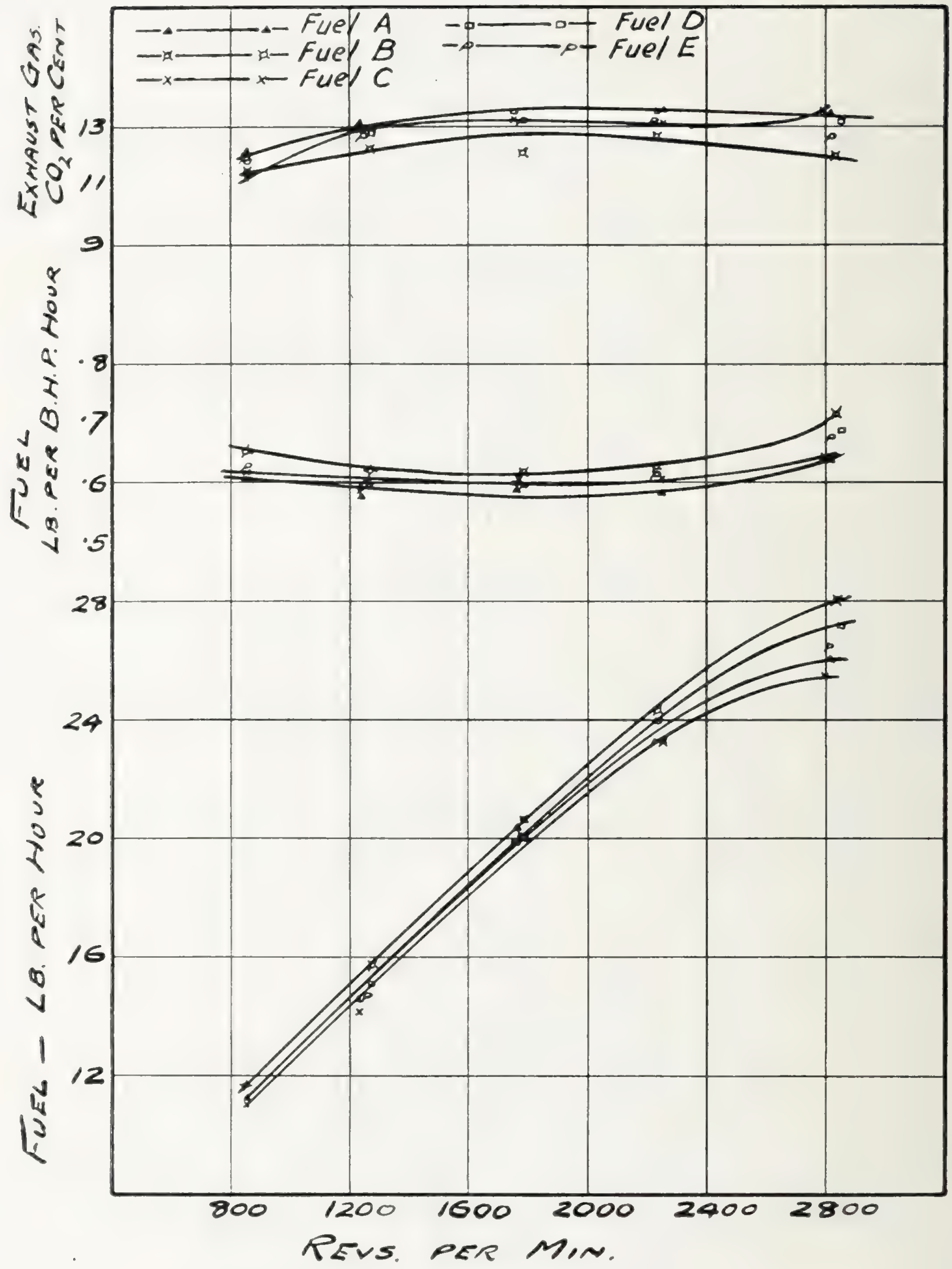

Fig. 5.—Different Fuels.

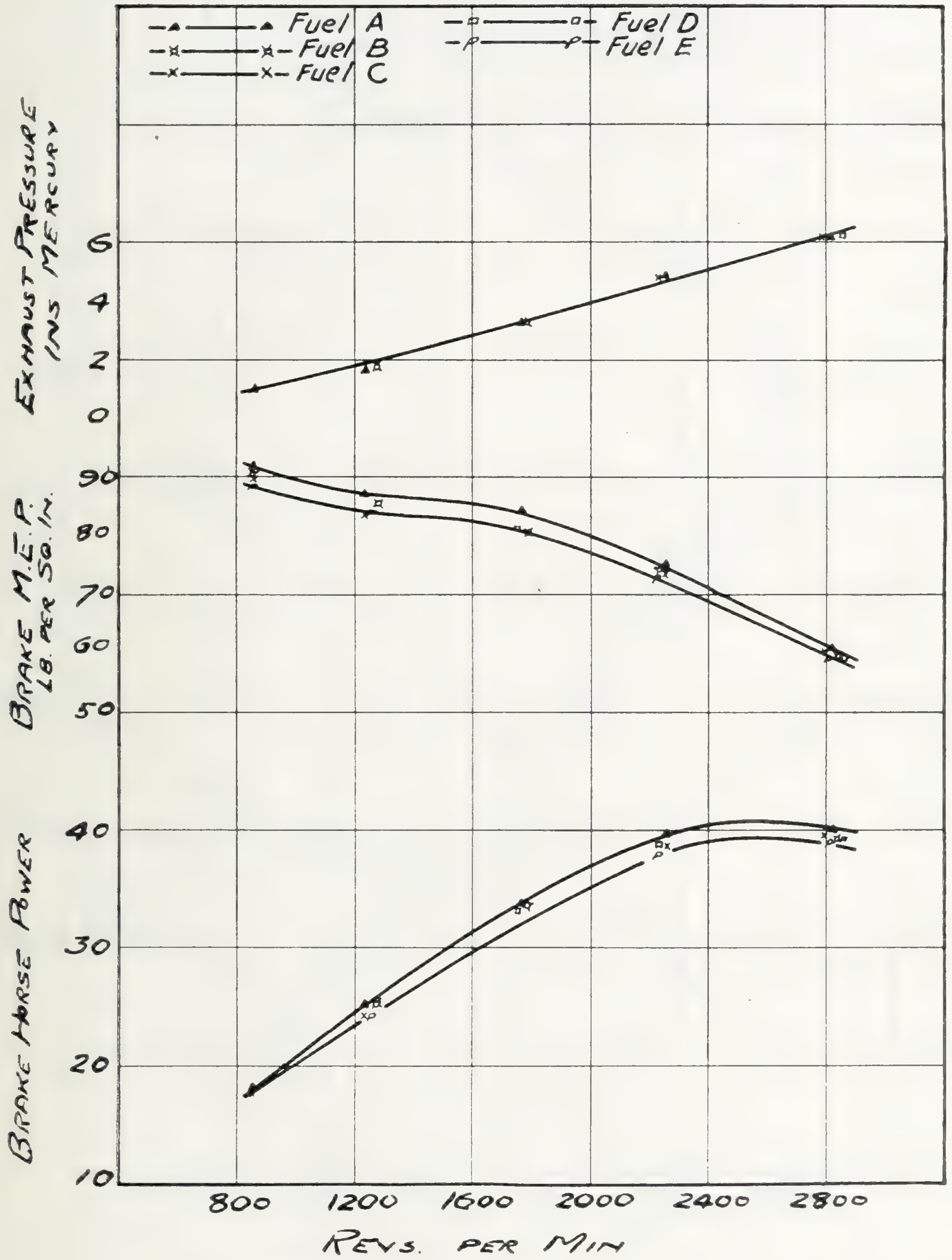

Fig. 6.—Different Fuels.

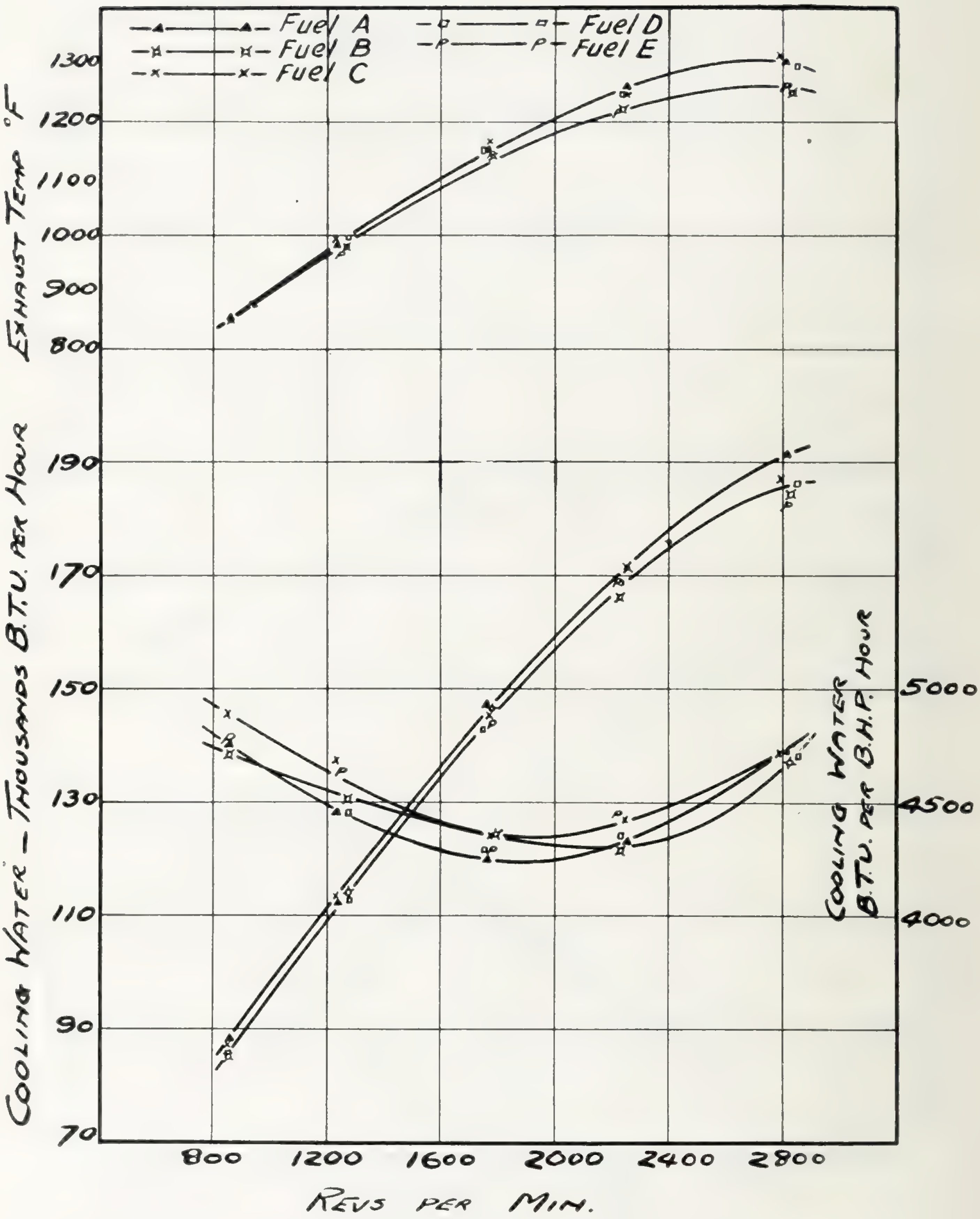

Fig. 7.—Different Fuels.

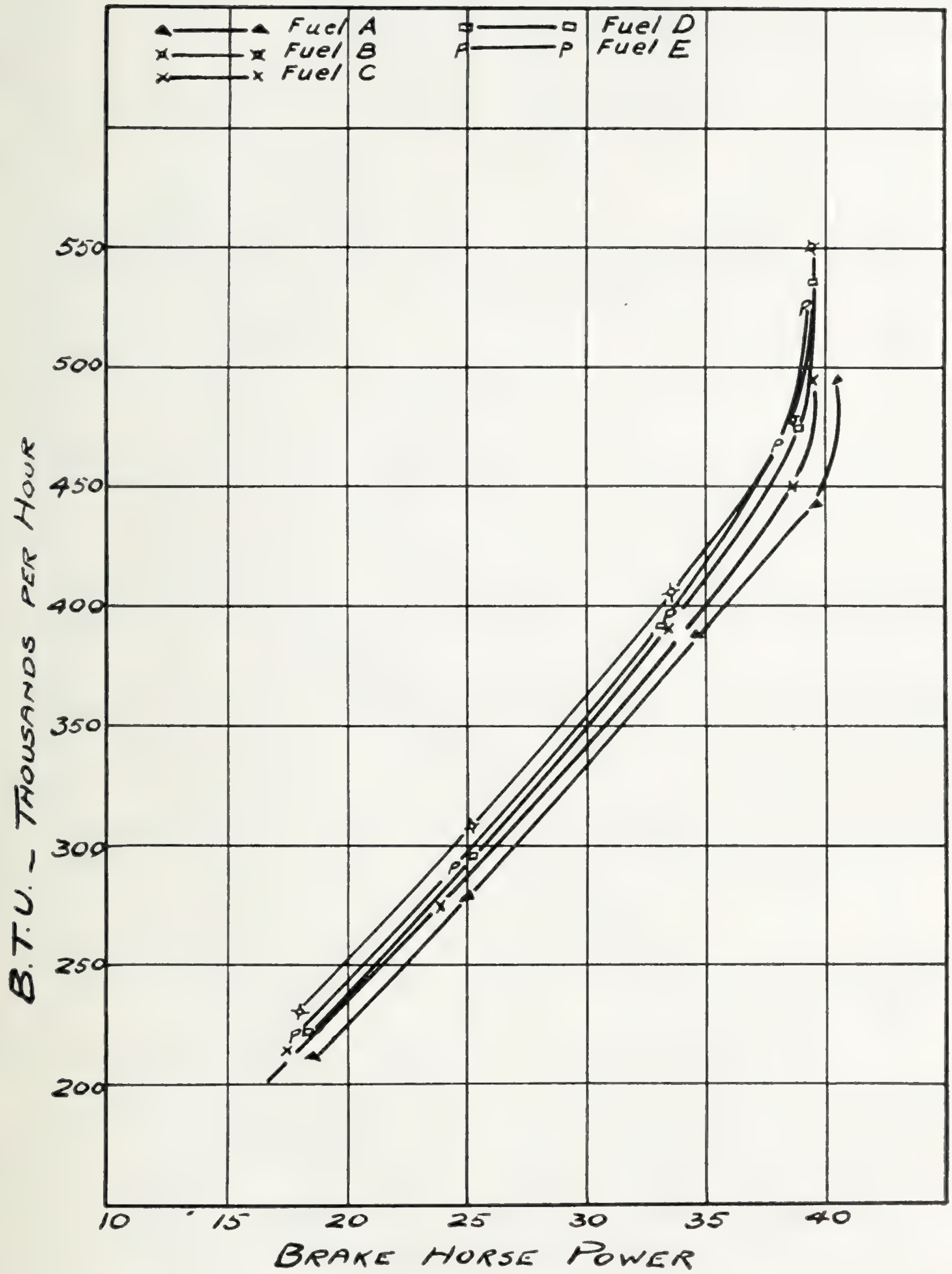

Fig. 8.—Hourly Heat Consumption with Different Fuels.

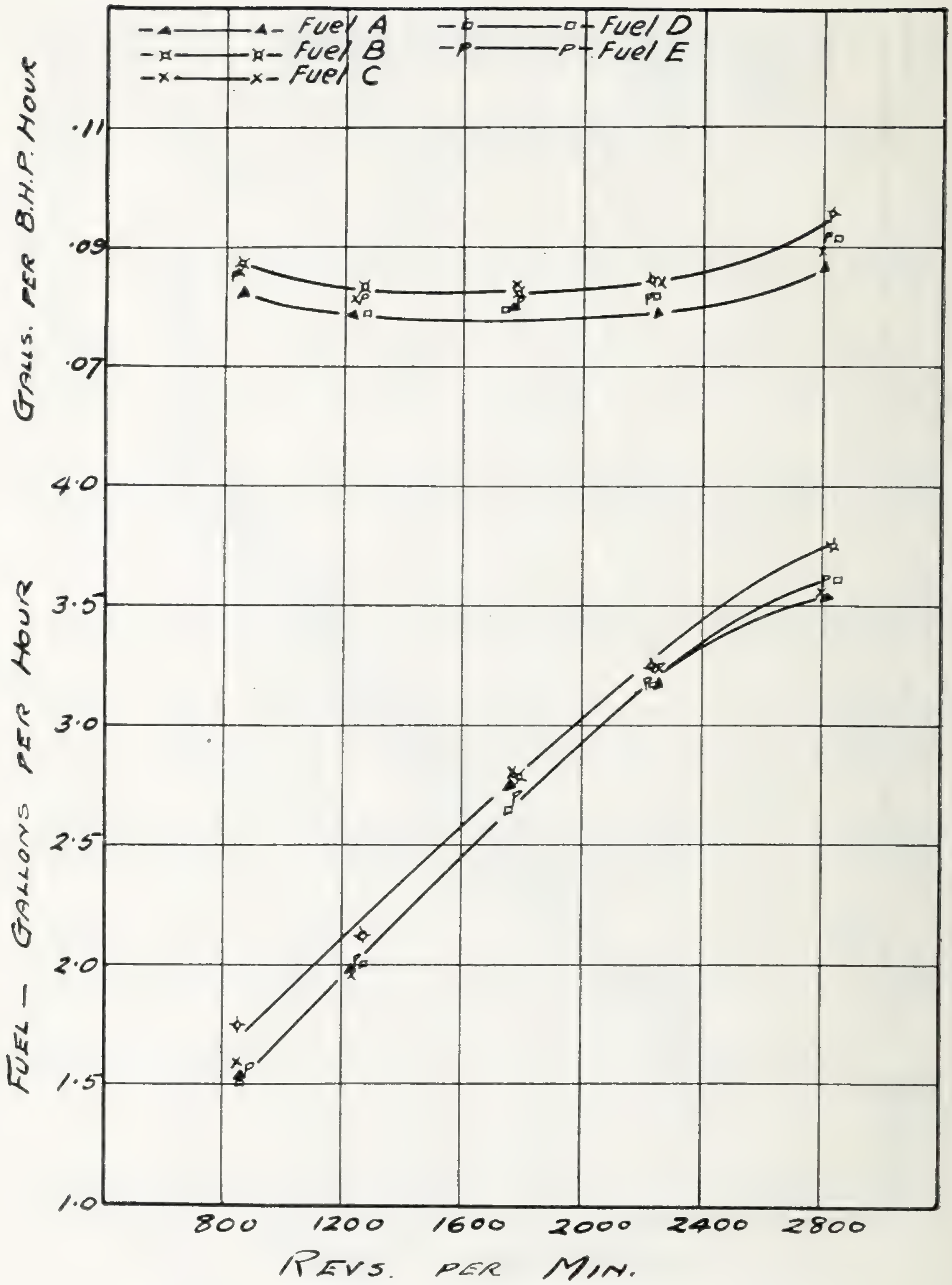

Fig. 9.—Different Fuels (Volumetric Comparison).

and economy of the engine might be studied at different speeds. The temperature range obtainable was not the same in each case, because the cooling water for the brake was taken off the same inlet pipe, and this could not be reduced below a certain quantity. However, the outlet temperature range in all cases was varied sufficiently to indicate any differences that might exist on this account.

The jacket temperature may be expected to influence five factors in engine economy and operation. These are:

(*a*) Cooling loss during combustion.
(*b*) Volumetric efficiency.
(*c*) Piston friction.
(*d*) Crank case dilution.
(*e*) Starting difficulties.

Ricardo[11] states that in all cases the cooling water loss during the combustion period does not exceed about 13% of the heat in the fuel, and therefore the saving achieved by using high water temperatures can only be a portion of this amount. Also, as the difference in temperature between the hot gases and water is in the neighbourhood of 3500° F., the greatest possible difference in water temperature can only be small by comparison, and therefore the difference in (*a*) is probably negligible.

On the basis that the change in temperature of the entering gases is about 1/6 that of the cylinder walls, and that the power output is proportional to the absolute temperature of the gases, Ricardo also calculates an advantage of 2-1/2 to 3-1/2% in power in favour of a low cooling water temperature. This is confirmed by Taub[12], who found that with a water outlet temperature of 110-140° F., the volumetric efficiency of an engine was 81%. From 140-180° F. the volumetric efficiency fell, until at 180-200° F. it was 77%.

As the wall temperature rises, the viscosity of the lubricating oil falls, thus improving the mechanical efficiency, so that the power output may be increased as much as 3-8%, due to this factor alone.

It follows, therefore, that factors (*b*) and (*c*) work in opposite directions, and according as the change in (*c*) is large or small, the use of high outlet temperatures may have a beneficial effect, or no effect at all, on the actual power produced.

It is certain, however, that a low cylinder temperature will increase crank case dilution. This is confirmed by Sparrow & Eisinger,[13] who indicate a reduction of dilution from 5% to 0 as the jacket temperature increases from 50° to 212° F. Taub states that 16% crank case dilution

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[11] "The Internal Combustion Engine," Vol. II, p. 72.

[12] "High Average Operating Temperature," S.A.E. Jrl., March, 1926.

[13] "Recent Co-operative Fuel Research Progress," S.A.E. Jrl., Feb., 1925.

TABLE No. 11.

DURANT—*Continental Motor*

Test No	A 7	A 3	A 6	B 7	B 1	B 6
Date	June 11	June 11	June 11	June 25	June 25	June 25
Air Temp. F	79	72.2	78.2	81.5	75	80
Exhaust Temp. F	1000	1040	1005	1156	1138	1156
Fuel per hour, lbs	27.48	27.18	27.08	19.53	20.68	19.52
Cooling water per hour, lbs	1594.5	1105	856.5	2586	1436	960
“ “ Inlet temp., F	50	49	50	56	55	56
“ “ Outlet temp., F	130.8	165.4	194	110	157	202
“ “ Temp. rise, F	80.8	116.4	144	54	102	146
“ “ B.T.U. per hour	129000	128700	123300	139900	146400	140100
Brake load, net lbs	54.7	54.0	54.2	53.1	52.8	52.4
Brake M.E.P., lbs. per sq. inch	83.6	82.4	82.9	81.1	80.8	80.1
“ “ torque, lbs. ft	102.9	101.5	102	99.8	99.3	98.5
Speed, r.p.m	1745	1760	1735	1760	1780	1752
Brake horse-power	34.1	33.9	33.6	33.4	33.6	32.8
Gasoline—S.G	(“A”)	.738		(“B”)	.744	
Cal. value	—19000 B.T.U./lb.—			—19635 B.T.U./lb.—		
Fuel per B.H.P. hour, lbs	.806	.801	.806	.585	.615	.595
Thermal eff'y., per cent	16.65	16.75	16.65	22.15	21.1	21.8
Cooling loss B.T.U. per B.H.P. hour	3785	3795	3670	4190	4360	4275
Cooling loss, per cent	24.75	24.95	24.00	36.5	36.1	36.6
Exhaust and radiation, per cent	58.60	58.30	59.35	41.35	42.8	41.6
Ignition advance	....	....	....	....	....	....
Throttle	——Full open——			——Full open——		
Oil temp. in crank case, °F	....	....	....	....	....	....
Barometer—Ins. mercury	29.85	29.83	29.85	29.40	29.40	29.40
Exhaust pressure—Ins. mercury	3.7	3.7	3.6	3.3	3.3	3.2
Humidity, per cent	53	58	48	70	73	69
Exhaust analysis: CO_2	....	8.63	....	....	12.1	....
“ “ O	....	0.20	....	....	0.2	....
“ “ CO	....	7.63	....	....	2.4	....
“ “ N	....	83.54	....	....	85.3	....

is necessary to facilitate starting with a cold engine, but this would appear to be dependent on the kind of oil used and its temperature.

Cold cylinder walls obviously make it more difficult to get sufficient fuel into the cylinder when starting the engine, on account of the precipitation of fuel in the liquid state.

The question of cylinder wall temperature and its influence on power and economy is important in connection with the proposed adoption of steam cooling, which produces a uniform temperature of 212° F. Numerous papers have been written on the advantages of this system to prove

TABLE No. 11.—(*Continued*)

DURANT—*Continental Motor*

Test No	C 5	C 4	C 6	D 7	D 5	D 6
Date	June 27	June 27	June 27	June 29	June 29	June 29
Air Temp. F	80	76.25	81	69	70	70
Exhaust Temp. F	995	992	1000	848	850	847
Fuel per hour, lbs	14.86	14.07	14.73	11.72	11.31	11.15
Cooling water per hour, lbs	2017.5	1092	762	2800.5	837	547.5
" " Inlet temp., F	54	53	54	50.5	51	51.5
" " Outlet temp., F	113	157	202	82	153	198
" " Temp. rise, F.	59	104	148	31.5	102	146.5
" " B.T.U. per hour	119000	113700	112800	88200	85400	80200
Brake load, net lbs	55.5	54.8	54.7	61.4	60.0	59.3
Brake M.E.P., lbs. per sq. inch	85.0	83.8	83.6	94.1	91.8	90.6
" " torque, lbs. ft	104.3	103	103	115.4	112.9	111.5
Speed, r.p.m	1249	1236	1258	855	853	850
Brake horse-power	24.8	24.2	24.6	18.8	18.28	18.0
Gasoline—S.G.	("C")	.717		("D")	.754	
Cal. value	—19485 B.T.U. lb.—			—19655 B.T.U. lb.—		
Fuel per B.H.P. hour, lbs	.599	.581	.599	.624	.618	.619
Thermal eff'y., per cent	21.75	22.45	21.75	20.75	20.95	20.9
Cooling loss B.T.U. per B.H.P. hour	4800	4695	4590	4690	4670	4460
Cooling loss, per cent	41.0	41.50	39.2	38.20	38.50	36.6
Exhaust and radiation, per cent	37.25	36.05	39.05	41.05	40.55	42.6
Ignition advance	....	....	....	....	....	....
Throttle	——Full open——			——Full open——		
Oil temp. in crank case, °F	....	....	....	....	....	....
Barometer—Ins. mercury	29.68	29.68	29.68	29.40	29.40	29.40
Exhaust pressure—Ins. mercury	1.8	1.7	1.8	1.1	1.1	1.1
Humidity, per cent	61	61	58	78	80	78
Exhaust analysis: CO_2	....	12.95	....	....	11.90	....
" " O	....	0.35	....	....	0.35	....
" " CO	....	0.20	....	....	1.60	....
" " N	....	86.50	....	....	86.15	....

the benefits obtainable in practical operation, but steam cooling is advancing very slowly, if at all.[14]

In connection with this investigation, a previous series of tests on a tractor engine had indicated little or no difference in power and economy when the cooling water outlet temperature was varied from 100° to 200° F. In the present case, tests were made at 1750 (A and B series),

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[14] Except in aircraft work, where steam cooling is being used on the "Tornado" engine, used in the large British airship R-101 for transatlantic work. Wing-Commander Cave-Browne-Cave, on "The Evaporative Cooling of Aircraft Engines." British Association, Section G, Sept. 10, 1928.

TABLE No. 11—(*Continued*).

DURANT—*Continental Motor*

Test No	E 6	E 3	E 7
Date	July 4	July 4	July 4
Air Temp. F	86	83	88
Exhaust Temp. F	1220	1219	1220
Fuel per hour, lbs	23.44	23.38	22.6
Cooling water per hour, lbs	2812.5	1552	1132.5
" " Inlet temp., F	57	56	56
" " Outlet temp., F	118	165	202
" " Temp. rise, F	61	109	146
" " B.T.U. per hour	171500	169000	165200
Brake load, net lbs	47.1	47.9	47.5
Brake M.E.P., lbs. per sq. inch	72.0	73.2	72.6
" " torque, lbs. ft	88.5	90.1	89.4
Speed, r.p.m	2226	2220	2220
Brake horse-power	37.5	38.0	37.7
Gasoline—S. G	("E")	.737	
Cal. value	19830 B.T.U./lb.		
Fuel per B.H.P. hour, lbs	.626	.615	.595
Thermal eff'y., per cent	20.5	20.9	21.6
Cooling loss B.T.U. per B.H.P. hour	4575	4450	4390
Cooling loss per cent	36.9	36.50	37.2
Exhaust and radiation, per cent	42.6	38.05	41.2
Ignition advance	....	....	....
Throttle	Full open		
Oil temp. in crank case, °F	....	....	....
Barometer—Ins. mercury	29.65	29.59	29.65
Exhaust pressure—Ins. mercury	4.7	4.8	4.7
Humidity, per cent	60	65	55
Exhaust analysis: CO_2	....	13.25	....
" " O	....	0.20	....
" " CO	....	0.65	....
" " N	....	85.90	....

1250 (C series), 855 (D series), and 2250 r.p.m. (E series), so that different fuels were investigated, as well as different speeds. The results are given in Table 11.

In most cases, a slight fall of power was observed at the higher temperatures, and at 855 and 2250 r.p.m. there was a slight increase of economy, but this did not hold good for the other speeds, at which the economy was unaffected.

The total heat lost per hour to the cooling water decreased generally as the temperature increased, but the cooling loss, expressed as a percentage of the total heat entering the engine, remained very nearly

TABLE No. 12

DURANT—*Continental Motor*

Test No	F 1	F 2	F 3	F 4
Date	July 9	July 9	July 9	July 9
Air Temp. F	81.5	82.5	85.5	84
Exhaust Temp. F	1155	1165	1165	1161
Fuel per hour, lbs	20.0	19.5	19.4	19.6
Cooling water per hour, lbs	1170	1140	1253	1285
" " Inlet temp., F.	59	59	58	58
" " Outlet temp., F.	183.5	184	174	171.5
" " Temp. rise., F.	124.5	125	116	113.5
" " B.T.U. per hour	145800	142500	145300	145890
Brake load, net lbs	51.8	51.7	51.5	51.7
Brake M.E.P., lbs. per sq. inch	79.1	79.0	78.7	79.1
" " torque, lbs. ft	97.4	97.2	96.9	97.2
Speed, r.p.m	1784	1793	1793	1789
Brake horse-power	33.0	33.1	33.0	33.1
	"E"	Mixtrol	"C"	Mix-Ur-Oil
Gasoline—S.G	.737	.851	.717	.865
Cal. value	19830 B.T.U./lb.		19485 B.T.U./lb.	
Fuel per B.H.P. hour, lbs.	.606	.589	.588	.592
Thermal eff'y., per cent	21.2	....	22.2	....
Cooling loss B.T.U. per B.H.P. hour	4420	4310	4410	4410
Cooling loss, per cent	36.8	....	38.5	....
Exhaust and radiation, per cent	42.0	....	39.3	....
Ignition advance	....	....	....	....
Throttle	——Full open——		——Full open——	
Oil temp. in crank case, °F	203	215	200	220
Barometer—Ins. mercury	29.72	29.72	29.72	29.72
Exhaust pressure—Ins. mercury	3.3	3.3	3.3	3.3
Humidity, per cent	78	77	70	73
Exhaust analysis: CO_2	12.95	13.55	13.45	13.50
" " O	0.75	0.20	0.35	0.25
" " CO	0.00	0.00	0.00	0.00
" " N	86.30	86.25	86.20	86.25

constant, and did not show any progressive decrease. The figures are so close that it may be said that the cooling water temperature has no appreciable effect on the brake horse-power or on the brake thermal efficiency.

SPECIAL LUBRICANTS IN FUEL

Various special lubricants are now on the market, and it is claimed that if these are mixed with the fuel entering the carburettor, remarkable benefits in power, economy and mechanical running will be obtained. The "F" series (Table 12) was therefore devoted to an investigation of

TABLE No. 13.

DURANT—*Continental Motor*

(Tests on engine No. 10045, stripped)

Test No	A 19	A 21	A 20	A 22	A 23
Date	July 26	July 26	July 26	July 26	July 26
Air Temp. F	78.7	82	82	85	87
Exhaust Temp. F	1200	1285	1330	1305	1225
Fuel per hour, lbs	20.25	23.35	26.05	27.48	32.3
Cooling water per hour, lbs	....	....	....	....	....
" " Inlet temp., F	....	....	....	....	....
" " Outlet temp., F	146	154	161	161	155
" " Temp. rise, F	....	....	....	....	....
" " B.T.U. per hour	....	....	....	....	....
Brake load, net lbs	55.8	50.2	44.0	44.3	44.2
Brake M.E.P., lbs. per sq. in	85.2	76.8	67.2	68.7	67.4
" " torque, lbs. ft	105	94.4	82.6	83.5	83.1
Speed, r.p.m	1758	2265	2710	2714	2735
Brake horse-power	35.0	40.6	42.6	42.9	43.1
Gasoline—S.G. ("A")	.738				
Cal. value		19000 B.T.U./lb.			
Fuel per B.H.P. hour—lbs	.58	.575	.612	.641	.75
Thermal eff'y., per cent	23.2	23.3	21.8	20 9	17.9
Cooling loss B.T.U. per B.H.P. hour	....	....	....	....	....
Cooling loss, per cent	....	....	....	....	....
Exhaust and radiation, per cent	....	....	....	....	....
Ignition advance	....	....	....	....	....
Throttle	Full open				
Oil temp. in crank case, °F	....	....	....	....	....
Barometer—Ins. mercury	29.82	29.82	29.82	29.82	29.82
Exhaust pressure—Hg	None	1.1	1.3	1.5	1.5
Humidity, per cent	74	78	78	80	75
Exhaust analysis: CO_2	14.2	14.4	14.0	13.2	10.0
" " O	.6	.5	.4	.4	.4
" " CO	0	0	0	.9	5.4
" " N	85.2	85.1	85.6	85.5	84.2

these claims. The running time was not sufficiently long to test the influence of these lubricants on the life or wear of the running surfaces, but it enabled the power and economy claims to be investigated.

Test F-1 was run at nearly 1800 r.p.m. with "E" gasoline, and at the end of this test the quantity of Mixtrol, prescribed in the instructions, was added to the gasoline and the engine allowed to run for some time before test F-2 was made. The power was unaltered, but there was a fall of about 2-1/2% in fuel consumption. The comparative costs are shown in Table 10, from which it is seen that although the actual fuel cost per unit of power is reduced, the extra cost of the oil more than swamps the economy obtained.

A similar series (F 3 and F 4) with "C" gasoline and "C"+Mix-Ur-Oil, produced similar results. It would appear, therefore, that there is no gain in power or economy by using these lubricants, but there may possibly be beneficial results, as far as the running surfaces are concerned.

This research was made under the auspices of the School of Engineering Research in the University of Toronto.

The thanks of the author are due to the Durant Company of Canada, who supplied the engine; to Messrs G. H. Harlow, B.A.Sc., and R. A. Westervelt, B.A.Sc., who assisted in taking observations and making the necessary calculations; and to Mr. F. Hickey, who erected the test apparatus and was in charge of the engine while the tests were in progress.

THE UNIFLOW STEAM ENGINE

GENERAL NOTES INCLUDING AN HISTORICAL REFERENCE, RESULTS OF TESTS, ITS APPLICATION AND A BIBLIOGRAPHY OF THE SUBJECT

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In this paper the principal object is to discuss from the thermal standpoint the history and development of the uniflow principle as applied to steam engines. The mechanical details will only be described, therefore, insofar as they influence the heat consumption of the engine for a given amount of work done. It is true that such items as valve gear, governing and cylinder construction are essential to the consideration of the uniflow engine as a power-producing mechanism, but an adequate discussion of all these points would transform the paper into a volume. For this reason, one part of the problem only is covered.

History

The credit for the first application of the uniflow principle to steam engine work is due to Jacob Perkins of London, England, who patented the system in 1827 after two years of working tests.[1] He also made a compound uniflow engine which was tested in 1928, the next example of this application being the compound uniflow engine exhibited at the Wembley Exhibition in 1924. It is interesting to note that Perkins also anticipated the recent use of high steam pressures by generating his steam at 2,000 lbs. per square inch, and this at a time when the Boulton and Watt engines used steam at pressures of 5 to 10 lbs. per square inch. By these means, Perkins was able to work a pumping engine with a fuel consumption of 1.7 lb. per pump horse power hour. The fuel used was a mixture of coal and coke. These patents, however, were impounded by the creditors of Perkins' partner, and so could not be followed up.

Further experiments along these lines were made by Eaton (U.S.A.), in 1857, and patent No. 7301 was taken out by Todd (England), in 1885,[2] the graduation of temperature from hot inlets to cold outlet and the reduction of condensation being specifically noted.

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[1] Power, March 10th, 1925, and The Power Engineer, August, 1925.

[2] "The Uniflow Steam Engine," Pilling (Manchester Assoc. of Engineers, March 27th, 1920).

In 1849, the South Eastern Railway (England), had a locomotive running on the uniflow principle which continued in operation for three years, but this was taken off on account of mechanical trouble in 1852.[3]

A Belgian patent was also taken out by Prof. A. Rateau on May 29th. 1894.[4] No great commercial development took place, however, until the matter was taken up by Prof. J. Stumpf of Charlottenburg in 1908, and to him the modern development of the uniflow engine is very largely due. Since that time the application of the uniflow engine has been extending rapidly, and it is now generally recognized as having a definite and important place in the field of power generation.

It seems probable that the revival of the idea in 1885 was partly due to the work done by Sir Dugald Clerk (1881), and others, on the two-stroke cycle gas engine, and certainly the cylinder design of the two-stroke cycle double-acting Körting gas engine bears a striking similarity to that of the uniflow steam engine.

CYLINDER CONDENSATION

One of the principal defects in the ordinary or "counterflow" type of steam engine cylinder results from the interchange of heat between the

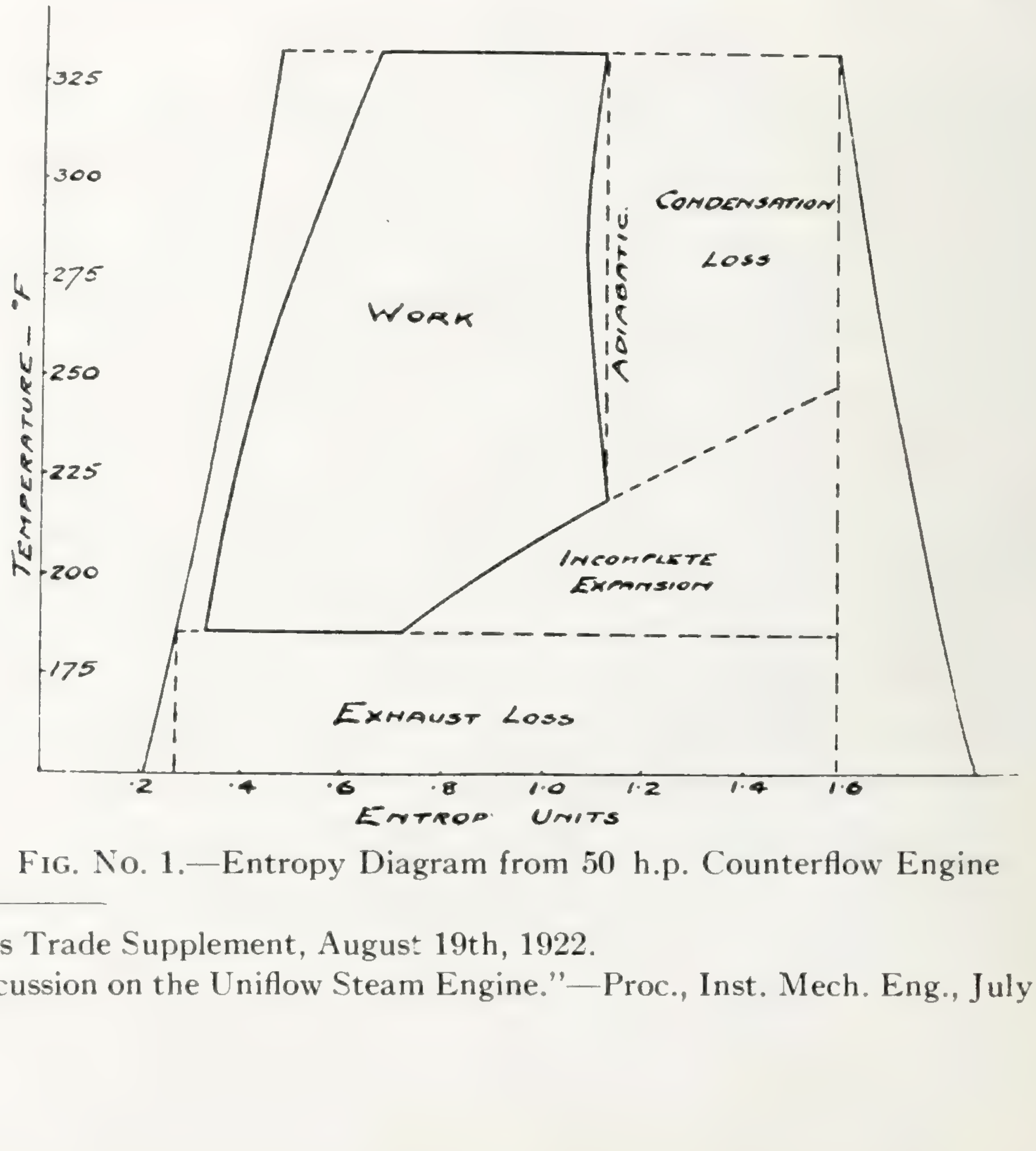

FIG. No. 1.—Entropy Diagram from 50 h.p. Counterflow Engine working fluid and the metallic wall which surrounds it. The Carnot and Rankine cycles are both based on the assumption that the cylinder walls are non-conducting, but this condition does not, and cannot, obtain in practice, with the result that some loss of heat must necessarily take place. With steam originally superheated, this loss results in lower superheat, or none at all, in the cylinder, and if the steam is initially saturated or wet, considerable condensation takes place during the admission period. The amount of this loss naturally depends upon, (*a*) the ratio of surface to volume in the cylinder during the admission period; (*b*) the facilities for the interchange of heat between the steam and the cylinder walls.

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[3] Times Trade Supplement, August 19th, 1922.

[4] "Discussion on the Uniflow Steam Engine."—Proc., Inst. Mech. Eng., July, 1920.

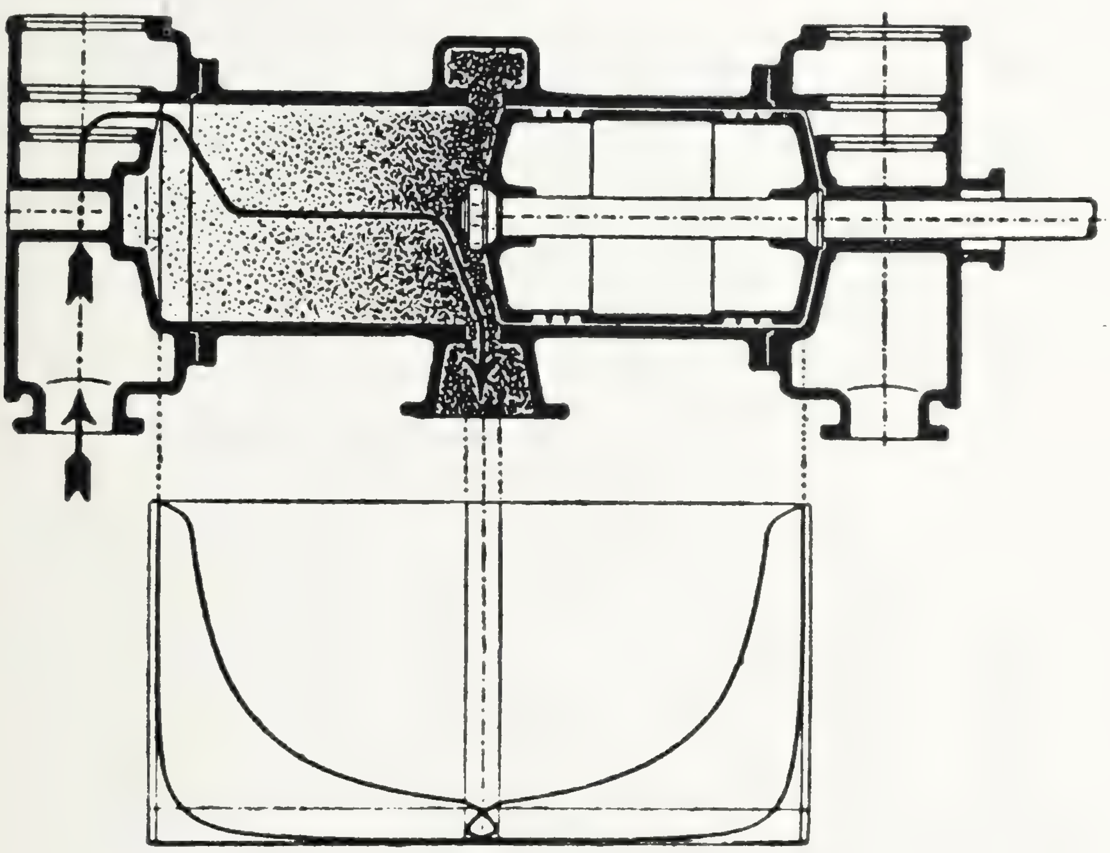

FIG. No. 2.—Uniflow Engine Cylinder

With early cut off, the conditions for heat loss are ideal, as the ratio of surface to volume is large, so that a large ratio of expansion, desirable as it is from a thermal standpoint, may actually be a source of loss.

Apart from this, two factors contribute to a high rate of heat exchange per unit of surface, namely (1), a large temperature difference between the incoming and exhaust steam; (2) a large percentage of water in the steam itself.

The "counterflow" type of cylinder thus contributes to low thermal efficiency by allowing the cool exhaust steam to pass over the surface of the cylinder during approximately half the cycle. This results in the condensation of 40 to 50 per cent. of the incoming steam before the point of cut off (figure No. 1), and the very fact that this steam is wet causes the heat interchange between steam and cylinder walls to be more rapid by substituting a medium of comparatively high conductivity for the previously dry steam.

The steam turbine does not suffer from this defect, as the live steam enters at one end and the exhaust steam goes out at the other. The uniflow engine is an attempt to reproduce this condition in an engine cylinder, so that the hot steam enters at each end and the cool, wet exhaust steam leaves through ports in the middle of the cylinder. The flow in all cases, therefore, is from the ends to the centre (figure No. 2), and in place of the temperature fluctuations characteristic of the counterflow type there is a steady fall of temperature from the admission to the exhaust end of the stroke.[5] The ports in the middle of the cylinder through which the steam is exhausted are 1/10 stroke long, and this means that the subsequent compression must occupy 90 per cent. of the return stroke. This is advantageous in keeping the ends of the cylinder hot, but has the disadvantage of reducing the mean effective pressure and of producing a dangerously high compression pressure in non-condensing engines or in cases where the vacuum falls. The uniflow

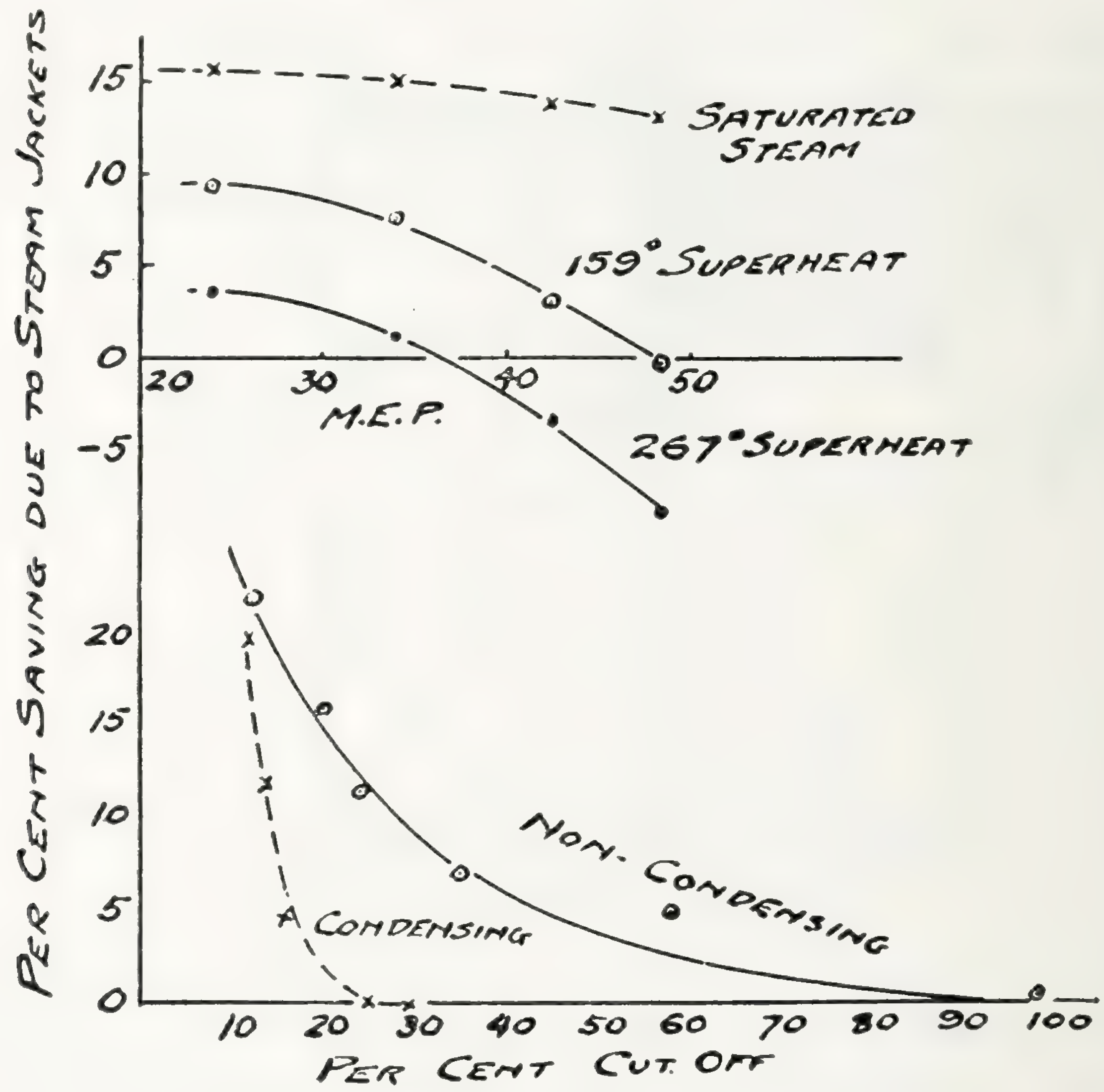

FIG. NO. 3.—Curves showing Gain Due to Steam Jacketing (Counterflow Engine)

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[5] This was investigated experimentally by Prof. Dr. Nägel, of Dresden (Z.V.D.I., July 5th, 1913, and the "Uni-flow Steam Engine," by J. Stumpf).

engine is thus essentially a high vacuum engine, but in small installations the attainment of this condition is not always feasible. There are two ways of avoiding the difficulty. One is to provide a large clearance volume, permanently or temporarily, but this is always uneconomical. The other is to delay the start of compression by the use of automatic or mechanically operated auxiliary exhaust valves. This is a departure from the uniflow principle and really makes the engine a counter-uniflow engine, with a consequent loss of economy. The position in which these auxiliary exhaust valves are placed determines the extent to which the hot surfaces are cooled by the passage over them of exhaust steam, and, other things being equal, the further these valves are placed from the end of the cylinder, the better.[6]

The influence of wet steam on heat transmission has already been referred to, but another means of reducing the amount of water in the cylinder is to heat it externally by means of a steam jacket. Uniflow engine cylinders are usually steam jacketed, so that in the following pages considerable attention will be given to the action of the jacket.

FACTORS INFLUENCING CYCLE EFFICIENCY

In addition to cylinder condensation there are other factors which need to be balanced so that losses may be reduced to a minimum. These are steam pressure, back pressure, clearance volume, compression ratio and mean effective pressure. An exhaustive analysis of the interaction of these was made by Professor Stumpf,[7] and among his conclusions may be noted the following:—

(1) With proper proportioning of the length of compression, the clearance volume must be kept as small as possible.

(2) For given steam pressure, back pressure, m.e.p. and length of compression, the clearance volume must be such that the change of pressure during expansion is equal to the change of pressure during compression.

(3) Change of compression in case of high vacuum has no material effect upon the steam consumption.

It has already been stated, and the statement is susceptible to proof, that large clearance volumes are uneconomical, and the principles enumerated above show that different clearance volumes are required for best economy at different steam pressures, exhaust pressures and expansion ratios. It is evident, therefore, that a compromise must be made in practice whereby the minimum compression volume may be used and the compression ratio either adjusted to suit the conditions of working or chosen to give the best economy at the normal working load. Both systems are used in commercial engines, and both have disadvantages. A further method is that of compressing the steam to a pressure higher than that in the steam chest and delivering the excess through a spring loaded valve back to the steam pipe, but the desirability of generally applying this arrangement is doubtful.[8] Another engine uses a double compression curve. The compression volume is in two parts, both of which are used during the first part of the stroke. At a predetermined point communication between these is cut off by a valve and compression is completed in the small cylinder clearance. The "spill over" in the auxiliary clearance volume is then allowed to re-expand on the other side of the piston. A special design of uniflow engine has two sets of exhaust ports and a director valve, so that each set may be

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[6] For a full discussion of this factor, see Power, August 29th, 1922.

[7] "The Una-flow Steam Engine," Second Edition, p. 13-43.

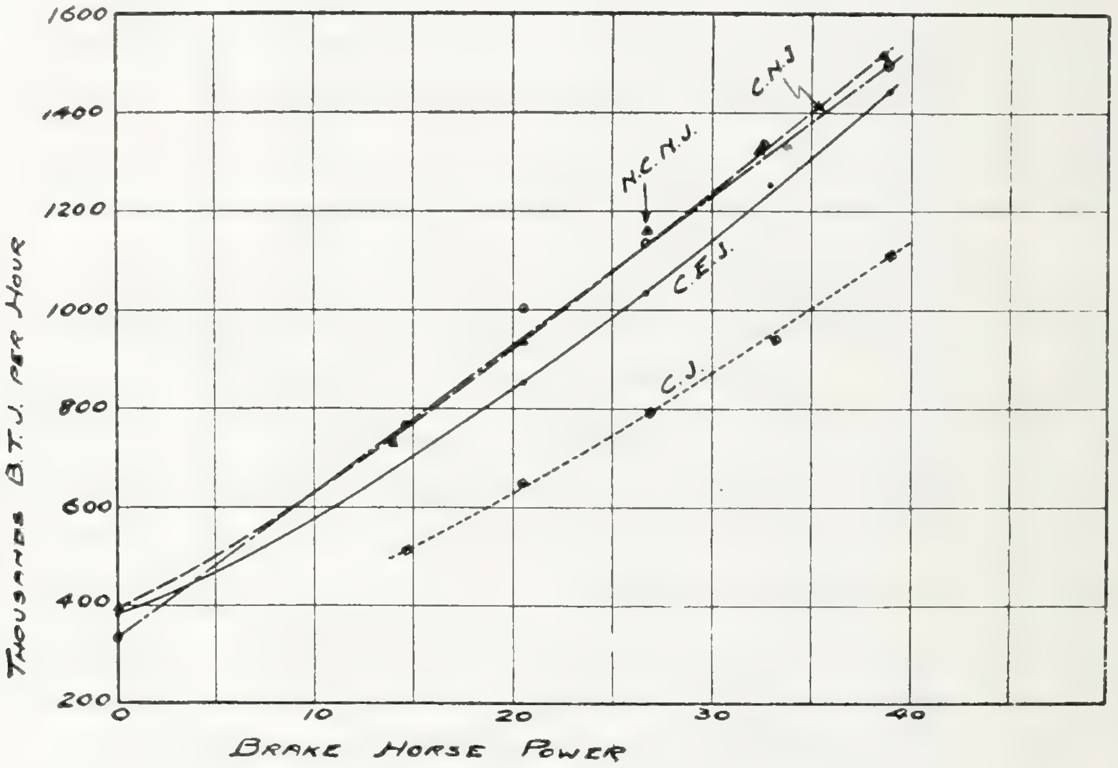

FIG. No. 4.—Heat Consumptions of 40 h.p. Uniflow Engine

TABLE No. 1—ANALYSIS OF TWENTY OF THE PRINCIPAL MAKES OF AMERICAN AND EUROPEAN UNIFLOW ENGINES

Steam admission valves	Drop	15
	Piston drop	3
	Piston	3
Jackets	End only	16
	End and body	4

Auxiliary clearance volume with automatic relief valves in most condensing engines. Mechanically operated exhaust valves with most non-condensing engines.

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[8] It has been adopted, however, in a 30- x 60-inch four-cylinder mill engine, but in this case auxiliary exhaust valves are also provided as a safety device, Power, June 22nd, 1926.

used alternately to delay the point of compression.[9] An analysis of twenty of the principal makes of American and European uniflow engines is given in Table No. 1.

The question of valve leakage is important in all engines, but is particularly so in the uniflow engine, in view of the early cut off and large pressure differences employed. This explains the almost universal use

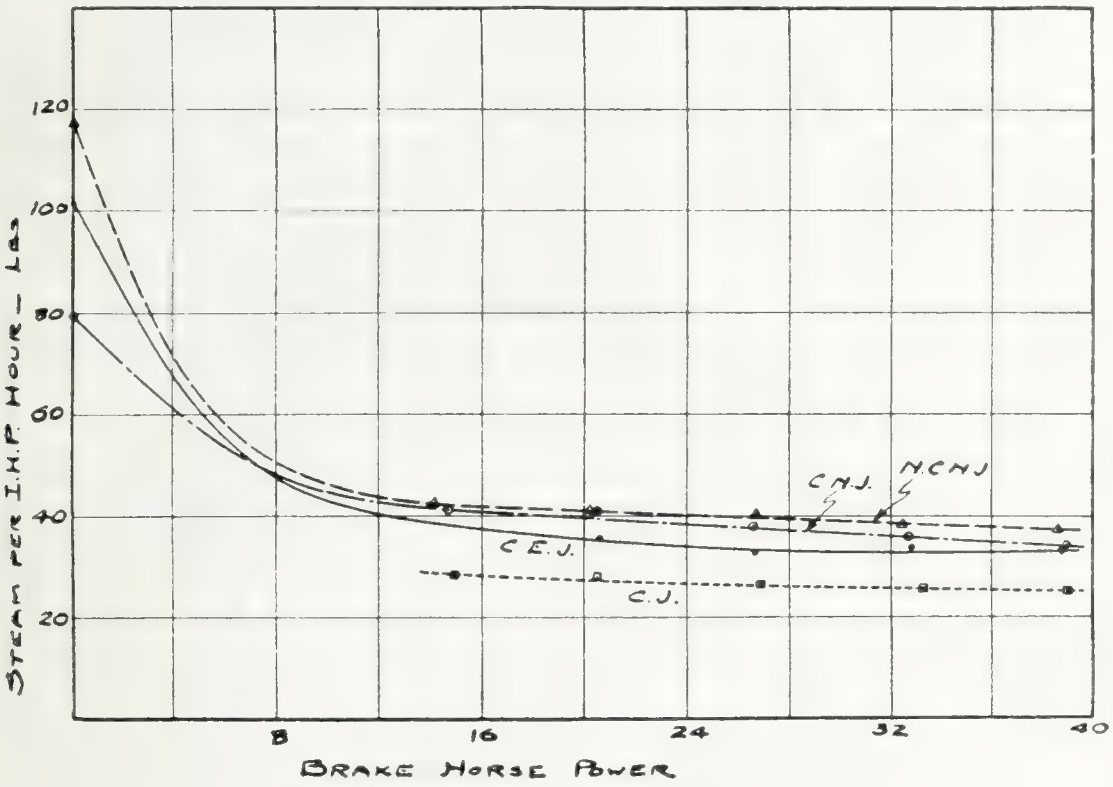

FIG. No. 5.—Steam Consumption per I.H.P. Hour of 40-h.p. Uniflow Engine

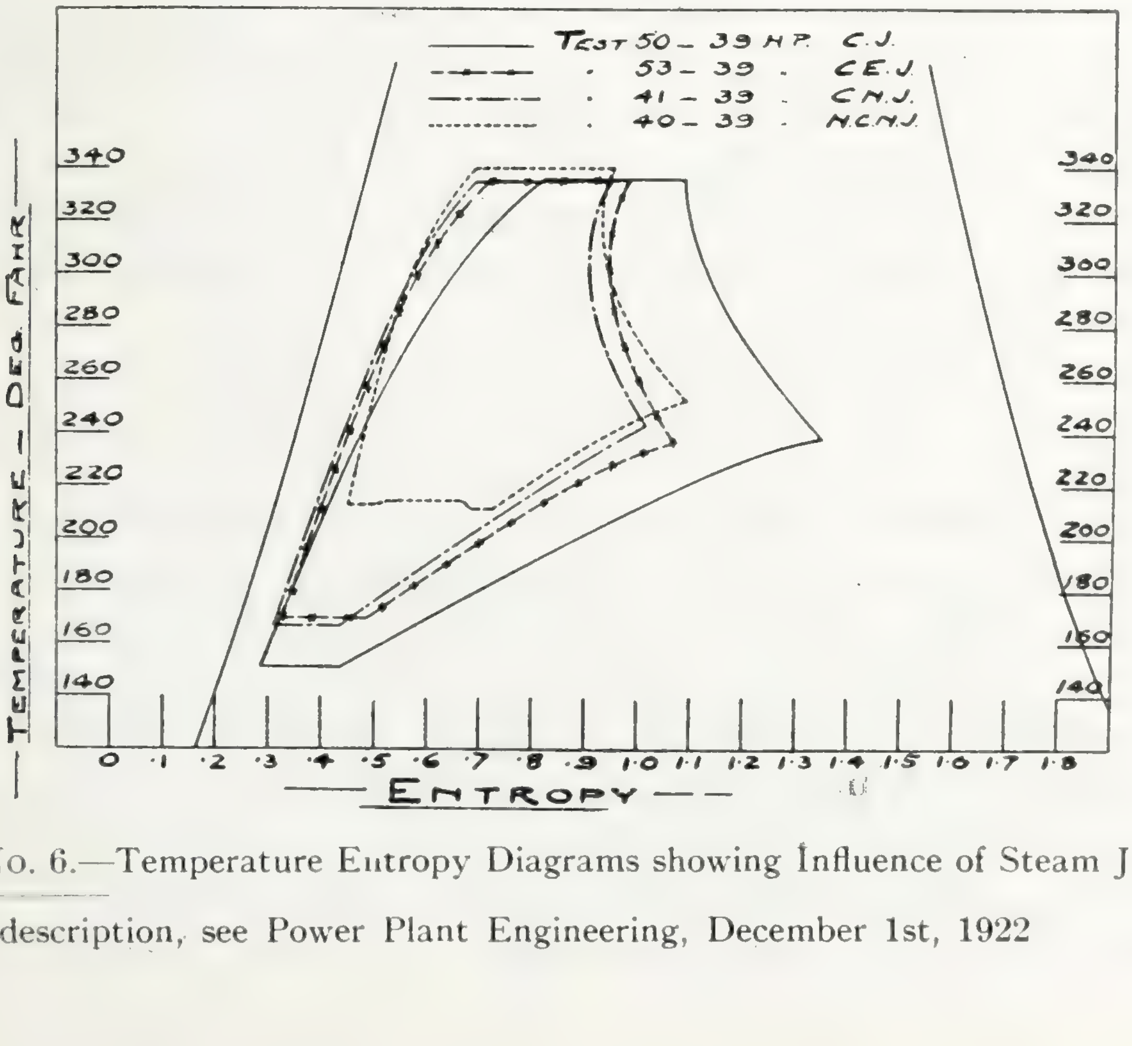

FIG. No. 6.—Temperature Entropy Diagrams showing Influence of Steam Jackets of the drop or poppet valve for large engines, the piston valve being only employed in small sizes. The double beat drop valve, however, can only be made tight for one set of pressure and temperature conditions, and therefore some form of automatic adjustment, usually of the spring type, must be provided in the valve or valve seat when high temperatures or pressures are used. Piston and Corliss valves are usually leaky, and their use results in lower thermal efficiencies.

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[9] For description, see Power Plant Engineering, December 1st, 1922

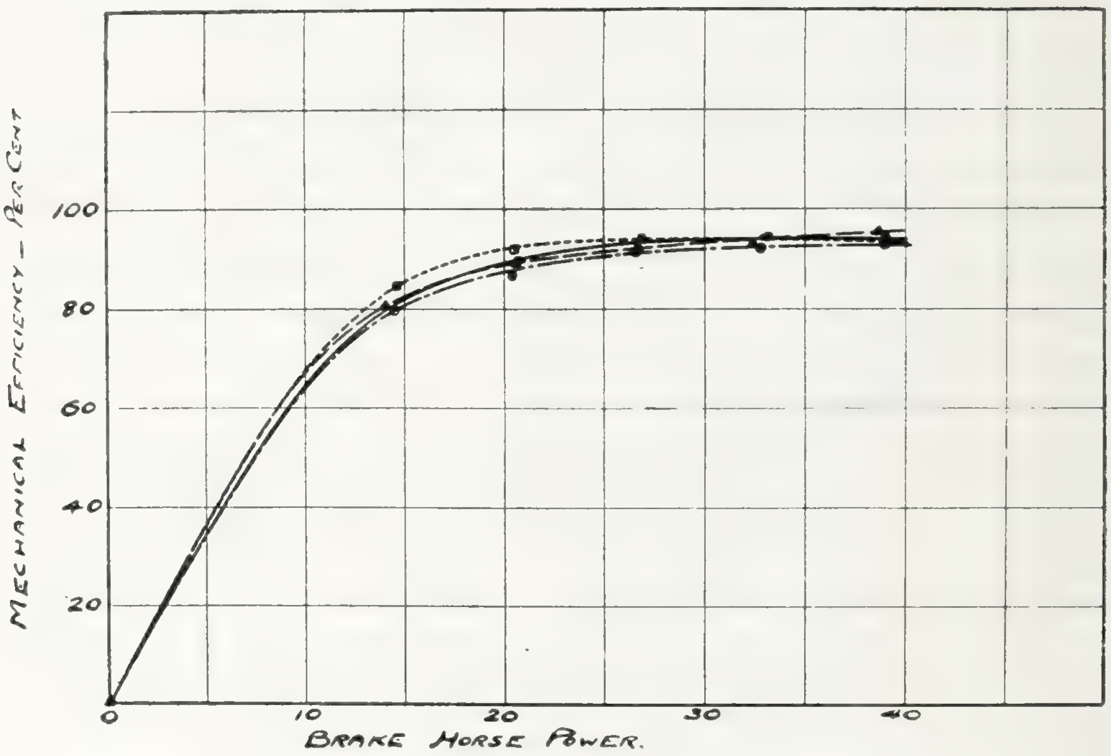

FIG. No. 7.—Mechanical Efficiency Curves of 40-h.p. Engine

Increased speeds of rotation tend to diminish the amount of surface condensation by reducing the time of the temperature cycle, and therefore the amount of heat passing through the surface. As the uniflow engine uses drop valves, heavy reciprocating parts and high ratios of expansion, it is usually a slow speed engine. Few uniflow engines of large size run at speeds exceeding 200 r.p.m., but some small engines are constructed for speeds up to 400 r.p.m.

STEAM JACKETING

Perhaps the most important of James Watt's many contributions to steam engine theory and practice was his statement of the principle that the engine cylinder must at all times be kept as hot as possible, and that cold bodies must be kept away from it. The design of the uniflow cylinder itself may be considered as a natural application of this law, not only because it produces a minimum of temperature fluctuation at each point in the cylinder, but also because it conserves the driest steam for compression. The wettest steam is usually that near the piston, and therefore is the first to be exhausted. As a direct consequence, the heat loss during compression is reduced by the lower heat conductivity of the fluid, and the steam compressed is frequently in the superheated state when the admission point is reached.

Another method of keeping the cylinder hot is to surround it wholly or partly with a steam jacket. Table No. 1 shows that every uniflow engine is supplied with end jackets and that many of them (chiefly the smaller engines), have also jackets surrounding some part of the cylinder body. The jackets do not usually extend beyond the point of cut off because cylinder condensation is practically complete at that point, and also because a long body jacket entails a constant and considerable heat leakage to the exhaust. The inefficiency of the theoretical steam jacketing cycle is indicated by figure No. 10, where the efficiency of the Rankine cycle is represented by $\frac{ABCD}{ABCEF}$. If it be assumed that the steam jacketing is such as to keep the steam dry during expansion, the extra work done is CDG, but the extra heat loss is $DGHE$, so that the resulting efficiency $\frac{ABCG}{ABCGHF}$ is lower. The work done per pound of steam is increased, but the heat loss is relatively much greater.

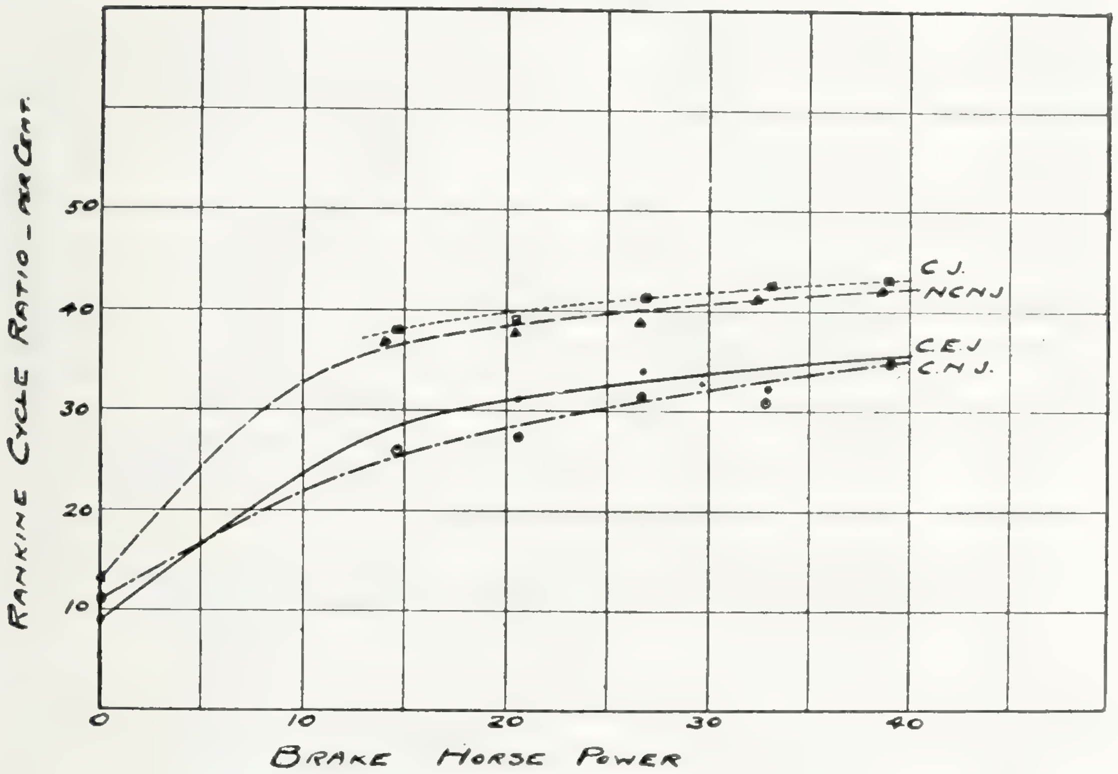

FIG. No. 8.—Rankine Cycle Ratios of 40 h.p. Engine

In practice, however, the steam jacket is frequently beneficial in transferring condensation from the inside to the outside of the cylinder. The influence of the steam jacket in reducing steam consumption naturally varies with the type of engine and condition of the steam. Tests made by Delafond (1883),[10] indicate the saving in the case of low pressure steam in a slow speed horizontal Corliss engine at Creusot. Some of these results are plotted in the form of a curve in figure No. 3. Other tests made on an engine at the Massachusetts Institute of Technology indicate the saving in compound and triple expansion engines.[11] The experiments of Callendar and Nicolson[12] at McGill University also indicate the importance of the piston face in promoting condensation. This is a factor which cannot be controlled by outside jacketing, but an attempt has recently been made in the "Prosser" engine to make the

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[10] "Thermodynamics of the Steam Engine," Peabody, p. 248-256.

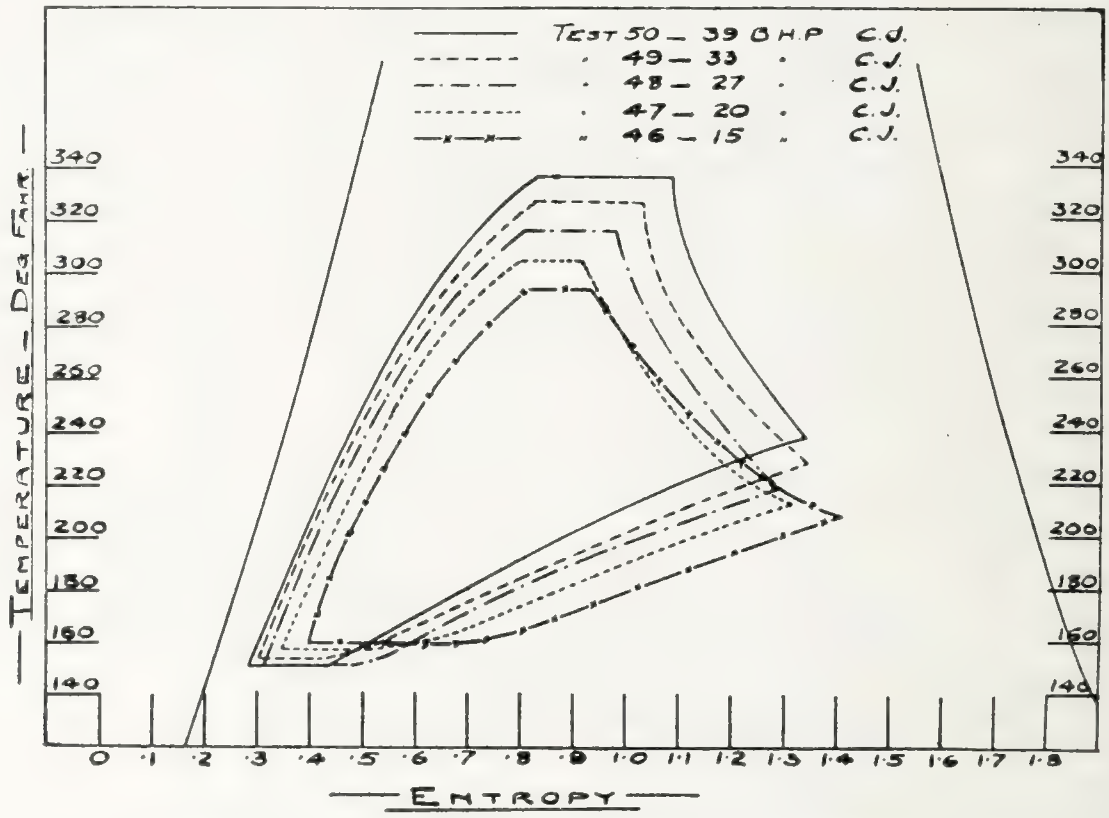

FIG. NO. 9.—Temperature Entropy Diagrams for Jacketed Tests at Different Loads

TABLE NO. 2.—PROF. STUMPF'S RESULTS ON A 23.6″ x 31.5″ SULZER UNIFLOW ENGINE RUNNING AT 155 R.P.M. ON A STEAM PRESSURE OF 120 LBS. PER SQUARE INCH GAUGE.

Mean Effective Pressure lbs. per sq. in.	Saving (per cent.) due to use of Steam Jackets		
	Saturated Steam (350° F.)	159° Superheat Steam temp. 509° F.	267° Superheat Steam temp. 617° F.
24	15.7	9.4	3.5
34	15.1	7.5	1.2
42.6	13.8	3.0	—3.5
48.3	13.0	0	—8.5

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[11] Trans., A.S.M.E., 1892-94, and Peabody, p. 261-268.

[12] Proc., Inst. C.E., vol. 132

inside of the piston a steam pocket. This is not a uniflow engine, and so will not be described in detail, but is interesting as indicating what results can be obtained with complete steam jacketing of all surfaces.[13] Rankine cycle efficiency ratios (non-condensing), of 82 to 92 per cent. are claimed for this engine, and it is stated that tests made in 1916 with and without jackets showed Rankine cycle ratios of 83.7 and 64.7 per cent., respectively, under the same test conditions.

An examination of the various test results obtained under different conditions, and of the opinions of various designers and makers of uniflow engines, indicates that apart from the fact that all recommend the use of end jackets (and produce test results to justify this), there are considerable divergencies of opinion on the subject of jacketing. Generally, it is agreed that the gain due to jacketing decreases as the cut off is made later, and that saturated steam shows more gain than superheated steam (Table No. 2).

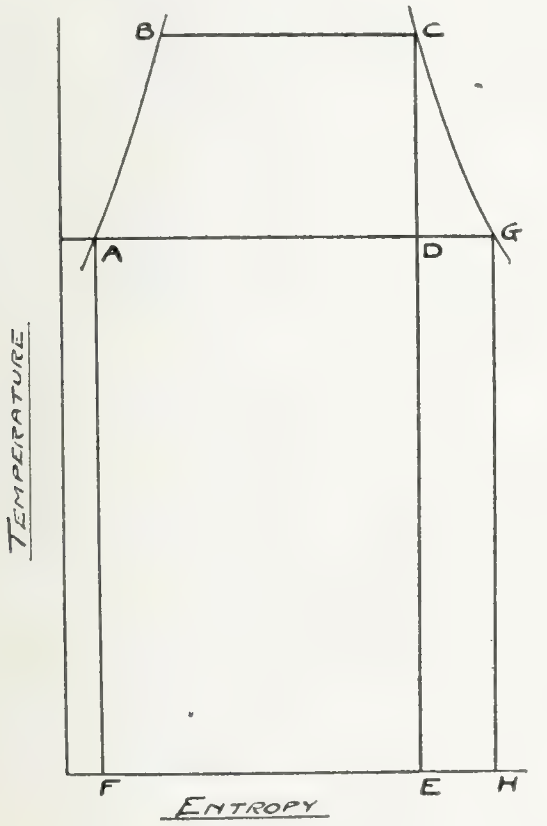

FIG. No. 10. — Temperature Entropy Diagram for Steam Jacketing Cycle

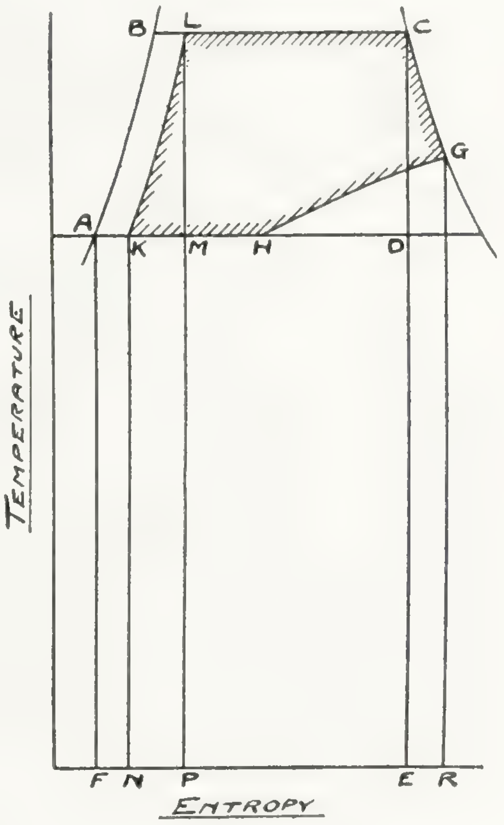

FIG. No. 11. — Temperature Entropy Diagram with Restricted Expansion

The latter is to be expected, as the heat conductivity of superheated steam is lower than that of saturated steam. Also, it may be expected that the gain due to jacketing will decrease with increasing cylinder diameter, due to the smaller ratio of surface to volume in large engines. The practically unanimous adoption of the steam chest as an end jacket seems peculiar, in view of the fact that it has generally been held as axiomatic that the steam should not pass through the jacket on its way to the engine, as the beneficial effects of transferring the condensation from the inside to the outside of the cylinder would thereby be lost. It is true that these jackets may be drained to remove the water before the steam enters the cylinder, but it is questionable whether the steam can be completely dried by this means, as it must be in a state of turbulence during the admission period.[14]

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[13] For test results, see Mechanical Engineering, April, 1925.

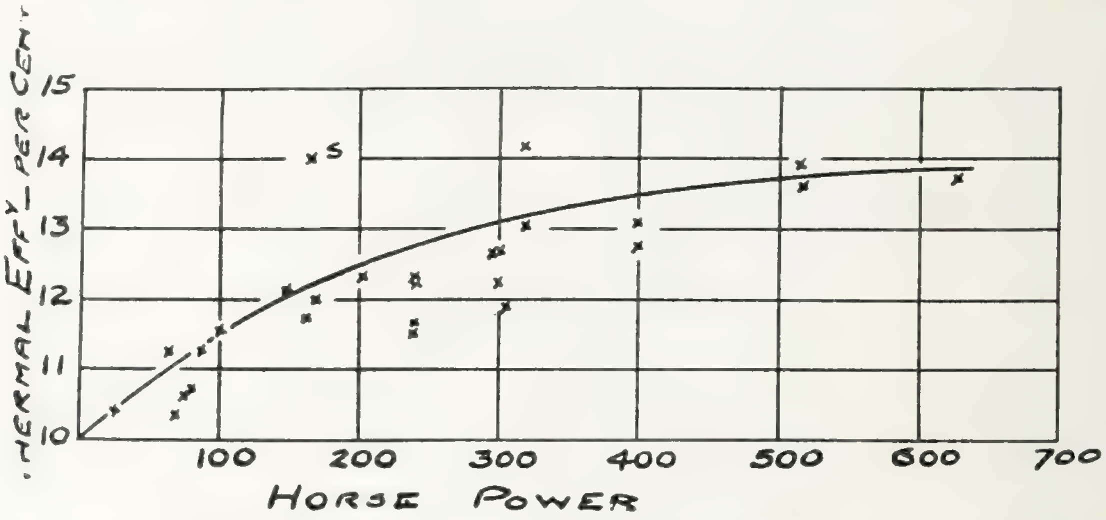

FIG. NO. 12.—Thermal Efficiencies of Non-condensing Uniflow Engines

To shed some light on these and other questions, a series of 60 tests was made on the 10- by 15-inch uniflow engine in the mechanical engineering laboratory at the University of Toronto. Different loads and jacket conditions were taken, and the results are summarized in table No. 3 and figures Nos. 4 to 9. The actual figures obtained are of no particular significance, but their relative values indicate a large saving

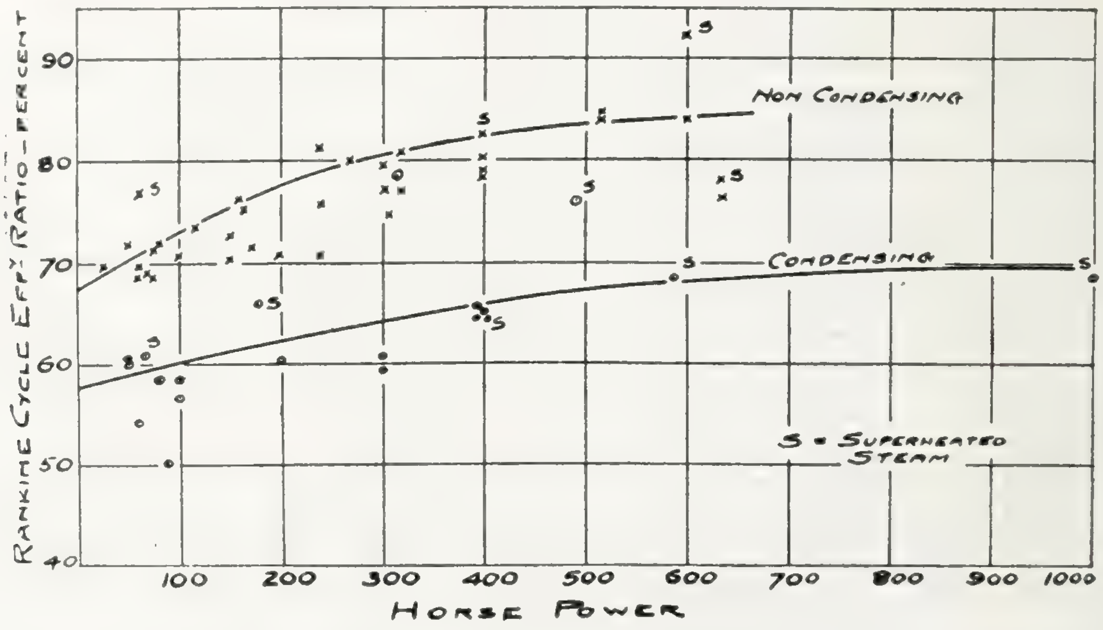

FIG. NO. 13.—Rankine Cycle Efficiency Ratios of Uniflow Engines

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[14] This is confirmed by Barrus (A.S.M.E. paper No. 1968, May, 1925), in tests on a 4-cylinder vertical uniflow engine (para. 3).

TABLE NO. 3.—TEST RESULTS ON 10 BY 15 INCH UNIFLOW ENGINE

Test No.	36	37	38	39	40	58	45	44	43	42	41	59
Speed r.p.m.	171.2	171.6	171.5	169.2	169.5	173.7	171.3	171.8	171.0	170.4	170.8	175
B.H.P.	14.1	20.4	26.7	32.5	38.7	0	14.6	20.4	26.6	32.7	38.9	0
I.H.P.	17.47	23.0	28.9	34.8	40.6	3.37	18.12	23.5	29.1	35.5	42.2	3.19
Mechanical Efficiency (per cent.)	80.6	88.9	92.3	93.4	95.1	0	80.5	86.9	91.4	92.1	92.4	0
Steam press. lbs. per sq. in gauge	121.3	120.2	119.6	119.5	119.3	120.4	122	118.6	118.4	118.2	119.4	120.4
Dryness Fraction	0.964	0.969	0.965	0.964	0.962	0.973	0.975	0.961	0.962	0.960	0.961	0.977
Steam per I.H.P hour—lbs.	42.4	41.3	40.8	38.7	37.9	117.5	41.6	41.2	38.8	36.4	34.5	79.6
Steam per B.H.P. hour—lbs.	52.55	46.6	44.2	41.4	39.9	...	51.1	47.5	42.4	39.5	37.4	...
Vacuum at engine—ins.	2.0	2.0	1.8	1.0	1.2	2.8	21.7	21.8	20.3	21.6	20.0	13.3
Rankine cycle eff'y ratio	0.37	0.38	0.39	0.414	0.422	0.165	0.26	0.275	0.32	0.31	0.35	0.10
Jacket steam per cent. of total consumption	...	...	...	...	...	...	...	...	...	...	...	...
Condition of test	Non-condensing non-jacketed (N.C.N.J.)						Condensing—non-jacketed (C.N.J.)					

TABLE NO. 3—*Cont.*

Test No.	55	54	52	53	57	46	47	48	49	50
Speed r.p.m.	172.6	172.6	171.4	170	176	172.3	172.1	172.6	173.1	171
B.H.P.	20.5	26.8	32.8	38.8	0	14.7	20.5	26.8	33.2	39.0
I.H.P.	23.0	30.6	35.0	41.4	3.06	17.3	22.3	28.6	35.2	41.9
Mechanical efficiency (per cent.)	89.0	87.5	94.0	93.7	0	84.6	92.0	93.7	94.4	93.1
Steam press. lbs. per sq. in. gauge	120.0	120.0	120.0	120.0	116.8	121.6	120.8	120.9	120.3	119.8
Dryness Fraction	0.967	0.967	0.967	0.962	0.973	0.974	0.972	0.972	0.972	0.974
Steam per I.H.P. hour—lbs.	36.0	33.4	34.7	33.9	110.5	28.9	28.5	26.9	25.9	25.6
Steam per B.H.P. hour—lbs.	40.5	37.4	37.0	36.2	...	34.1	30.9	28.7	27.5	27.5
Vacuum at engine—ins.	21.6	21.4	21.4	20.7	13.6	22.3	21.7	21.9	22.0	21.8
Rankine cycle eff'y ratio	0.313	0.34	0.325	0.354	0.09	0.38	0.39	0.41	0.43	0.43
Jacket steam per cent. of total consumption	3.1	2.47	2.18	1.6	7.33	14.0	11.0	9.2	7.8	6.6
Condition of Test	Condensing end jacket only (C.E.J.)					Condensing—Jacketed (C.J.)				

due to the body jackets, and the relative unimportance of the end jackets under these conditions. This is indicated by the curves in figures Nos. 4 and 5, which give the hourly heat consumption and steam used per i.h.p. hour, respectively, at different loads. The saving is shown still more clearly by figure No. 6, which gives the temperature-entropy diagrams under the four conditions with the same load on the engine in each case. Figure No. 7 shows that none of these changes has an appreciable effect on the mechanical efficiency, and figure No. 8 indicates that the use of jackets, when running under condensing conditions, raised the Rankine cycle efficiency ratio to a value approximate to that in non-condensing non-jacketed operation. Figure No. 9 is a temperature entropy diagram showing the effect of jacketing (condensing operation), when the load was reduced from 39 to 15 b.h.p. The increased amount of re-evaporation due to the heat received from the jackets at light loads is clearly shown, so that the dryness fraction of the steam at the end of expansion varies little at different loads.

TEST RESULTS AND EFFICIENCIES

An analysis of a large number of tests on uniflow engines of different makes and sizes confirms generally the claim that high efficiencies can be obtained, and the steam consumption curves at different loads in most cases are very flat. This clearly indicates the fact that this type of engine is well suited to varying load conditions, as the economy is usually not greatly affected when the engine is run at light loads for comparatively long periods. Practically all the tests referred to were made at steam pressures between 100 and 160 lbs. per square inch gauge, and, in view of the present tendency towards the use of higher pressures, it would be interesting to have some results on the performance of these engines under such conditions. Consumptions as low as 10 lbs. steam per i.h.p. hour with 250° F. superheat and 28 inches vacuum have been obtained in large engines (500 h.p. and over), but the number of test results available under condensing conditions and their variable natures do not justify any general conclusions on this point beyond the general observation that thermal efficiencies up to 20 per cent. have been obtained. (See Table No. 4.)

Figure No. 12 shows the thermal efficiencies obtained on a large number of engines using saturated steam and running non-condensing, the limit being about 14 per cent. in such cases. Piston speeds range from about 600 feet per minute for cylinders of 15 inches diameter to 800 feet per minute for 43-inch cylinders.

The ratio of the thermal efficiency actually obtained to that of the Rankine cycle under the same conditions (called the Rankine cycle

TABLE NO. 4.—HIGH THERMAL EFFICIENCIES OBTAINED ON UNIFLOW ENGINES USING SUPERHEATED STEAM AND HIGH VACUA

I.H.P.	R.P.M.	Steam Press. lbs. sq. in. gauge	Steam Temp. °F.	Vacuum Ins. Mercury	Steam Consumption lbs. I.H.P. hr.	Thermal Efficiency per cent.
300	155	135	617	26	8.8	23.3
317	129	157	387	26	11.0	20.8
500	121	185	627	28	10.1	19.9
402	...	140	487	26.3	10.8	20.0
588	...	142	580	26.7	10.9	19.0
1109	126	168	590	27	10.4	19.8

efficiency ratio), gives a better idea of the relative performance of such engines, particularly as far as their success in reducing cylinder condensation is concerned. The test results for engines of various sizes are plotted in figure No. 13, showing that under non-condensing conditions efficiency ratios of 70 to 85 per cent. may be expected and under condensing conditions efficiencies varying from 60 to 70 per cent. are usual. In this connection it is interesting to sum up the possibilities of the steam jacketing cycle with different clearance volumes. In figure No. 11 the Rankine cycle $ABCD$ and its efficiency $\frac{ABCD}{ABCEF}$ are again indicated. Assuming that the steam remains dry during expansion (due to the action of the steam jackets), the expansion line is CG. At G release takes place, followed by the exhaust line GHK. The line KL is taken as roughly parallel with the water line, its position varying with the clearance assumed, so that the work done is $LCGHK$ and the efficiency $\frac{LCGHK}{LCGRNK}$. Plotting these efficiencies on the basis of clearance percentages gives the line marked "actual cycle" in figure No. 14.[15] If it be assumed that compression is adiabatic along ML, the cycle efficiency is increased and follows the line marked "adiabatic cycle" in figure No. 14. These lines indicate the fall of efficiency as the clearance volume increases, and also show the closeness with which the efficiency ratio of the actual uniflow engine approaches that of the ideal cycle.

The influence of superheat in reducing steam consumption and raising thermal efficiencies has already been referred to, and under these conditions the effect of the jacketing is reduced.

APPLICATION TO PRACTICE

The uniflow engine is generally adopted in cases where moderate amounts of power are required, and the usual limit is about 1,000 to 1,200 h.p., but within recent years large units capable of generating up to 14,000 h.p. have been installed for rolling mill work.[16] For fluctuating loads of this nature, the flat steam consumption curve of the uniflow engine would seem to be very suitable, particularly as the time of the load cycle is very short (2 to 3 minutes). It is claimed that the engine will take up to 100 per cent. overload, and this flexibility is a decided asset for such work, particularly as a low speed must necessarily be used in any case.

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[15] The figures for this study were taken from the acceptance tests of a 500-h.p. engine, 140 lbs. steam pressure, atmospheric exhaust, 200 r.p.m.

[16] See Power, July 28th, 1925, and June 22nd, 1926.

The application of the uniflow engine to locomotive work hardly comes within the scope of this paper, but it is interesting to note that many experiments have been made along these lines by Professor Stumpf and others.[17] Tests on the Prussian State Railways showd an economy of 19 per cent over piston valve engines and 28 per cent. over Lentz valve locomotives. A steam wagon using a vertical uniflow engine in which the admission valves consist of stainless steel balls is also being made in England. The three cylinders are each 6¾ inches diameter and the stroke is 10 inches. The speed of the engine is 200 r.p.m.[18]

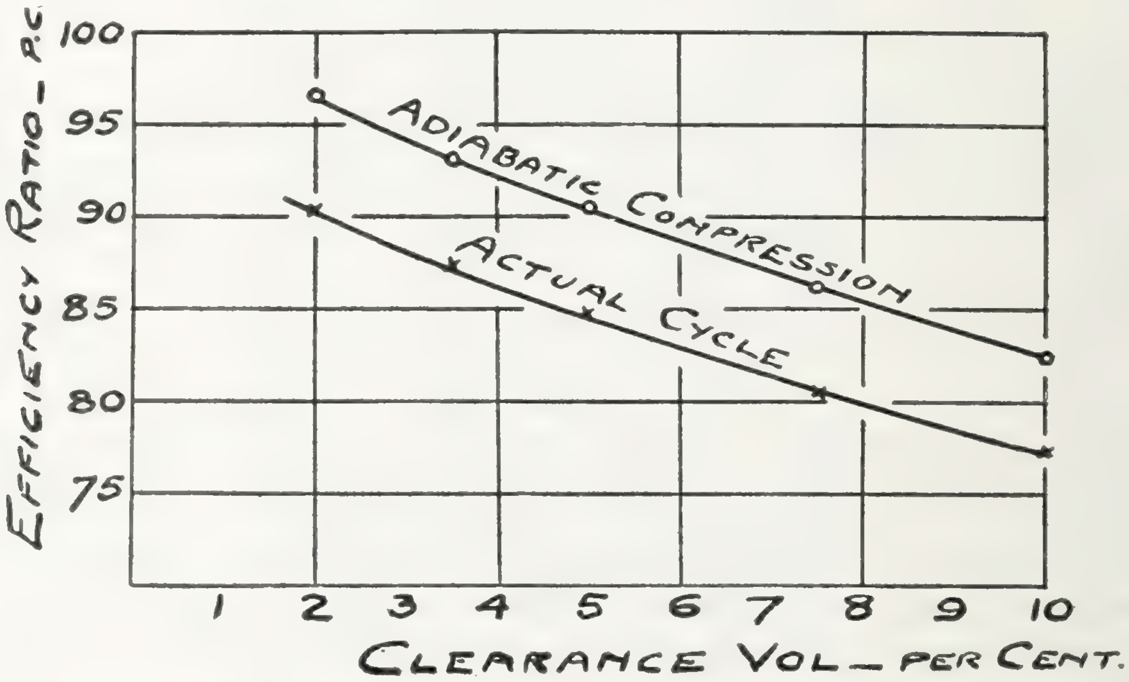

FIG. NO. 14.—Influence of Clearance on Possible Efficiency Ratios.

A single acting vertical engine with two cylinders each 4 inches diameter by 5 inches stroke is made by the Babcock-Wilcox and Goldie-McCulloch Company of Galt, Ont., for driving stokers, blowers, centrifugal pumps, etc., and on account of its simple construction (32 working parts), is claimed to be very suitable for such purposes where long hours of service and little attention are inevitable. This engine has a plain piston valve, and its maximum rating is 7.8 b.h.p. at 700 r.p.m.

Recently[19] a 1,500-h.p. three-cylinder marine engine has been put into service on the lake of Geneva. In this engine the mechanical construction of the cylinder is new and the valves are operated by oil pressure.

Multiple cylinder engines of this type have not been adopted to any considerable extent, but they offer the possible advantage of using the exhaust from one cylinder to produce suction in another. It is claimed that as much as 18 inches of vacuum can be produced in this way.

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[17] "The Una Flow Steam Engine," by Prof. J. Stumpf, 2nd ed., and Times Trade Supplement, August 10th, 1922.

[18] British Engineers Export Journal, August, 1927.

[19] Mechanical Engineering, September, 1927.

A number of compound uniflow engines have also been made, but in such cases the high pressure cylinder is usually of the counterflow type and the low pressure is uniflow.[20] Such engines are usually combined with a heat extraction or bleeding system, and where uniflow engines are used in connection with process work using bled or exhaust steam, remarkably low steam consumptions can be obtained. A new type of compound uniflow engine has recently been developed by Professor Stumpf[21] for the purpose of avoiding the high crank pin pressures obtained with large single cylinder engines. This engine has an uncommon arrangement of crank angles and it is claimed to give a very uniform torque curve and better balancing than is usually obtained in such, engines.

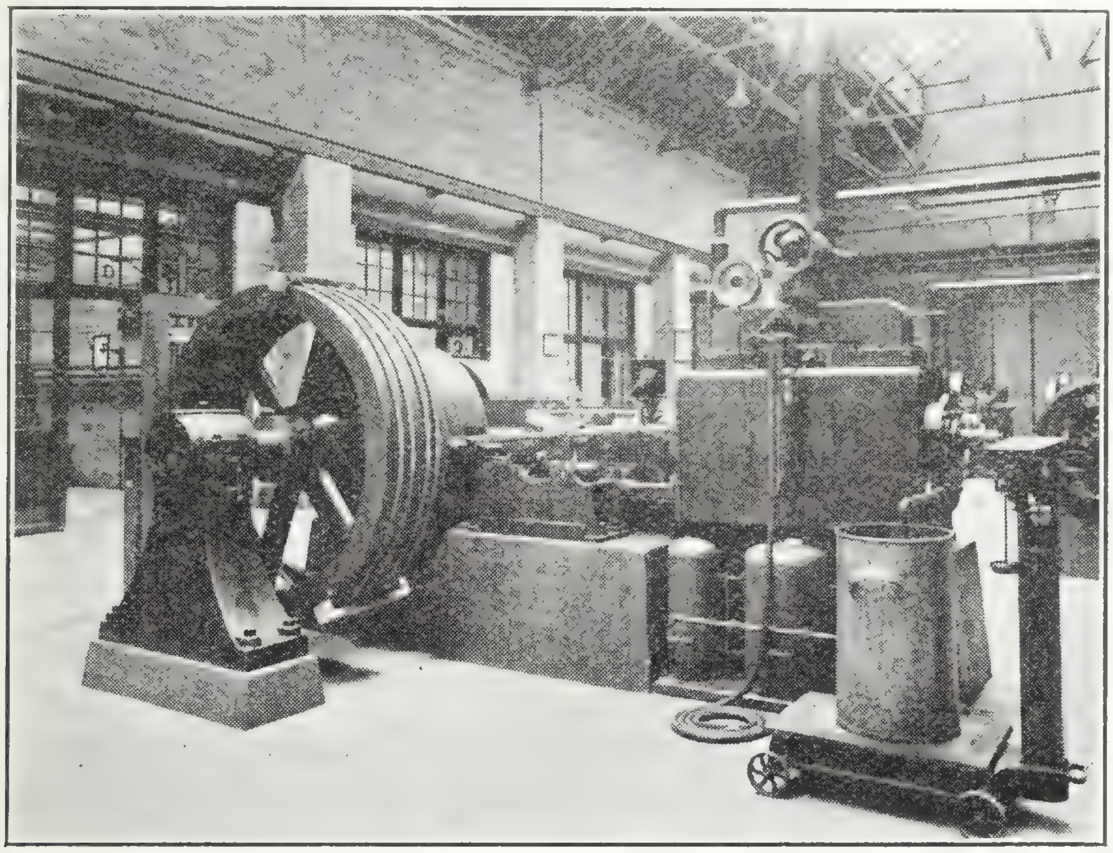

FIG. No. 15.—40-h.p. Uniflow Engine at the University of Toronto.

The other disadvantages of the uniflow engine are due primarily to the heavy reciprocating parts which introduce large inertia forces, and to the long piston, which necessitates a long cylinder or a short stroke, or both. The reduction of mean effective pressure due to the large amount of power used for compression also necessitates the use of larger cylinder diameters than are customary in counterflow engines producing the same powers.

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[20] Power Plant Engineering, November 1st, 1926; Engineering, November 10th 1922, etc.

[21] Werft Reederei Hafen, June 22nd, 1927, and Appendix.

In spite of these disadvantages, however, the reduced fuel consumption of the uniflow engine is so important a factor that this engine is likely to find a considerable market among small power users.

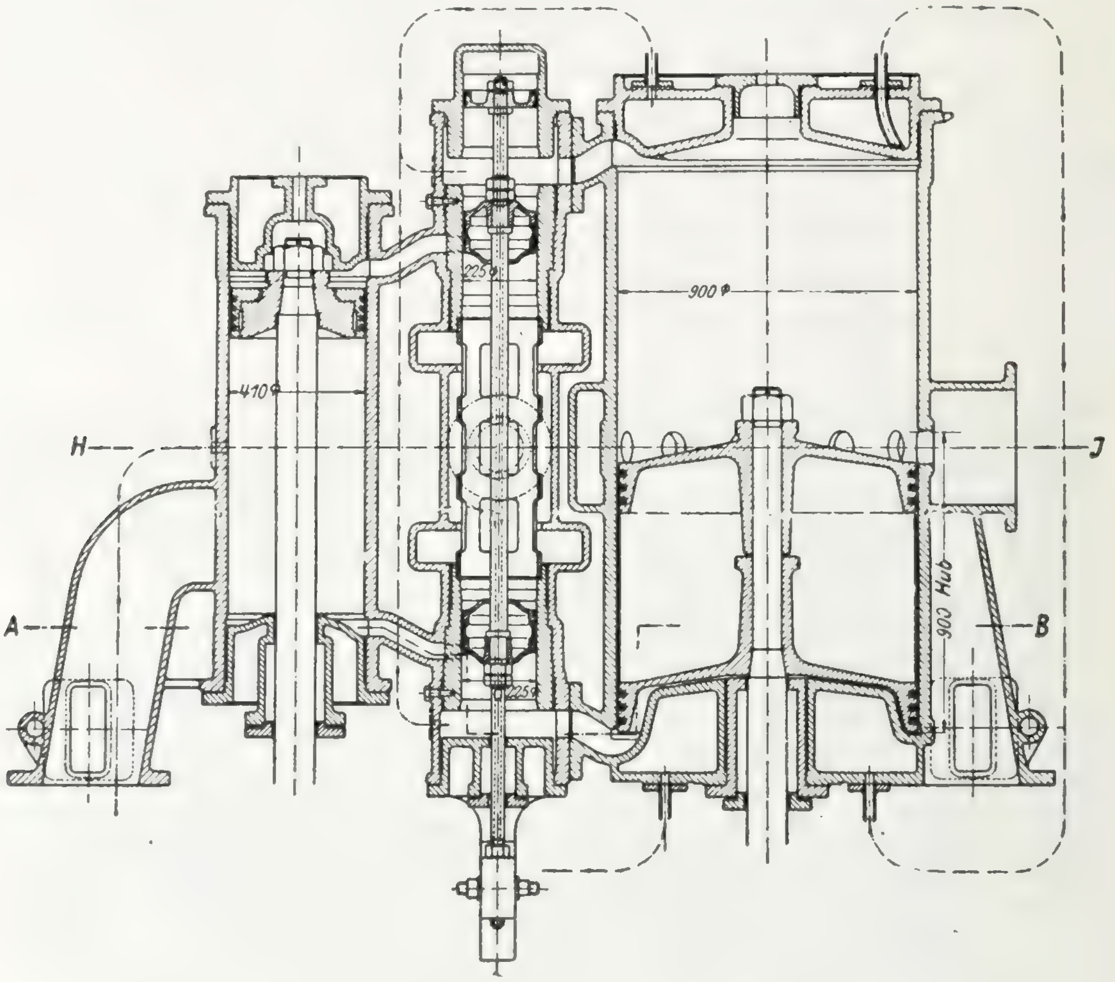

FIG. No. 16.—Compound Marine Engine (Stumpf).

The thanks of the author are due to the authorities of the University of Toronto and to Professor R. W. Angus, M.E.I.C., for providing facilities and assistance for the tests made at the University of Toronto. Also to Messrs. G. H. Harlow, S.E.I.C., B.A.Sc., H. E. Saunders, B.A.Sc., and G. L. DeLaplante, B.A.Sc., for their help in carrying out the tests and in making the necessary calculations.

APPENDIX

The latest type of compound uniflow engine designed by Professor Stumpf for marine work contains several features of interest, so that the following description may be useful. Several three-cylinder uniflow marine engines have been made by Messrs. Burmeister and Wain, and it is claimed that their steam consumption is lower than that of the ordinary triple expansion type. In single-cylinder expansion, however, particularly when the steam pressure is high, the crankpin pressure per cylinder tends to become excessive, and for this reason, and also to avoid heavy losses due to incomplete expansion, the compound system is used in the new engine.[22] (See figure No. 16.)

The diameters of the high and low pressure cylinders are 16.1 and 35.4 inches respectively and the stroke is 35.4 inches. The high pressure cylinders are made counterflow on account of the higher superheat used and the low pressure cylinders are uniflow. The steam is controlled by a single piston valve between the two cylinders, the Woolf system of compounding being used. It is claimed that the piston valve can be made as tight as the drop valve if properly designed and manufactured, and also has the advantages of simplicity, rapid motion at admission and cut off and less throttling loss.

The valve is perfectly balanced, receives live steam for the high pressure cylinder inside and exhausts to the low pressure cylinder outside the pistons. Four cylinders are used to produce 1,250 h.p., and the arrangement is so compact that the overall length of the cylinders is twelve feet. The high pressure clearance volume is eight per cent. and that of the low pressure cylinder six per cent. of the stroke volume. Cylinder volume ratios of one to five and one to seven are advised for different exhaust pressures. It is claimed that under these conditions the small pressure drops at the end of expansion in the high pressure and low pressure cylinders, respectively, produce little throttling loss and allow high vacua to be used, so that the steam consumption is expected to be below that of a combined triple expansion engine and exhaust steam turbine.

The arrangement of the cranks is also unusual. The high pressure cranks are spaced 90° apart and the respective low pressure cranks are placed 150° behind them. This arrangement is claimed to give better balancing and a more even crank effort curve than when the cranks are 180° part. Specimen curves are drawn to substantiate this.

Another smaller engine is also described in which the piston valve is placed before the high pressure cylinder and free exhaust takes place to the low pressure cylinder, the cranks in this case being 120° apart.

The trials of these engines will be watched with interest, particularly as piston valves have generally been held to be inferior in economy to drop valves.

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[22] Werft Reederei Hafen, June 22nd, 1927.

Bibliography of Uniflow Engines

1927

Sandfields Pumping Station	Engineering, April 22.
Sulzer Uniflow Marine Engine	Mechanical Engineering, Sept.
A Novel Rail Tractor	British Engineers Report Jl., Aug.
Una Flow Engines	R. Trautschold Paper Trade Jl., July 21.

1926

Bleeder Uniflow	Power, Sept. 14, p. 396-7.
Four Cylinder Condensing	Power, June 22, p. 968-72.
Uniflow for Steel Mill	Power Plant Eng., July 1, p. 746-50.
Bleeder Uniflow	Power Plant Eng., Nov. 1, p 1145-6.
Compound Tandem Heat Extraction Engine	Engineering, March 26.

1925

Developments in Uniflow Engine	Power Engr., Aug., p. 290-1.
Uniflow for Steel Mill	Power, July 28, p. 122-25.
Improvements in Uniflow Engine Design	Iron and Coal Trades Rev., May 15, p. 796-7.
Test of Uniflow Engine	Barrus, Mech. Eng., May, p. 440-1.
Uniflow Engine Practice of a Century Ago	Power, Mar. 10, p. 368-70.
Uniflow Engine Driving Compressors	Power, Dec. 15, p. 940-1.
Uniflow Engine Calculations	Power Plant Eng., Oct. 1, p. 997-8.
Uniflow and Compound Engines	Mech. Eng., March, p. 191-2.

1924

Auxiliary Exhaust Valves vs. Additional Clearance	Power, June 10, p. 937-39.
Four-Valve and Two-Valve Uniflow	Power, Aug. 19, p. 289-90.
Carmichael Uniflow	Engineering, June 20, p. 808-10.
Uniflow Engine	Inst. Mar. Eng., Mar., p. 627-42.

1923

1,000-H.P. Sulzer Engine	Power Engr., May, p. 185-88.
Principle of Uniflow Engine	Power House, Jan. 5, p. 21-23.
Watson Vertical Uniflow	Engineering, June 15, p. 742.
Philadelphia Pumping Engines	Power, Apr. 17, p. 584-7.
Uniflow Principle and Design	Power Plant Eng., Dec. 1, 15, 1922, Jan. 15, 1923.
Valve Gears for Uniflow	Power, Feb. 13.

1922

200-H.P. Galloway Engine	Power House, Sept. 20-21, p. 21-3.
Chuse Uniflow	Power, Nov. 14, p. 766-7.
Harrisburg Engine	Power, Dec. 26, p. 1024-5.
Murray Engine	Power, Oct. 17, p. 600-1.
Skinner Valve Gear	Power, Nov. 7, p. 718-19.

1922—*Continued*

Ridgeway Engine	Power, Dec. 12, p. 942-3.
Tandem Extraction Engine	Engineering, Nov. 10.
Modern Development of Uniflow Engine	Zeit fur Dampfkessel, Nov. 18-21.
Nordberg Uniflow Engine	Power, Nov. 21, p. 800-1.
Efficiency of Uniflow Engine (Skinner)	Power, Aug. 29, p. 327-9.
Uniflow Steam Engine Industry	Power, June 20, p. 960-4.
300-H.P. Uniflow Engine	Engineering, Feb. 3, p. 136-7.
Uniflow Engine Guarantees and Tests	Power, Jan. 17, p. 98-99.
Worthington Engine for Compressors	Power, Dec. 19, p. 978-9.
Filer and Stowell Engine	Power, Dec. 5, p. 880-1.
Blooming Mill Engine	Eng. Progress, Apr., p. 69-71.
Application to Locomotives	Times Trade Supp., Aug., p. 467.
Steam Wagon with Uniflow Engine	Engineering, Feb. 10.

1921

Uniflow Engine at Slough	Elec. Times, Aug. 11, p. 123-4.
Further Tests of Uniflow Pumping Engine	New Eng. Waterworks Assoc., Sept., 1920.
New Uniflow Engine Design	Power, June 28, p. 1042-3.
Straight Flow Steam Engine	Mech. Eng., Aug., p. 541-2.
200-H.P. Uniflow Steam Engine	Engineer, Aug. 26, 0p. 222-4.

1920

Discussion of Uniflow Engine	Power House, June 5 and 21, and Manchester Assoc. of Engineers.
The Uniflow Steam Engine	Proc., Inst. Mech. E., Perry.

1919

Modern Advancement in Steam Engine Prime Movers	Utah Soc. Engrs., Mar., p. 45-54.
Uniflow Pumping Engine	Can. Engr., June 19, p. 557-8.

1918

Notes on the Uniflow Steam Engine	Inst. Min. Eng., Jan.

1916

Uniflow Engines as Rolling Mill Drives	Blast F. & Steel Plant, Dec.
Characteristics of Uniflow Engine Generating Units	Elec. World, Feb. 10.
High Pressure Uniflow Engine	Sibley Jl. of Eng., April.
Uniflow Engine for Rod Mill	Iron Age, July 20.

1915

Uniflow Steam Engines	Ingenieure, April 24.
Recent Developments in the Construction of Uniflow Engine	Z.V.D.I. and Power, March 23.
The Uniflow Steam Engine	Power, April 27.
First Uniflow Engine on Pacific Coast	Power, June 1.

1914

The Una Flow Steam Engine..................Jl. Am. Soc. Naval Eng., Nov.
First Stumpf Uniflow Engine Built in America....Power, Nov. 17.
American Uniflow Engine Installation...........Prac. Engineer, Dec. 1.
Uniflow Undertype Engine....................Engineer, Dec. 4.
Universal Una Flow Engine...................Power, Dec. 1.
Developments in Construction of Uniflow Engine.Z.V.D.I., July 18.
The Uniflow Engine in Europe.................Power, Oct. 20.

1913

Recent Developments in the Reciprocating Engine Art.....................................Sci. American, July 12.
An Interesting Steam Engine Conversion........Engineer, July 18.

1912

Kerchove and Uni-Directional Flow Engines.....Z.V.D.I., Oct. 7. 1911, and Power, Apr. 16, 1912.
American Type of Uniflow Steam Engine........Power, June 11.
Direct Current Steam Engine..................Prac. Engr., Aug. 29.

1911

Improved Uni-Directional Steam Engine........Prac. Engr., Nov. 3.
The Straight Flow Steam Engine...............Power, Jan. 31.
The Stumpf Uni-Directional Flow Steam Engine..Zeit. Oest. Ing., Sept. 8.
A Double Piston Uni-Directional Flow Engine....Alliance Industrielle, July.

1910

The Stumpf Uni-Directional Flow Steam Engine..Alliance Industrielle, Oct.
The Stumpf Uni-Direction Flow Engine.........Engineering, June 10.

1906

A Heated Piston Engine.......................Machinery, N.Y., April.

THE SYSTEM POTASSIUM CARBONATE, SODIUM CARBONATE AND WATER AT 40° AND THE TRIHYDRATE OF SODIUM CARBONATE

By J. W. Bain

Professor of Chemical Engineering

Reprint from the Journal of the American Chemical Society, **49**, 2734 (1927).

An investigation of this system has been proceeding in this Laboratory for a number of years and brief statements of progress have appeared elsewhere.[1] The excellent investigations of Hill and Miller[2] have thrown a flood of light upon the various phases which exist between 20 and 40° and have explained some of the difficulties encountered in the work here; conflicting results which were met with early in our study have focused attention on the isotherm of 40°. This communication gives additional data which had escaped the notice of previous investigators.

At the outset of our work a number of determinations confirmed the accuracy of the results of Osaka[3] and of Kremann and Zitek[4] regarding the composition of the saturated solution; the residues were not analyzed and the existence of the double salt $K_2CO_3.Na_2CO_3.12H_2O$ was assumed, instead of the solid solution $(K_2Na_2)CO_3.6H_2O$ found by Hill and Miller. In order to be well beyond the transition point of the supposed double salt, the temperature of 40° was selected for the next isotherm. An inspection of the solubility curves for potassium and sodium carbonates showed that there was only a slight variation in solubility in both cases with a change of temperature of several degrees, and for this reason an ordinary thermometer divided to single degrees was considered sufficiently accurate to show the temperature of the bath; this choice gave much trouble subsequently.

Experimental Methods

The carbonates used were of c. p. quality and each lot was analyzed before use; the samples conformed to the usual standards. Weighed quantities of the components were placed in L-shaped glass tubes which were rocked to and fro in the bath, the movement of a large bubble of air producing a stirring action. From time to time the tubes were removed and shaken vigorously to break up crusts, and in some instances the solid phase was filtered off, ground in a mortar and returned to the tube. The samples were withdrawn by a weighing pipet and, after dilution, the sodium and potassium were determined as chlorides in one portion, an operation in which wide glass tubes with bent-up, drawn-out ends[5] were almost indispensable; in another portion potassium was determined as perchlorate. All determinations were made in duplicate and the agreement was excellent.

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[1] (a) Bain and Oliver, *Trans. Roy. Soc. Canada*, **10**, III, 65-66 (1916); (b) Bain, **18**, III, 273-274 (1924).

[2] Hill and Miller, This Journal, **49**, 669 (1927).

[3] Osaka, *Mem. Coll. Sci. Eng. Kyoto Imp. Univ.*, **3**, 51 (1911).

[4] Kremann and Zitek, *Monatsh.*, **30**, 323 (1909).

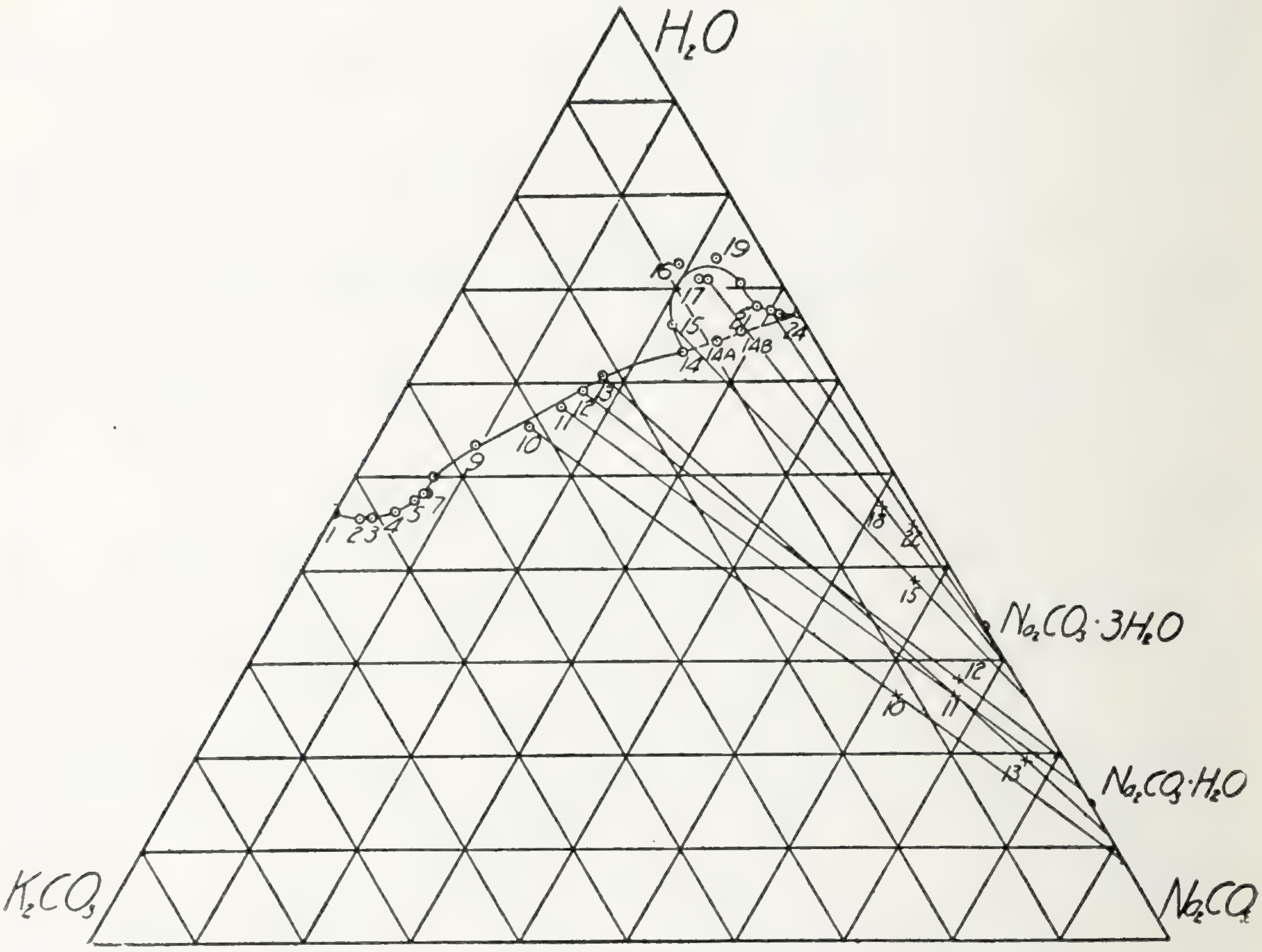

Fig. 1.

The solid phases were filtered off, sucked dry, dissolved and analyzed as above. Hill and Miller's method of centrifuging, which was only published toward the conclusion of our work, has been tried and is undoubtedly a great improvement. During the course of the investigation sixty-seven separate experiments have been carried out, the large number being due to the difficulty in securing solutions in equilibrium with a stable phase; from these the following results have been selected; temperature, 40°.

The curves lying to the left of Point 14 were thoroughly established by April, 1924, but no concordant results could be obtained in the remainder of the field. Variable results from solutions of the same composition led to the conclusion that equilibrium was only very slowly attained and, consequently, the period of shaking was increased first

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[5] Ostwald-Luther, "Physiko-chemischer Messungen," 4th edition, p. 309.

TABLE I

RESULTS OF EXPERIMENTS

	Solution		Residue		Sp. gr.,	Days	Solid
No.	K_2CO_3	Na_2CO_3	K_2CO_2	Na_2CO_3	40°/20°	shaken	phase†
1	53.9	....	....	....	....	..	$K_3/_2$
2	51.9	2.7	....	....	1.582	7	$K_3/_2$
3	50.7	3.8	....	....	1.581	7	$K_3/_2$ + KN
4	48.1	5.6	....	....	1.586	5	KN
5	45.8	6.8	....	....	1.563	15	KN
6	44.7	7.2	....	....	1.557	89	KN + N_1
7	44.2	7.2	....	....	1.567	6	KN + N_1
8	42.8	7.2	....	....	1.550	6	N_1
9	37.1	9.5	....	....	1.508	56	N_1
10	31.1	13.6	11.8	62.0	1.496	58	N_1
11	27.1	15.5	6.6	67.5	1.451	61	N_1
12	24.3	16.7	5.0	67.2	1.424	94	N_1
13	21.6	17.8	3.6	77.3	1.447	99	N_1
14	12.9	24.0	....	....	1.387	91	N_1 + N_3
14A	9.1	26.6	....	....	1.374	15	N_1
14B	6.2	28.5	....	....	1.366	47	N_1
15	12.2	21.6	4.0	57.5	1.341	104	N_3
16	8.4	19.0	....	....	1.441	107	N_3
17	7.2	21.6	....	....	1.290	111	N_3
18	6.6	22.5	2.6	50.7	1.291	113	N_3
19	4.8	22.1	....	....	1.272	113	N_3
20	3.7	25.8	....	....	1.292	115	N_3
21	3.6	28.6	....	....	....	34	N_3
22	2.3	30.0	0.8	54.6	1.328	120	N_3
23	1.8	31.1	....	....	1.326	12	N_3
24	....	32.8	....	....	....	..	N_3

†For convenience, the symbols of Hill and Miller have been used. $K_3/_2 = K_2CO_3 \cdot 3/2H_2O$; $KN = K_2CO_3 \cdot Na_2CO_3$; $N_1 = Na_2CO_3 \cdot H_2O$; $N_3 = Na_2CO_3 \cdot 3H_2O$. The solid phases in Nos. 3 to 7 were not determined and are assigned on the basis of Hill and Miller's isotherm at 36°.

to a fortnight and then to greater lengths as indicated in the table. The conclusion was finally reached that the temperature of the bath oscillated about the temperature of a transition point, and that a new hydrate of sodium carbonate was present. It is very easy to obtain solutions supersaturated with respect to the monohydrate and of the numerous results which were obtained in consequence, numbers 14A and 14B are placed on the diagram upon a dotted line indicating the metastable prolongation of the curve stretching from 7 to 14.

As is shown in the diagram, the projection of the tie lines between the points 14, 18 and 22, indicating the composition of the solution and of the moist residue, intersect the side of the triangle at very obtuse angles and consequently little reliance can be placed upon them. It was, therefore, necessary to investigate the composition of the solid phase in equilibrium with solutions lying to the right of point 14 by some other method.

A sample of sodium carbonate which had been shaken with an amount of water sufficient only to partly dissolve it, was filtered in a tube with a Jena glass disk surrounded by water at 42°; after a few moments' suction the moist salt was washed with a very small quantity of ether, poured on dropwise, and was then rapidly removed to a stoppered weighing bottle.

Anal. 1.0702 g. titrated with 0.9562 *N* HCl, using methyl orange, required 14.0 cc., $=0.7102$ g. Na_2CO_3, whence $Na_2CO_3:H_2O=1:2.98$. 0.6768 g. heated at 270-300° gave 0.4507 g. Na_2CO_3, whence $Na_2CO_3:H_2O=1:2.95$.

To confirm this result, 50.0 g. of Na_2CO_3, 4.0 g. of NaCl and 60.18 g. of H_2O were shaken for a number of days at 41°; in the filtered solution there were found: 0.6771 g. of NaCl $=0.5045$ g. of $Na_2CO_3+0.1202$ g. of NaCl in 1.9355 g. of solution, on the assumption that all the NaCl present was in solution; the NaCl alone $=0.1202$ g.; H_2O as solution $=$ 43.64 g.; Na_2CO_3 in solid phase, 33.20 g.; hence $Na_2CO_3:H_2O=1:2.93$.

Samples of the solid phase were also centrifuged as suggested by Hill and Miller and it was found after a number of trials that great care had to be taken to prevent the sample from falling below 40°, since otherwise the results were unsatisfactory.

Anal. 0.5479 g. of the centrifuged salt lost on heating 0.1850 g. of H_2O, whence $aN_2CO_3:H_2O=1:3.02$.

The results of these independent methods, taken together with the intersections of the tie lines on the $Na_2CO_3.H_2O$ side of the triangle, are evidence of the existence of a trihydrate of sodium carbonate, the solubility of which is represented by the curve from Points 14 to 24. Solutions prepared by agitating sodium carbonate for a few hours at 40° or over are saturated with respect to the metastable monohydrate but are supersaturated with respect to the stable trihydrate; since the change from one form to the other is slow, the investigators of the solubility of this salt, even such careful and experienced workers as Wells and McAdam,[6] have overlooked the existence of the trihydrate. The present data on solubility above 40° are undoubtedly in error and will be further investigated in this Laboratory.

It is not yet possible to state definitely how long a sample must be shaken at 41° or slightly higher in order that the complete conversion to the trihydrate may be assured, but at present it is believed that not less than six days should be allowed.

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[6] Wells and McAdam, THIS JOURNAL, **29,** 721 (1907).

An attempt was made to determine with some precision the transition point by the thermal method, using the procedure of Richards.[7] A series of runs was made with slowly rising and falling temperatures which yielded definite indications of a transition point although the constancy of temperature lasted only for a few minutes. The change of one hydrate into the other proceeds slowly, and the existence of the monohydrate above the transition point for days as a metastable form is indicative of this sluggishness; the trihydrate appears to pass over into the monohydrate more rapidly but the rate is still low. The transition of the trihydrate into the monohydrate is accompanied by the evolution of heat. When these difficulties had developed during the first few runs, attempts were made to obtain more favourable conditions, that is, the monohydrate prepared at 37° was added to the trihydrate at 41° and the mixture slowly cooled and then heated again. The converse procedure was also tried but no method of accelerating the change from one hydrate to the other could be found. In all, seventeen runs were made; the arrests with rising temperature occurred from 39.60 to 39.98° and on cooling at 39.97 to 39.85°. The transition temperature is, therefore, assumed for the present to be 39.8°±0.1°.

The compound $Na_2CO_3.3H_2O$ has not been previously described except as associated with the efflorescent soda in the dried river beds of the Cordilleras,[8] and as a deposit from hot springs near Saxby River, Queensland.[9]

The greater portion of the experimental work in this investigation has been carried out by a number of students, whose assistance is gratefully acknowledged.

SUMMARY

1. The system $K_2CO_3+Na_2CO_3+H_2O$ has been studied at 40°.

2. The existence of the trihydrate of sodium carbonate has been proved.

3. The transition point in the system $Na_2CO_3.3H_2O-Na_2CO_3.H_2O$ is 39.8±0.1°.

4. The solubility data for sodium carbonate at 40° and for some unknown higher range in temperature, as at present recorded, refer to the composition of solutions in equilibrium with the metastable monohydrate and require revision.

TORONTO, CANADA

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[7] Richards, *Am. J. Sci.*, [4] **6**, 201 (1898).

[8] Schickendantz, *Ann.*, **155**, 359 (1870).

[9] Daintree, *Quart. J. Geol. Soc.*, **28**, 285 (1872).

THE ELASTIC CATENARY APPLIED TO BASE MEASUREMENT

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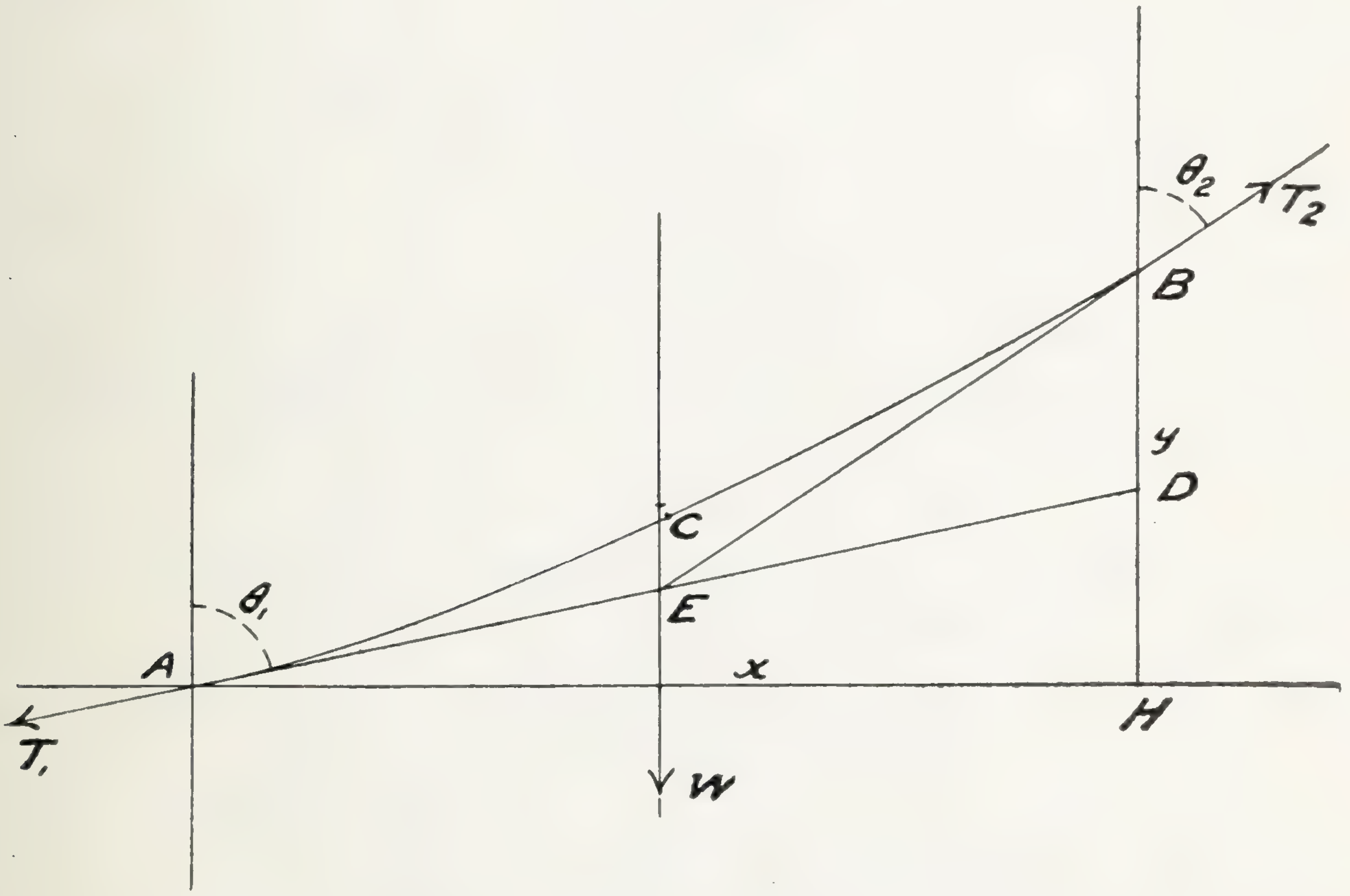
In a former article, published in Bulletin No. 7 of the School of Engineering Research, the author of the present article gave an account of a method of base line measurement which requires no further equipment than that used by the engineer in his ordinary practice. For the details of the method reference may be made to that article. In the derivation of the formulæ for the reduction of the field measurements the weight per unit of length of the tape was assumed to be constant, notwithstanding the fact that the tension to which it is subjected varies continuously from point to point throughout its length, when stretched in the usual way between two supports in measuring a base. The error arising from this assumption is extremely small.



In the investigation which follows, however, the tape is considered to be extensible, in accordance with Hooke's law, account being taken of the varying tension throughout the length of the tape. The weight per unit of length is therefore assumed to vary continuously, and to be at any point inversely proportional to the tension at that point. A constant tension is assumed to be applied at the forward end of the tape, in measuring a base, the two ends resting on posts or tripods; and formulæ are derived by which the length of the projection of the tape on a horizontal plane may be computed, thus correcting in one operation for the combined effect of extension, sag, and grade,

General Equations.—In the figure, AB represents a tape, stretched as in measuring a base. It is in equilibrium under the action of the tensions T_1 and T_2, applied at its extremities, and its weight W acting vertically through its centre of gravity. The directions of these three forces pass through some point E. θ_1 and θ_2 denote the angles which T_1 and T_2 make with the vertical direction, and x and y the rectangular coordinates of B referred to horizontal and vertical axes through A as origin.

The sides of the triangle $B\ E\ D$ are parallel to, and, taken clockwise, in the same sense as, the three forces T_1, T_2 and W; that triangle then gives at once the relations,

$$T_1 = W \frac{\sin \theta_2}{\sin (\theta_1 - \theta_2)} \tag{1}$$

$$T_2 = W \frac{\sin \theta_1}{\sin (\theta_1 - \theta_2)}\ ; \tag{2}$$

and from these we have:

$$T_1 \sin \theta_1 = T_2 \sin \theta_2 \tag{3}$$

If T denote the tension at any point of the tape, and θ its direction at that point with reference to the vertical, we should obtain in a similar manner the relation:

$$T_1 \sin \theta_1 = T \sin \theta;$$

whence it follows that the horizontal component of the tension at any point of the tape is constant. Denoting it by τ, we have:

$$\tau = T \sin \theta = W \frac{\sin \theta_1 \sin \theta_2}{\sin (\theta_1 - \theta_2)}\ , \tag{4}$$

$$= \frac{W}{\cot \theta_2 - \cot \theta_1} \tag{5}$$

Similarly, for a point C, which divides the weight of the tape equally, we have,

$$\tau = \frac{1}{2} \frac{W}{\cot \theta' - \cot \theta_1}\ ,$$

θ' denoting the angle which the tape at that point makes with the vertical, from which we have:

$$2(\cot\theta' - \cot\theta_1) = \cot\theta_2 - \cot\theta_1,$$

or,

$$\cot\theta' = \tfrac{1}{2}(\cot\theta_1 + \cot\theta_2) \tag{6}$$

Again, if T_m denote the mean of the end tensions, and θ_m the corresponding direction of the tape, we have by (1) and (2),

$$T_m = \tfrac{1}{2}W\frac{\sin\theta_1 + \sin\theta_2}{\sin(\theta_1 - \theta_2)}\ ; \tag{7}$$

$\therefore$ by (3),

$$\frac{\tau}{T_m} = \frac{T_m \sin\theta_m}{T_m} = \frac{2\sin\theta_1, \sin\theta_2}{\sin\theta_1 + \sin\theta_2},$$

or,

$$\sin\theta_m = \frac{2}{\operatorname{cosec}\theta_2 + \operatorname{cosec}\theta_1},$$

or,

$$\operatorname{cosec}\theta_m = \tfrac{1}{2}(\operatorname{cosec}\theta_1 + \operatorname{cosec}\theta_2) \tag{8}$$

Resolving vertically the three concurrent forces T_1, T_2 and W, we find

$$T_2\cos\theta_2 - T_1\cos\theta_1 = W; \tag{9}$$

then, applying this to the point C, we have,

$$T'\cos\theta' - T_1\cos\theta_1 = \tfrac{1}{2}\ W,$$

and,

$$T_2\cos\theta_2 - T'\cos\theta' = \tfrac{1}{2}\ W; \tag{10}$$

whence,

$$T'\cos\theta' = \tfrac{1}{2}(T_1\cos\theta_1 + T_2\cos\theta_2). \tag{11}$$

From (9), (10), and (3),

$$\begin{aligned} 2\ W\ T'\cos\theta' &= T_2^2\cos^2\theta_2 - T_1^2\cos^2\theta_1\ , \\ &= T_2^2 - T_1^2\ , \end{aligned}$$

or,

$$T'\cos\theta' = \frac{T_2^2 - T_1^2}{2W} \tag{12}$$

From (10) and (12) we have,

$$\begin{aligned} T_1\cos\theta_1 &= \frac{T_2^2 - T_1^2}{2W} - \tfrac{1}{2}W = \frac{T_2^2 - T_1^2 - W^2}{2W}\ , \\ T_2\cos\theta_2 &= \frac{T_2^2 - T_1^2}{2W} + \tfrac{1}{2}W = \frac{T_2^2 - T_1^2 + W^2}{2W}\ ; \end{aligned} \tag{13}$$

and, from the first of these,

$$T_2^2 - T_1^2 = W^2 + 2\ W\ T_1\cos\theta_1\ . \tag{14}$$

Similarly, for the point C, we have,

$$T'^2 - T_1^2 = \tfrac{1}{4}W^2 + WT_1 \cos \theta_1 \ ,$$

$$T_2^2 - T'^2 = \tfrac{1}{4}W^2 + WT' \cos \theta' \ . \qquad (15)$$

Derivation of expressions for x and y, and the length s of the tape—

From (1) and (2) we have,

$$T_2 - T_1 = W \frac{\sin \theta_1 - \sin \theta_2}{\sin (\theta_1 - \theta_2)} \ , \qquad (16)$$

which, if applied to an element of the curve, gives

$$dT = d\,W \, \frac{\sin \theta - \sin (\theta - d\theta)}{\sin d\theta} \ ,$$

$$= w.\ ds \, \frac{d \ \sin \theta}{d\theta} = w.\ ds \cos \vartheta,$$

$$= w.\ dy \ , \qquad (17)$$

in which w denotes the weight of a unit of length of the tape at the point. To find an expression for w, let—

s_0 denote the natural or unextended length of the tape,

w_0 the corresponding weight of a unit of length, and

e the extension of a unit of length due to a unit tension;

then ds_0 becomes $ds_0\ (1+e\ T)$, due to the tension T;

$$\therefore \qquad w_0 ds_0 = w.\ d\ s_0\ (1+e\ T)$$

or

$$w = \frac{w_0}{1+e\ T} \ . \qquad (18)$$

Equation (17) then becomes,

$$dT = \frac{w_0 dy}{1+eT}$$

or

$$w_0\ dy = (1+eT)\ dT \ .$$

Integrating, we have,

$$w_0 y = \int_{T_1}^{T_2} (1+eT)\ dT,$$

$$= T_2 - T_1 + \tfrac{1}{2}e\ (T_2^2 - T_1^2) \ ,$$

$$= (T_2 - T_1) \ \left\{ \ 1 - \tfrac{1}{2}e\ (T_2 + T_1) \ \right\} \ , \qquad (19)$$

$$=(T_2-T_1)\ (1+eT_m),$$

or

$$y=\frac{1+eT_m}{w_0}\ (T_2-T_1),$$

$$=\frac{T_2-T_1}{w_m}\ , \tag{20}$$

or

$$T_2-T_1=w_m\ y\ ,$$

by (18), where w_m denotes the weight of a unit of length of the tape due to the tension T_m.

Again, we have,

$$dx=\tan\theta\ dy.$$

Also, from (4),

$$\sin\ \theta=\frac{\tau}{T}$$

Also,

$$\tan\ \theta=\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}=\frac{\tau}{\sqrt{T^2-\tau^2}}\ ;$$

and by (17) and (18), $dy=\frac{dT}{w}=\frac{(1+eT)\ dT}{w_0}.$

Therefore,

$$dx=\frac{\tau}{\sqrt{T^2-\tau^2}}\ .\ \frac{(1+eT)\ dT}{w_0}$$

$$=\frac{\tau}{w_0}\cdot\frac{(1+eT)\ dT}{\sqrt{T^2-\tau^2}}\ ;$$

and ∴

$$x=\frac{\tau}{w_0}\int_{T_1}^{T_2}\frac{(1+eT)\ dT}{\sqrt{T^2-\tau^2}}$$

$$=\frac{\tau}{w_0}\left\{\ \log\frac{T_2+\sqrt{T_2^2-\tau^2}}{T_1+\sqrt{T_1^2-\tau^2}}+e\sqrt{T_2^2-\tau^2}-e\sqrt{T_1^2-\tau^2}\right\}. \tag{21}$$

But,

$$\frac{\tau}{w_0}=\frac{T_1\ \sin\ \theta_1}{w_0}=\frac{W}{w_0}\frac{\sin\theta_1\sin\theta_2}{\sin\ (\theta_1-\theta_2)}$$

$$=s_0\ \frac{\sin\theta_1\sin\theta_2}{\sin\ (\theta_1-\theta_2)}\ \ ; \tag{22}$$

and,

$$\sqrt{T_2^2-\tau^2}=\sqrt{T_2^2-T_2^2\ \sin\ ^2\theta_2}=T_2\ \cos\ \theta_2,$$

and,

$$\sqrt{T_1^2-\tau^2}=T_1\ \cos\ \theta_1.$$

Therefore,
$$\frac{T_2+\sqrt{T_2^2-\tau^2}}{T_1+\sqrt{T_1^2-\tau^2}}=\frac{T_2(1+\cos\theta_2)}{T_1(1+\cos\theta_1)}=\frac{\sin\theta_1\cos^2\frac{1}{2}\theta_2}{\sin\theta_2\cos^2\frac{1}{2}\theta_1}$$
$$=\frac{\tan\frac{1}{2}\theta_1}{\tan\frac{1}{2}\theta_2}.$$

Also,
$$e(\sqrt{T_2^2-\tau^2}-\sqrt{T_1^2-\tau^2})=e(T_2\cos\theta_2-T_1\cos\theta_1).$$
$$=eW,$$

by (9) Therefore,

$$x=s_0\frac{\sin\theta_1\sin\theta_2}{\sin(\theta_1-\theta_2)}\left(\log\frac{\tan\frac{1}{2}\theta_1}{\tan\frac{1}{2}\theta_2}+eW\right). \tag{23}$$

This may also be written.

$$x=\frac{\tau}{w_0}\log\frac{\tan\frac{1}{2}\theta_1}{\tan\frac{1}{2}\theta_2}+e\tau s_0; \tag{24}$$

or
$$x=s_0\frac{\tau}{W}\log\frac{\tan\frac{1}{2}\theta_1}{\tan\frac{1}{2}\theta_2}+e\tau s_0. \tag{25}$$

Again, we have

$$dx=\frac{\tau}{w_0}\frac{(1+eT)dT}{\sqrt{T^2-\tau^2}}\qquad dy=\frac{(1+eT)\,dT}{w_0};$$

and
$$ds=\sqrt{dx^2+dy^2},$$
$$=\sqrt{\frac{\tau^2}{T^2-\tau^2}+1}\,.\,\frac{(1+eT)dT}{w_0},$$
$$=\sqrt{\frac{T^2}{T^2-\tau^2}}\,.\,\frac{(1+eT)\,dT}{w_0}=\frac{T(1+eT)}{w_0\sqrt{T^2-\tau^2}}dT,$$

$$\therefore\qquad s=\frac{1}{w_0}\int_{T_1}^{T_2}\frac{(1+eT)TdT}{\sqrt{T^2-\tau^2}}.$$

But
$$\int\frac{TdT}{\sqrt{T^2-\tau^2}}=\sqrt{T^2-\tau^2};$$

and
$$\int\frac{T^2dT}{\sqrt{T^2-\tau^2}}=\int T\frac{d}{dT}\sqrt{T^2-\tau^2}$$
$$=T\sqrt{T^2-\tau^2}-\int\sqrt{T^2-\tau^2}\,dT$$
$$=T\sqrt{T^2-\tau^2}-\int\frac{T^2-\tau^2}{\sqrt{T^2-\tau^2}}\,dT$$

$$\therefore \qquad 2\int \frac{T^2dT}{\sqrt{T^2-\tau^2}} = T\sqrt{T^2-\tau^2} + \int \frac{\tau^2 dT}{\sqrt{T^2-\tau^2}},$$

$$= T\sqrt{T^2-\tau^2}+\tau^2 \log\left(T-\sqrt{T^2-\tau^2}\right).$$

Therefore,

$$s = \frac{1}{w_0}\left(\sqrt{T_2^2-\tau^2}-\sqrt{T_1^2-\tau^2}\right)$$

$$+\frac{e}{w_0}\left(\frac{T_2}{2}\sqrt{T_2^2-\tau^2}-\frac{T_1}{2}\sqrt{T_1^2-\tau^2}\right)$$

$$+\frac{e\tau^2}{2w_0}\log\frac{T_2+\sqrt{T_2^2-\tau^2}}{T_1+\sqrt{T_1^2-\tau^2}}$$

$$=\frac{1}{w_0}\left(T_2\cos\theta_2-T_1\cos\theta_1\right)$$

$$+\frac{e}{w_0}\left(\frac{T_2}{2}T_2\cos\theta_2-\frac{T_1}{2}T_1\cos\theta_1\right)+\frac{e\tau^2}{2w_0}\log\frac{\tan\frac{1}{2}\theta_1}{\tan\frac{1}{2}\theta_2};$$

or,

$$s=s_0+\frac{e\tau^2}{2w_0}\left(\frac{\cos\theta_2}{\sin^2\theta_2}-\frac{\cos\theta_1}{\sin^2\theta_2}+\log\frac{\tan\frac{1}{2}\theta_1}{\tan\frac{1}{2}\theta_2}\right), \tag{26}$$

by (9) and (3).

The above equation for x, while concise, and interesting from a mathematical standpoint, is not suitable for numerical computation. We proceed therefore to place it in more convenient forms.

Writing,

$$m=\frac{\tan\frac{1}{2}\theta_1}{\tan\frac{1}{2}\theta_2}, \tag{27}$$

we have,

$$\log m = 2\left\{\frac{m-1}{m+1}+\frac{1}{3}\left(\frac{m-1}{m+1}\right)^3+\frac{1}{5}\left(\frac{m-1}{m+1}\right)^5+\right\};$$

or again writing,

$$M=\frac{m-1}{m+1} \tag{28}$$

we have,

$$\log m = 2(M+\tfrac{1}{3}M^3+\tfrac{1}{5}M^5+).$$

Also,

$$M=\frac{\tan\frac{1}{2}\theta_1-\tan\frac{1}{2}\theta_2}{\tan\frac{1}{2}\theta_1+\tan\frac{1}{2}\theta_2}=\frac{\sin\frac{1}{2}(\theta_1-\theta_2)}{\sin\frac{1}{2}(\theta_1+\theta_2)}. \tag{29}$$

Also, by (1) and (2)

$$T_1+T_2=W\frac{\sin\theta_1+\sin\theta_2}{\sin(\theta_1-\theta_2)}$$

$$=W\frac{\sin\frac{1}{2}(\theta_1+\theta_2)}{\sin\frac{1}{2}(\theta_1-\theta_2)}=\frac{W}{M},$$

$$\therefore \qquad M=\tfrac{1}{2}\frac{W}{T_m} \tag{30}$$

Equation (25) then becomes:

$$x=s_0\frac{\tau}{T_m}(1+\tfrac{1}{3}M^2+\tfrac{1}{5}M^4+). \tag{31}$$

We must now find expressions for τ and T_m. Returning to eq. (20), and writing,

$$s_m=s_0\,(1+eT_m), \tag{32}$$

we have, by (1) and (2),

$$\begin{aligned} y&=\frac{W}{w_m}\,\frac{\sin\ \theta_1-\sin\ \theta_2}{\sin\,(\theta_1-\theta_2)}\,, \\ &=s_m\frac{\sin\ \theta_1-\sin\ \theta_2}{\sin\,(\theta_1-\theta_2)}\,; \end{aligned} \tag{33}$$

whence $$\frac{s_m}{y}=\frac{1+\tan\,\frac{1}{2}\,\theta_1\,\tan\,\frac{1}{2}\theta_2}{1-\tan\,\frac{1}{2}\,\theta_1\,\tan\,\frac{1}{2}\,\theta_2}\,;$$

and $\therefore$ $$\frac{s_m+y}{s_m-y}=\frac{1}{\tan\,\frac{1}{2}\,\theta_1\,\tan\,\frac{1}{2}\theta_2}$$

or $$\tan\,\tfrac{1}{2}\,\theta_1\,\tan\,\tfrac{1}{2}\,\theta_2=\frac{s_m-y}{s_m+y}\,. \tag{34}$$

Again eliminating $\tan\,\frac{1}{2}\theta_2$ from (29) by means of (34); we find after a little reduction

$$M=\frac{y-s_m\cos\theta_1}{s_m-y\cos\theta_1}\,. \tag{35}$$

This also gives, $\cos\,\theta_1=\dfrac{y-Ms_m}{s_m-My}\,;$ $\quad$ (36)

from which we have,

$$\sin\,\theta_1=\frac{\sqrt{(s_m{}^2-y^2)\,(1-M^2)}}{s_m-My}\,.$$

Then, by (3) and (30),

$$\begin{aligned} \sin\,\theta_m&=\sin\,\theta_1\,\frac{T_1}{T_m}=\sin\,\theta_1\,\frac{2\,T_1M}{W}\,; \\ &=\frac{2T_1\sqrt{(s_m{}^2-y^2)\,(1-M^2)}}{s_0w_0\left(\dfrac{s_m}{M}-y\right)}\,. \end{aligned}$$

Also, $$\begin{aligned} w_0\,\frac{s_m}{M}-w_0y&=\frac{w_0s_0\,(1+eT_m).\ 2\,T_m}{W}-(T_2-T_1)\,(1+eT_m), \\ &=2\,T_1\,(1+eT_m)\,, \end{aligned}$$

by (20) ; ∴ $$\sin\theta_m=\frac{\sqrt{(s_m{}^2-y^2)(1-M^2)}}{s_0(1+eT_m)}. \tag{37}$$

Also, $$\tau=T_m\sin\theta_m=\frac{T_m\sqrt{(s_m{}^2-y^2)(1-M^2)}}{s_0\ (1+eT_m)} \tag{38}$$

Substituting the values of τ/T_m and τ given by (38) in (31), we have,

$$\begin{aligned} x=&\frac{\sqrt{(s_m{}^2-y^2)\ (1-M^2)}}{1+eT_m}(1+\tfrac{1}{3}\ M^2+\tfrac{1}{5}\ M^4+) \\ &\qquad +eT_m\,\frac{\sqrt{(s_m{}^2-y^2)\ (1-M^2)}}{1+eT_m} \\ =&\frac{\sqrt{s_m{}^2-v^2}}{1+eT_m}\quad\{(1-M^2)^{\frac{1}{2}}(1+\tfrac{1}{3}M^2+\tfrac{1}{5}M^4+) \\ &\qquad +eT_m(1-M^2)^{\frac{1}{2}}\} \end{aligned}$$

Then expanding, multiplying out, and reducing, this becomes,

$$\begin{aligned} x=&\frac{\sqrt{s_m^2-y^2}}{1+eT_m}\{1-\tfrac{1}{6}\ M^2-\tfrac{11}{120}M^4- \\ &\qquad +eT_m(1-\tfrac{1}{2}\ M^2-\tfrac{1}{8}\ M^4-)\}. \end{aligned} \tag{39}$$

By (32), and writing, $$y_0=\frac{y}{1+eT_m}, \tag{40}$$

this becomes,

$$\begin{aligned} x=&\sqrt{s_0^2-y_0^2}\{1-\tfrac{1}{6}M^2-\tfrac{11}{120}M^4-. \\ &\qquad +eT_m\ (1-\tfrac{1}{2}\ M^2-)\} \end{aligned} \tag{41}$$

If extended to terms of a higher order the first series in this expression becomes:

$$1-\tfrac{1}{6}M^2-\tfrac{11}{120}M^4-\tfrac{103}{1680}M^6-\tfrac{1823}{40320}M^8-;$$

but the terms after the third will seldom affect the ninth decimal place in x.

If the ends of the tape are at the same level eq. (41) takes the form:

$$\begin{aligned} x=\ s_0\Bigg\{&1-\tfrac{1}{24}\frac{W^2}{T^2}-\tfrac{11}{1920}\frac{W^4}{T^4}- \\ &\qquad +eT\ (1-\tfrac{1}{8}\frac{W^2}{T^2}-)\Bigg\} \end{aligned} \tag{42}$$

An expression for T_m is thus obtained: If T denote the tension applied at the forward end of the tape, we have

$$T_m=\tfrac{1}{2}\,(T_2-T_1)+T_1,$$

and

$$T_m=T_2-\tfrac{1}{2}\,(T_2-T_1),$$

or, by (20)

$$T_m=\tfrac{1}{2}\,\frac{w_0 y}{1+eT_m}+T_1\ ,$$

and

$$T_m=T_2-\tfrac{1}{2}\,\frac{w_0 y}{1+eT_m}$$

or, whether the forward end of the tape is the higher or the lower,

$$T_m=T-\tfrac{1}{2}\,\frac{w_0 y}{1+eT_m}\ , \tag{43}$$

y being negative if the forward end of the tape is the lower. By (40) this becomes,

$$T_m=T-\tfrac{1}{2}w_0 y_0 \tag{44}$$

and (40) may be written,

$$y_0=\frac{y}{1+e\,(T-\tfrac{1}{2}\,w_0 y_0)} \tag{45}$$

In this equation y_0 in the right-hand number may be replaced by y, and the resulting value of y_0 regarded as a first approximation if for any purpose higher precision is required. In the example given below the first approximation to y_0 is only in error by a unit in the ninth decimal place, which is sufficiently precise for all practical purposes.

Equations (45), (44), (30), and (41), give a practically rigorous determination of x, as they serve to determine that quantity with as high a precision as the most exacting conditions can require. They are rather cumbersome in that form, however, for general use, but they serve for working out numerical results that may be used as checks upon those obtained by other less rigorous methods. They will now be employed to determine the values of x corresponding to the four values of y: 5, 10, 20, and 40 feet, taking as the constants of the tape:

$$s_0=100 \text{ feet}$$

$$w_0=0.012 \text{ lb.}$$

$$e=0.000\ 009\ 11$$

$$T=25 \text{ lbs.}$$

The results are given in greater detail for $y=10$ feet.

Thus we have,

$T-\frac{1}{2}w_0y$ = 24.94
$e(T-\frac{1}{2}w_0y)$ = 0.000 227 203 4
y_0 (1st approx.) = 9.997 728 482
$T-\frac{1}{2}w_0y_0$ = 24.940 013 629 108
$e(T-\frac{1}{2}w_0y_0)$. = 0.000 227 203 524
y_0 (2nd approx.) = 9.997 728 480 856
T_m = 24.940 013 629 115
M = 0.024 057 725 426
M^2 = 0.000 578 774 153
M^4 = 0.000 000 334 980
$\frac{1}{6}M^2+\frac{11}{120}M^4$ = 0.000 096 493 065
$\frac{1}{2}M^2+\frac{1}{8}M^4$ = 0.000 289 428 949
eT_m = 0.000 227 203 524
$eT_m(1-\frac{1}{2}M^2-\frac{1}{8}M^4)$ = 0.000 227 137 755
0.000 130 644 700
$\sqrt{s_0^2-y_0^2}$ = 99.498 971 980 735
0.012 999 013 310
x = 99.511 970 994 045

The value of the terms in M^6 is:

0.000 000 001 183

so that it has an effect in the ninth decimal place in x.

The tabulated results for the four values of y are as follows:

$y=5$— $T-T_m$ = 0.029 993 177 249
y_0 = 4.998 862 874 859
M = 0.024 028 828 036
x = 99.888 077 278
$y=10$— $T-T_m$ = 0.059 986 370 885
y_0 = 9.997 728 480 856
M = 0.024 057 725 426
x = 99.511 970 994
$y=20$— $T-T_m$ = 0.119 972 807 318
y_0 = 19.995 467 886 281
M = 0.024 115 729 270
x = 97.993 216 132
$y=40$— $T-T_m$ = 0.239 945 876 826
y_0 = 39.990 979 470 985
M = 0.024 232 580 309
x = 91.667 145 175

For the reduction of field measurements made with a given tape, to which a constant tension is applied, equation (41) may be placed in a very convenient form, as y will then be the only variable quantity in that equation, so that it may be written in the form:

$$x=\sqrt{s_0^2-y_0^2}\ (1+\mathrm{A}+By+Cy^2+),$$

in which A, B, C, etc., are numerical constants. We proceed to illustrate the process by applying it to the tape whose constants, and the tension to which it is subjected, are given above.

As a first step the equations for y_0 and M must be placed in suitable forms. Thus eq. (45) may be written in the form:

$$\tfrac{1}{2}\, e\, w_0\, y_0^2-(1-eT)\, y_0+y=0,$$

the solution of which is,

$$y_0=\frac{1+eT\pm\sqrt{(1+eT)^2-2e\, w_0\, y}}{ew_0}$$

$$=\frac{1+eT}{ew_0}-\frac{1+eT}{ew_0}\left\{1-\frac{2ew_0y}{(1+eT)^2}\right\}^{\frac{1}{2}},$$

taking the lower sign. Expanding, we obtain,

$$y_0=\frac{y}{1+eT}+\tfrac{1}{2}\frac{ew_0\, y^2}{(1+eT)^3}+\tfrac{1}{2}\frac{(ew_0)^2y^3}{(1+eT)^5}+. \tag{46}$$

This may also be written,

$$y_0=\frac{y}{1+eT}+\tfrac{1}{2}\frac{ew_0}{1+eT}\frac{y^2}{(1+eT)^2}+\tfrac{1}{2}\frac{(ew_0)^2}{(1+eT)^2}\frac{y^3}{(1+eT)^3}+. \tag{47}$$

The value of T_m then follows from (44).

Then, to derive a convenient expression for M^2, we write ΔT for $T-T_m$ and we have by (30),

$$M^2=\tfrac{1}{4}\frac{W^2}{(T-\Delta T)^2}=\tfrac{1}{4}\frac{W^2}{T^2}\left(1-\frac{\Delta T}{T}\right)^{-2}$$

$$=\tfrac{1}{4}\frac{W^2}{T^2}\left\{1+2\frac{\Delta T}{T}+3\left(\frac{\Delta T}{T}\right)^2+4\left(\frac{\Delta T}{T}\right)^3+\right\} \tag{48}$$

The numerical work in outline follows.

$$\begin{aligned}
1/(1+eT) &= 1-.000\ 227\ 698\ 142 \\
\{1/(1+eT)\}^2 &= 1-.000\ 455\ 344\ 437 \\
\{1/(1+eT)\}^3 &= 1-.000\ 682\ 938\ 898 \\
ew_0 &= 0.000\ 000\ 109\ 32 \\
ew_0/1+eT &= 0.000\ 000\ 109\ 295 \\
y_0 &= y-0.000\ 227\ 698\ 142.y \\
&\quad +0.000\ 000\ 054\ 623.y^2 \\
T_m &= 25-0.005\ 998\ 633\ 811.y \\
&\quad -0.000\ 000\ 000\ 328.y^2 \\
\tfrac{1}{4}\ W^2/T^2 &= 0.000\ 576 \\
\Delta T/T &= 0.000\ 239\ 945\ 352.y \\
&\quad +0.000\ 000\ 000\ 013.y^2 \\
3(\Delta T/T)^2 &= 0.000\ 000\ 172\ 721.y^2 \\
4(\Delta T/T)^3 &= 0.000\ 000\ 000\ 055.y^3 \\
M^2 &= 0.000\ 576 \\
&\quad +0.000\ 000\ 276\ 417.y \\
&\quad +0.000\ 000\ 000\ 099.y^2 \\
M^4 &= 0.000\ 000\ 331\ 776 \\
&\quad +0.000\ 000\ 000\ 318.y \\
eT_m &= 0.000\ 227\ 75 \\
&\quad -0.000\ 000\ 054\ 648.y
\end{aligned}$$

Finally—

$$\begin{aligned}
x=\sqrt{s_0^2-y_0^2}\ (1&+0.000\ 131\ 653\ 986 \\
&-0.000\ 000\ 100\ 762.y \\
&-0.000\ 000\ 000\ 016\ \ 6.y^2-)
\end{aligned} \qquad (49)$$

Applying this equation to the same data as given above, the results are as follows, the errors being in units of the ninth decimal place:

y	x	*Error*
5	99.888 077 278	0
10	99.511 970 995	1
20	97.993 216 136	5
40	91.667 145 208	33

The term in (49) involving the radical is a little inconvenient. If logarithms are used, seven or eight figure tables will insure precision in the fifth decimal place, and nine figure tables in the sixth decimal place. The following shows the result with nine figure tables:

$y=5$, y_0	=	4.998 862 875	
s_0	=	100	*log*
s_0+y_0	=	104.998 862 875	2.021 184 595
s_0-y_0	=	95.001 137 125	1.977 728 804
			2) 3.998 913 399
		99.874 978 61	1.999 456 699
		0.000 131 654 0	
	−	0.000 000 503 8	
	−	0.000 000 000 4	
	−	0.000 000 504 2	
		0.000 131 149 8	
		0.013 098 583 5	
x	=	99.888 077 19	Error = 0.000 000 09

The radical term of (49) may if preferred be developed in series, and that form is convenient for small values of y; for large values, however, the series converges so slowly that it entails more labour than the use of logarithms. An example illustrating this is given below.

Developing that term in series, we have,

$$\sqrt{s_0^2-y_0^2}=s_0\left(1-\frac{y_0^2}{s_0^2}\right)^{\frac{1}{2}}.$$

Then writing u for y_0^2/s_0^2 gives:

$$(1-u)^{\frac{1}{2}}=1-\tfrac{1}{2}u-\frac{1.1}{2^2\,\underline{|2}}u^2-\frac{1.1.3}{2^3\,\underline{|3}}u^3-\frac{1.1.3.5}{2^4\,\underline{|4}}u^4$$
$$-\frac{1.1.3.5.7}{2^5\,\underline{|5}}u^5- \tag{50}$$

We have then—

$$\sqrt{s_0^2-y_0^2}=s_0\left\{1-\tfrac{1}{2}\left(\frac{y_0}{s_0}\right)^2-\tfrac{1}{8}\left(\frac{y_0}{s_0}\right)^4-\tfrac{1}{16}\left(\frac{y_0}{s_0}\right)^6-\right\} \tag{51}$$

We apply this to the example: $y=10$, thus—

$$\begin{array}{lr} y_0/s_0 & = 0.099\ 977\ 284\ 809 \\ \frac{1}{2}(y_0/s_0)^2 & = 0.004\ 997\ 728\ 739 \\ \frac{1}{8}(y_0/s_0)^4 & = 0.000\ 012\ 488\ 646 \\ \frac{1}{16}(y_0/s_0)^6 & = 0.000\ 000\ 062\ 415 \\ \cline{2-2} & 0.005\ 010\ 279\ 800 \\ & 0.994\ 989\ 720\ 200 \\ \cline{2-2} \sqrt{s_0^2-y_0^2} & = 99.498\ 972\ 020\ 002 \\ \cline{2-2} \end{array}$$

This value is in error by four units in the eighth decimal place.

Although eq. (49) will in theory serve to determine x with all necessary precision for large values of y, in practice, however, the effect of the inevitable error in y increases rapidly with y, which is determined by spirit levelling. This fact sets the limit of admissible slopes in a base line. To find the effect on x of a given error in y, we write—

$$x=\sqrt{s_0^2-y_0^2}\ ,$$

omitting the remaining terms which have little influence on the result. Differentiating, we have,

$$\frac{dx}{dy}=-\frac{y_0}{(s_0^2-y_0^2)^{\frac{1}{2}}}\frac{dy_0}{dy}=-\frac{y_0}{x}\frac{1}{1+eT},$$

nearly; or without material error we may write

$$\frac{dx}{dy}=-\frac{y}{x}\ .$$

This gives approximately the error in x due to a given error in y, and shows that for extreme values of y the error in x becomes comparable with that in y.

Further corrections, in addition to those involved in eq. (41) must be applied to base line measurements. These are the corrections for temperature, and change in the force of gravity with latitude.

Correction for Temperature.—This means finding the change in x caused by a small change in s_0. In eq. (41), if y is constant, then y_0, T_m, and M, are also constant, and we have,

$$\begin{aligned}\frac{dx}{ds_0}&=\frac{s_0}{\sqrt{s_0^2-y_0^2}}\left\{1-\tfrac{1}{6}M^2-\ \ .\ \ .\ \ .\ \ +eT_m(1-\text{etc.})\right\}\\ &=\frac{s_0x}{s_0^2-y_0^2}\ ;\end{aligned}$$

so that, treating the small changes in s_0 and x as differentials, we have,

$$ds_0 = \alpha(t-t_0)s_0,$$

in which,

t denotes the temperature of the tape during a measurement,
t° the temperature at which its length is standard when under no tension, and
α the coefficient of expansion.

The temperature correction to x then is:

$$c_t = \alpha\, x\, (t-t_0)\frac{s_0^2}{s_0^2-y_0^2} \tag{52}$$

The following is an example:

$y = 10 \qquad y_0 = 9.997\ 728\ 48$
$s_0 = 100$
$\alpha = 0.000\ 006\ 25$
$t-t_0 = 20$ F.
$x = 99.511\ 970\ 99$
Whence we find— $c_t = 0.012\ 564\ 587$

We take also the additional example:

$y = 40 \qquad y_0 = 39.990\ 979\ 47$
$x = 91.667\ 145\ 175$

the other data being unchanged; and we find—

$c_t = 0.013\ 639\ 774.$

The corrected value of x then is:

$x = 91.680\ 784\ 949.$

We may test the propriety of regarding as differentials the changes in s_0 and x due to change of temperature, by correcting s_0 for temperature and computing the corresponding value of x by means of (41). We shall apply this test to the second of the above examples

We find, s_0 becomes 100.0125; also—

$w_0 = 0.011\ 998\ 5,$
$y_0 = 39.990\ 979\ 459\ 971,$
$T_m = 24.760\ 084\ 112\ 726,$
$M^2 = 0.000\ 587\ 216\ 501,$
and finally, $x = 91.680\ 784\ 837,$

differing from the former value by a unit in the seventh decimal place.

In measuring a base the procedure may be as follows: The base, it is assumed, has been marked out on the ground by carefully aligned and firmly driven posts, spaced a trifle less than a tape length between centres, and their heights being about three feet above the ground. The spacing may be carried out by means of a tape similar to that to be used in the measurement of the base, a spring balance giving the required tension. Small plates of sheet metal are nailed to the tops of the posts, on each of which a fine line is ruled with a steel point at right angles to the direction of the base. These plates may be attached to the posts during the preliminary laying out of the base, or—which is the more usual practice—they may be attached as the measurement of the base proceeds, and the lines ruled on them at the same time. The base tape is then set up in position, with its ends resting lightly on two consecutive posts, and the rest of the tape unsupported, a tension being applied at each end by means of a weight attached to a cord which passes over a frictionless pulley. The pulleys are supported by tripods or other similar contrivances. The rear zero mark on the tape is now brought to coincide as nearly as possible with the line on its post, which, we shall assume, has been previously established; and the forward plate is then set, with its line as nearly as possible coinciding with the forward zero mark on the tape. Scale readings are then taken at the two ends of the tape, giving the intervals between the marks on the tape and those on the posts, either of which must be given the positive sign if outside the zero mark on the tape, and the negative sign if inside. The scale readings may thus be kept as small as possible, and their algebraic sum may be combined with the temperature correction to s_0, which by design it may be made partially to compensate, and the whole treated as a single correction.

Correction for Change in the Force of Gravity.—As the tension is applied to the tape by means of weights, any change in the force of gravity will affect the weights, and therefore the tension, and consequently the extension of the tape; but as the weight of the tape is changed in the same proportion as the tension, the sag of the tape is not affected by the change in the force of gravity. On the other hand, if the tension is applied by means of a spring balance, the tension applied to the tape, and therefore also the extension, is unchanged, but the sag of the tape is altered.

This last statement: that the extension is unchanged, when a spring balance provides the tension, is not strictly accurate, as the quantity T_m affects all the terms of eq. (41) except s_0, and T_m depends to a small extent upon the weight of the tape. The extension of the tape is therefore slightly affected in this case by the change in the force of gravity, but by an amount that is probably entirely negligible.

Bowie's formula for g may be written:

$$g = g_0(1 + 0.005\ 294 \sin^2\phi - 0.000\ 007 \sin^2 2\phi)$$

in which, g denotes the force of gravity at sea level in latitude ϕ, and g_0 its value at the equator.

Also— $$g_0 = 978\ 039.$$

Omitting the last term and introducing a term which makes allowance for height above sea level, this becomes:

$$g = g_0(1 + 0.005\ 294 \sin^2\phi)\left(1 - 2\frac{h}{\rho} + \frac{3}{4}\frac{h}{\rho}\right);$$

in which h denotes the height above sea level, and ρ the radius of curvature of the normal section containing the base. The last term in this expression contains the Bouguer correction for height, by which an attempt is made to allow for the attraction of the portion of the earth's crust between the surface of the ground and sea level. As it is found, however, in the reduction of observed values of g to sea level, that the inclusion of this term leads to larger gravity anomalies than when it is omitted, we shall omit it, and write the above equation,

$$g = g_0(1 + 0.005\ 294 \sin^2\phi)\left(1 - 2\frac{h}{\rho}\right).$$

in which ϕ and h denote the latitude and height of the base above sea level. If ϕ_1 and h_1 denote the values of those quantities for the place where the tape was standarized, we shall have,

$$g_1 = g_0(1 + 0.005\ 294 \sin^2\phi_1)\left(1 - 2\frac{h_1}{\rho_1}\right).$$

These equations may be written in the approximate form:

$$g = g_0\left(1 + 0.005\ 294 \sin^2\phi - 2\frac{h}{\rho}\right), \tag{53}$$

$$g_1 = g_0\left(1 + 0.005\ 294 \sin^2\phi_1 - 2\frac{h_1}{\rho_1}\right);$$

whence, writing Δg for $g - g_1$, we have,

$$\frac{\Delta g}{g_1} = \frac{g_0}{g_1}\left\{0.005\ 294\ (\sin^2\phi - \sin^2\phi_1) - 2\left(\frac{h}{\rho} - \frac{h_1}{\rho_1}\right)\right\}, \tag{54}$$

giving the ratio of the change in the value of g to the value of that quantity at the station of standardization. It is perhaps more convenient to compute g and g_1 by (53) and thence find $\Delta g/g_1$.

We take as an example:

$\phi_1 = 45°$ and $\phi = 65°$, and $h_1 = h = 500$ feet;

and we find— $g_1 = 980.581,$

$g = 982.245;$

whence— $$\frac{\Delta g}{g_1} = 0.00169695 \; \ldots$$

We must now determine the effect of Δg upon the quantity x. As it affects both the weight of the tape and the weights by which the tension is applied to it, it will have an effect on each term of eq. (41), with the exception of s_0, though in some cases the effect is negligible. The quantity e will evidently be unchanged, provided the weights used in its determination were correctly standardized. We shall examine the effect of Δg on each term in turn.

The quantity T_m is not proportional to T, so that a slight error will result from assuming that it is. Let—

T_m' denote the value of that quantity when a correct allowance is made for the change in gravity,

T_m'' the value found on the assumption that it varies proportionately to T, and

T' and w_0' the values of T and w_0 as affected by Δg; then, by (44) and (46), retaining only the first term of the latter equation, we have—

$$T_m' = T' - \tfrac{1}{2}w_0' \frac{y}{1+eT'} = T' - \tfrac{1}{2}w_0'y\,(1-eT')$$

$$T_m'' = T' - \tfrac{1}{2}w_0' \frac{y}{1+eT} = T' - \tfrac{1}{2}w_0'y\,(1-eT)$$

neglecting e^2. Therefore—

$$T_m' - T_m'' = \tfrac{1}{2}ew_0'y\,(T'-T).$$

Then, as $$T' = T.g/g_1$$

$$\therefore \qquad T' - T = T.\Delta g/g_1$$

and $$\therefore \qquad T_m' - T_m'' = \tfrac{1}{2}ew'_0yT\frac{\Delta g}{g_1}. \tag{55}$$

Taking as an example, $y = 20$ feet, we find,

$$T_m' - T_m'' = 0.000\ 000\ 046$$

As T_m in eq. (41) is multiplied by the small quantity e we may discard this term, and write:

$$T_m' = T_m'' = T_m\left(1+\frac{\Delta g}{g_1}\right).$$

The effect upon x of this change in T_m is—eq. (41)—

$$eT_m\frac{\Delta g}{g_1}(s_0^2-y_0^2)^{\frac{1}{2}}(1-\tfrac{1}{2}\ M^2-),$$

in which the last term may be neglected as inappreciable and the expression written—

$$eT_m x\frac{\Delta g}{g_1}$$

To find the effect of Δg on M, let M' denote the value of that quantity as affected by Δg and we have by (55)

$$M' = \frac{\frac{1}{2}W(1+\Delta g/g_1)}{T_m(1+\Delta g/g_1)+\frac{1}{2}ew_0'yT.\Delta g/g_1}.$$

This reduces readily to the approximate form:

$$M' = M\left(1-\tfrac{1}{2}ew_0'y\frac{\Delta g}{g_1}\right), \tag{56}$$

which gives, with the same data as before—

$$M'-M = -\ 0.000\ 000\ 001\ 858.$$

The largest neglected term in (56) only affects the twelfth decimal place, so that evidently the effect of Δg on M may safely be neglected.

Coming finally to the term y_0, let y_0' denote its value affected by Δg; then we have, approximately—

$$y_0 = \frac{y}{1+e(T-\frac{1}{2}w_0y)},$$

$$y_0' = \frac{y}{1+e(T-\frac{1}{2}w_0y)\ (1+\Delta g/g_1)}.$$

From these we obtain, by omitting unimportant terms.

$$y_0' = y_0\left\{1-eT\frac{\Delta g}{g_1}\left(1-\tfrac{1}{2}\frac{w_0y}{T}\right)\right\},$$

whence, approximately, omitting the last term—

$$s_0^2-y_0'^2 = s_0^2-y_0^2\left(1-2\ eT\frac{\Delta g}{g_1}\right),$$

$$= (s_0^2-y_0^2)\left(1+2\ eT\frac{\Delta g}{g_1}\frac{y_0^2}{s_0^2-y_0^2}\right).$$

From this we have—

$$x' = x\left(1+eT\frac{\Delta g}{g_1}\,\frac{y_0^2}{s_0^2-y_0^2}\right)$$

or, very nearly,

$$x'=x+eT\frac{y_0^2}{x}\cdot\frac{\Delta g}{g_1}.$$

The effect upon x, due to the change in y_0, is then

$$eT\frac{y_0^2}{x}\cdot\frac{\Delta g}{g_1}.$$

The total correction to x, due to the change in the force of gravity is, therefore:

$$c_g = eT\frac{\Delta g}{g_1}\left(x+\frac{y^2}{x}\right) \tag{57}$$

With the same data as before we find the value of this correction to be:

$$\begin{array}{lcl} \text{1st term} & = & 0.000\ 037\ 872 \\ \text{2nd } `` & = & 0.000\ 001\ 578 \\ \hline c_g & = & 0.000\ 039\ 450 \\ \hline \end{array}$$

The more precise value is:

$$\begin{array}{lcl} \text{1st term} & = & 0.000\ 037\ 675 \\ \text{2nd } `` & = & 0.000\ 001\ 570 \\ \hline c_g & = & 0.000\ 039\ 245 \\ \hline \end{array}$$

Correction for Deviation.—In aligning the posts or tripods used in measuring a base, there should be no difficulty in keeping the errors of alignment so small that they will have no appreciable effect upon the length of the base. To find the effect of a given error in alignment, let

d denote the deviation of a given tape length,

x' the value of x found as above described, and

x the length of its projection on the vertical plane of the base;

then,

$$x = x'\left(1-\frac{d^2}{x'^2}\right)^{\frac{1}{2}}$$

$$= x'\left\{1-\tfrac{1}{2}\left(\frac{d}{x'}\right)^2-\tfrac{1}{8}\left(\frac{d}{x'}\right)^4-\right\}$$

If $d=0.02$, and $x' = 100$, then

$$\left(\frac{d}{x'}\right)^2 = 0.000\ 000\ 04,$$

and the second term is—

$$0.000\ 002,$$

the third being unappreciable. This emphasizes the necessity for care in aligning the posts.

A STUDY OF THE DETERMINATION OF SULPHUR IN ORGANIC COMPOUNDS

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Although a great many methods for the quantitative estimation of sulphur in organic compounds have been proposed, it is not the intention of the authors to present a critical review, nor to bring forward any new procedure. The circumstances which led to the investigation here described may be briefly presented as follows. In the course of research work upon the barium salts of the lignosulphonic acids of spruce wood, it became necessary to carry out a number of ultimate analyses; since the formula weight of these compounds is large, the greatest precision in the determination of the single elements is obviously desirable.

Previous investigators in this field, Hönig and Spitzer[1] for example. had determined the sulphur in barium lignosulphonates by the Eschka method, and the same procedure was adopted in the earlier analyses which were made in this laboratory. At a later stage some suspicion arose as to the accuracy of the results obtained, and determinations by the Carius method for the sake of comparison, demonstrated conclusively that the Eschka method yielded low results. This discovery led to the search for a method which would be as accurate as and more convenient than the Carius. From the numerous methods described in the literature, four were selected because of their promise of convenience and accuracy, and brief descriptions of these will now be given. The results are presented in Table I.

ESCHKA'S METHOD

This was carried out with every conceivable precaution, such as the exclusion of the products of combustion of the burner during the heating, the oxidation of the aqueous extract by bromine, the precipitation of the $BaSO_4$ in a solution whose volume and acidity was carefully regulated, and the blank for sulphur in the reagents.

SODIUM PEROXIDE FUSION

The procedure here followed was that given by R. Sieber[2]. About 0.3 g. barium lignosulphonate was added to 5 c.c. of a concentrated solution of NaOH in a nickel crucible, and the liquid was evaporated to dryness. 3 g. each of Na_2O_2 and Na_2CO_3 were added and the mixture was gradually brought to fusion. A further gram of Na_2O_2 was added later and heating continued for half an hour. The fused mass was dissolved in water, bromine was added, then acid, and after the removal of excess bromine by boiling, the SO_4 was precipitated.

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[1] Honig and Spitzer—Monatsh. 39 (1918).

[2] R. Sieber—Zellstoff u. Papier, 2 (1922), 199.

KLASON'S METHOD[3]

About 0.2 gram of barium lignosulphonate in a platinum boat was introduced into a combustion tube 20 inches long. Before and after the boat were bundles of long-fibred asbestos, bound together with platinum wire, and soaked in fuming nitric acid. Rolls of platinum gauze were introduced before and behind the boat as described in the original communication. The end of the tube was drawn out and bent downwards dipping into water in a receiver. On heating gently and introducing a current of air, brown clouds of the oxides of nitrogen were evolved, and this continued for about one hour. It was, however, very difficult to oxidize the carbon completely, even after soaking the asbestos in nitric acid thrice in succession and repeating the heating which thus extended to four hours or more. The residue in the boat was treated with water and filtered. To the filtrate was added the solution in the receiver at the end of the combustion tube, and after acidifying with HCl, the liquid was evaporated to dryness. On redissolving the SO_4 was determined as usual.

APITZSCH'S METHOD[4]

About 0.2 gram of barium lignosulphonate in a platinum boat was introduced into a combustion tube, 24 inches long. Before and after the boat were placed rolls of platinum gauze and combustion was carried out in a stream of oxygen. The SO_3 formed was absorbed in sodium hypobromite solution, but the style of absorption flask proposed by the author was found to be inefficient. To arrest effectively the white fumes which escaped, a narrow tube 14 inches long, stuffed loosely with glass wool, was added vertically to the first absorber. The glass wool was thoroughly moistened with sodium hypobromite solution, and a further guard tube charged with iodine solution showed decisively that absorption was complete in the second absorber. The combustion required an hour. The residue in the boat was treated as described above under Klason's method and was united with the sodium hypobromite solution.

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[3] Klason—Ber. (1887), 3065.

[4] Apitzsch—Z. angew. Chem. 26 (1913), 503.

CARIUS METHOD

Conducted in the usual manner, using 0.2 gram barium lignosulphonate and 3 cc. fuming nitric acid; the tubes were gradually raised in temperature and were heated altogether for eight hours.

When oxidation was complete, there was present in the tube a greyish residue which was filtered off after dilution, and fused with Na_2CO_3. The fused mass was extracted with water, filtered, and the filtrate, united with the previous filtrate, was evaporated to dryness after the addition of HCl. After solution in water, the sulphur was determined in the usual way.

From the following table it is clear that the results by the Carius method are higher and more consistent than those yielded by other methods. When it is considered that no volatile sulphur compounds can escape from the sealed tube, and that a minimum of other reagents is present in the solution from which the $BaSO_4$ is precipitated, there is ample justification for the statement that the results by the Carius method are very close to the truth. We have accordingly adopted the Carius method as a standard for comparison.

TABLE I

Comparison of results of five methods for the determination of sulphur in a sample of barium lignosulphonate:

Experiment Number	Method	Weight of Sample grams	Barium Sulphate grams	per cent. of sulphur	Average
1	Eschka..........	0.6052	0.2115	4.70	
2		0.5000	0.1640	4.54	
3		0.5000	0.1570	4.40	4.59
4		0.5288	0.1810	4.70	
5	Peroxide..........	0.3556	0.1387	5.34	
6		0.3384	0.1228	5.15	5.25
7	Klason..........	0.2213	0.0871	5.40	
8		0.2661	0.0668	3.44	
9		0.2074	0.0582	3.86	
10		0.2009	0.0766	5.24	
11	Apitzsch..........	0.1866	0.0786	5.79	
12		0.2008	0.0862	5.88	
13		0.2004	0.0844	5.80	
14		0.2016	0.0846	5.77	5.81
15	Carius..........	0.2010	0.0860	5.87	
16		0.1910	0.0820	5.89	
17		0.1792	0.0774	5.94	
18		0.2454	0.1050	5.88	5.89

The results obtained by the Apitzsch method are slightly lower, but there is no doubt that this method may be used with confidence where rapidity and convenience are required.

The peroxide method yields results which vary considerably, and which are in addition much too low. The fusion must be conducted with great care, since deflagrations are apt to occur. The procedure cannot be carried out rapidly and these disadvantages, coupled with lack of accuracy, have led us to reject it entirely.

Klason's method has given very poor results in our hands. Omitting two of the results which are very low, the remaining two do not show good agreement, and are 0.6 per cent. too low. Since Klason has calculated the formulae of a number of derivatives of lignosulphonic acids from determinations made by this method, the published results are undoubtedly in error in some cases at least, and the analyses of similar compounds as made in this laboratory will be published at a later date.

Eschka's method has proved to be quite unreliable in our hands, and since the analyses of salts of lignosulphonic acids were made by Hönig and Spitzer[5] by this procedure, the figures for sulphur as published are unquestionably much too low.

SUMMARY

Five methods of determining sulphur in barium lignosulphonate have been studied. The Carius method gave the highest and most consistent results, followed closely by the procedure of Apitzsch. Eschka's method and that of Klason are unreliable and the sodium peroxide methods yield results which are decidedly low.

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[5] Hönig and Spitzer—loc. cit.

A GENERAL THEOREM ON QUANTIC DETERMINANTS

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Introduction

The paper here presented will be seen to be foreign to the range of subjects likely to be useful in egineering, being in fact typical of Pure Mathematics of the more abstract kind and it may be wondered why it is included in the output of a school of engineering research.

The explanation is that it is given as an example of how a by-product of Applied Science may be Pure Science.

It came about in the following way:

The writer, in making a study of the theory of the electrical transmission line, had occasion to write out the values of all products of quantities on the right hand side, using one from each of the following two sets of linear algebraic equations:

$$\left.\begin{aligned} E_1 &= \alpha\, E_0 + \beta\, I_0 \\ I_1 &= \gamma\, E_0 + \delta\, I_0 \end{aligned}\right\} \qquad \left.\begin{aligned} E'_1 &= \alpha' E'_0 + \beta' I'_0 \\ I'_1 &= \gamma' E'_0 + \delta' I'_0 \end{aligned}\right\}$$

in which all symbols stand for complex quantities and primes denote conjugates. Knowing that the determinant of the coefficients α, β, γ, δ of the first substitution is unity, that of the second must be so also. It thus occurred to the writer to be curious to know if the determinant of the sixteen coefficients of the equations so derived should be unity also. On writing this determinant out it factored as $(\alpha\delta-\beta\gamma)^2(\alpha'\delta'-\beta'\gamma')^2$ *i.e.*, the product of the squares of the two original determinants, thus proving the idea correct, since each of the factors is unity.

This factorization is a particular case of a Theorem of Kronecker, though unknown to be such by the writer at the time.

The question naturally arose as to the highest possible degree of generality for relations of this type.

In this instance there are three limitations, viz., the number of substitutions is limited to two, the number of variables in each also limited to two and the degree of the monomials in these variables limited to unity for each.

The case was therefore considered in which all these limitations were removed, that is to say, no restriction on the number of substitutions considered, the number of variables in any one of them, or the degree in these variables for the individual substitution. It appeared to the writer that the original determinants were the only possible factors of the resulting determinant and on this basis the result was readily obtained by elementary algebra.

However, Dr. Samuel Beatty, of the University of Toronto, pointed out to him the unsoundness of the principle invoked.

Faá di Bruno had made a study of the case of one substitution with two variables but unlimited degree in these variables. After a study of his method it was seen that by modifying the method of differential operators which he employed, it was possible to use such a method in the most general case.

The result of this study is the following Theorem, which, as is stated therein, contains as particular cases theorems by Faá di Bruno and Hunyady, as well as that of Kronecker, of which the first inciting instance mentioned above was itself a very simple particular case.

It should be added that in a letter received this year from Sir Thomas Muir of South Africa, the great authority on the history of determinants, he states that the theorems (F), (G), and (H) really belong to the 50's of last century.

A study of Sir Thomas Muir's monumental treatise on the "Theory of Determinants in the Historical Order of Development" would no doubt confirm this statement, but the present writer could not in the time at his disposal go more fully into a matter which is far from his usual field.

The paper which follows is reprinted by permission from the Bulletin of the American Mathematical Society, September-October, 1927, having been presented to the Society February 26th, 1927.

Below the paper is given a new reference obtained from Sir Thomas Muir which I regret I did not have at the time.

PAPER

Let there be a homogeneous linear substitution

$$x'_{rj} = \sum_{i=1}^{n_r} a_{rji}x_{ri}, \qquad (j = 1, 2, \ldots, n_r),$$

having M_r for the determinant of its coefficients, and let there be derived therefrom quantics of degree p_r giving values of all possible monomials in these n_r new variables of that degree. There will be

$$f_r = \binom{p_r + n_r - 1}{p_r}$$

such new monomials, and an equal number of the old will be involved in these quantics.

Now consider this to be but one, namely, the rth of a set of s such substitutions, each in its own set of variables of arbitrary (generally different) number n_r, similarly treated. Each new monomial of each of the s sets is now to be used, one only from each set, as a factor in a continued product which contains such factors.

It is now proposed to examine a direct method for finding the algebraic composition of the determinant M of the coefficients of the quantics so constructed, when placed in an array similarly ordered in the composition of the monomials of the new and old quantities. It is evident that this condition of similar ordering is sufficient to ensure the same value of M for any change in the order, since it effects the same permutation of the rows as of the columns.

The result of this examination will be found to be that

$$\log M = \sum_{r=1}^{s} u_r \log M_r$$

where

$$u_r = \frac{p_r}{n_r} \prod_{r=1}^{s} f_r.$$

It is desirable to agree first for descriptive purposes on some rule of order more definite than the mere essential condition of similar order. Let the factor-groups of new variables from all other substitutions than the rth, combined, appearing associated with factor-groups from the rth on the left side of the equations, be assigned an arbitrary order which is likewise applied to the corresponding factor-groups of old variables on the right.

There will then be

$$q_r = \frac{1}{f_r} \prod_{r=1}^{s} f_r$$

such factor-groups. Now let the value be supposed written out of the product of each of the f_r primed monomials of the rth group (taken in an assigned order) with the first of the other factor-groups; then the products of each of them in the same order with the second of the q_r other factor-groups, and so on. For convenience each set of f_r rows of the *coefficients* so obtained may be called a block. There will thus be q_r blocks.

The order of the columns of coefficients is fixed by that of the associated monomials of the old variables and is determined by application of the rule of similar order. Thus there will be blocks running horizontally as well as vertically in each of which the same order for rth substitution variables will apply. For convenience, let anything referring to variables or coefficients connected with the rth substitution be called

"rth stage". Considering now details of the rth stage coefficients within individual blocks horizontally and vertically, and the associated variables of that stage, and ignoring the factors from the other stages which are present, we proceed as follows:

There is a row in each block giving the value of a new term in which the factor

$$\prod_{j=1}^{n_r} (x'_{rj})^{t'_{rj}}$$

occurs. The ordinal number of this row in the block has been fixed in an assigned manner by the exponents of the system $(t'_{r1}, t'_{r2}, \ldots, t'_{r\,n_r})$, admitting zero values. The column in which the coefficient of the old term in which the factor

$$\prod_{j=1}^{n_r} (x_{rj})^{t_{rj}}$$

occurs has accordingly been fixed by the same rule of order by the exponents of the system $(t_{r1}, t_{r2}, \ldots, t_{rn_r})$ also admitting zero values.

Suppose now we examine the effect of the operator

$$\sum_{i=1}^{n_r} a_{rli} \frac{\partial}{\partial a_{rji}}$$

on M, the determinant under study, in case (1), $l=j$, and in case (2), $l \neq j$. If a determinant some of whose elements are functions of a variable be differentiated with regard to that variable the result is the sum of the products of the derivative of each element into the corresponding unchanged co-factor. Consequently an operator of the above form acting on a determinant gives the sum of the products of the result of its application to each element into the corresponding unchanged co-factor. Now these products may be grouped by rows, each row of the results of operation being associated with all the other rows unchanged, and thus a total of as many determinants as the order of the determinant may be so formed.

First consider, however,

$$\sum_{i=1}^{n_r} a_{rli} \frac{\partial}{\partial a_{rji}}$$

applied not to an individual element of the determinant M but to any one of the quantics in all the variables, containing, together with factors from all the other stages, the rth stage factor

$$\prod_{j=1}^{n_r} (x'_{rj})^{t'_{rj}}.$$

This quantic may be written, for a chosen value of j,

$$C(x'_{rj})^{t'_{rj}}$$

where C contains all factors independent of a_{rji} $(i=1, 2, \ldots, n_r)$ partly from its own stage and partly from the aggregate of the others. This is therefore

$$C\left\{\sum_{i=1}^{n_r} a_{rji}x_{ri}\right\}^{t'_{rj}}.$$

Now when $t'_{rj}\neq 0$, we have

$$\sum_{i=1}^{n_r} a_{rli}\frac{\partial}{\partial a_{rji}}\cdot C\left\{\sum_{i=1}^{n_r} a_{rji}x_{ri}\right\}^{t'_{rj}}$$
$$=t'_{rj}C\left\{\sum_{i=1}^{n_r} a_{rli}x_{ri}\right\}\left\{\sum_{i=1}^{n_r} a_{rji}x_{ri}\right\}^{t'_{rj}-1}.$$

Thus in case (1), $(l=j)$, this quantic is converted into t'_{rj} times itself by the operation (as in Euler's Theorem), while in case (2), $(l\neq j)$, it is converted into t'_{rj} times the quantic of another row, identifiable by the system of exponents in which t'_{rl} and t'_{rj} are respectively replaced by $t'_{rl}+1$ and $t'^{r}_{j}-1$. The case $t'_{rj}=0$ requires special consideration. In this case the a_{rji} not being present, the result of the operation both for $l=j$ and $l\neq j$ is zero. Hence *every coefficient* in a given row is, for $t'_{rj}=0$, in both cases converted into zero, and for $t'_{rj}\neq 0$, is converted either into t'_{rj} times itself, case (1): or, case (2), into t'_{rj} times the coefficient in the same column in another row identifiable as described above.

In case (2), since the operation yields either zero for each coefficient or else a constant multiple of the corresponding coefficient of another row, the value of each determinant so produced will be zero. That is

$$(1)\qquad \sum_{i=1}^{n_r} a_{rli}\frac{\partial M}{\partial a_{rji}} = 0,\ (l, j = 1, 2, \ldots, n_r),\ (l\neq j).$$

This denotes $n_r^2-n_r$ equations. But in case (1),

$$\sum_{i=1}^{n_r} a_{rli}\frac{\partial M}{\partial a_{rji}}$$

includes for every row a determinant having the value $t'_{rj}\,M$, the case $t'_{rj}=0$ being now included. Now the set of values of t'_{rj} for the individual rows in a block are the same for each block. The total contribution from any one block of rows is $M\sum t'_{rj}$ where it is easily seen that $n_r\sum t'_{rj}=f_rp_r$.

The contribution from one block of rows is thus f_rp_rM/n_r and from all the blocks, which are

$$\frac{1}{f_r}\prod_{r=1}^{s} f_r$$

in number, it is

$$\frac{p_r}{n_r} M \prod_{r=1}^{s} f_r = u_r M,$$

say. Hence

$$\sum_{i=1}^{n_r} a_{rji} \frac{\partial M}{\partial a_{rji}} = u_r M, \ (j=1, 2, \ldots, n_r); \tag{2}$$

that is, n_r equations. Now let (1) and (2) be combined in a single statement thus:

$$\sum_{i=1}^{n_r} a_{rli} \frac{\partial (u_r^{-1} \log M)}{\partial a_{rji}} = \begin{cases} 1, & l=j, \\ 0, & l \neq j, \end{cases} (l, j=1, 2, \ldots, n_r), \tag{3}$$

and compared with

$$\sum_{i=1}^{n_r} a_{rli} \frac{\partial \log M_r}{\partial a_{rji}} = \begin{cases} 1, & 1=j, \\ 0, & l \neq j, \end{cases} \quad (l, j=1, 2, \ldots, n_r), \tag{4}$$

which describes known properties of a simple determinant.

It is seen at once that the unique solution for the n_r^2 unknown partial derivatives in (3) is

$$\frac{\partial \log M}{\partial a_{rji}} = u_r \frac{\partial \log M_r}{\partial a_{rji}}, \ (i, j = 1, 2, \ldots, n_r),$$

that is

$$\frac{\partial (\log M - u_r \log M_r)}{\partial a_{rji}} = 0. \tag{5}$$

That is, the n_r^2 quantities a_{rji} occur in $\log M$ in such a manner that when $u_r \log M_r$ involving these n_r^2 quantities only has been subtracted from it they have all disappeared from it, and the remainder consists therefore only of an unaltered function of the coefficients of the other stages.

Taking account also of the fact that the simplest possible substitutions corresponding to unity along all principal diagonals and zero elsewhere would give a similar form to the determinant yielding M, and thus give $M_r=1$, $(r=1, 2, \ldots, s)$ and $M=1$, we conclude that

$$\log M = \sum_{r=1}^{s} u_r \log M_r,$$

where

$$u_r = \frac{p_r}{n_r} \prod_{r=1}^{s} \binom{n_r+p_r-1}{p_r}. \tag{6}$$

From this general result others less general may of course be obtained by successive specialization. For this purpose the use of a notation will be convenient. Let

$$\begin{bmatrix} p \\ n \end{bmatrix}$$

denote the theorem relating to the determinant of the coefficients of the quantics representing all monomials of degree p in the n variables given by n equations linearly in terms of others. Then

$$\Pi \begin{bmatrix} p_r \\ n_r \end{bmatrix}$$

may be understood to denote that for the case where all the monomials formed as products from different substitutions are expressed as quantics in variables from the other sets, namely, the general theorem proved above.

The relationship between the general theorem and others derivable as special cases may be expressed in the following diagram, in which the abbreviation Q.D. is used for Quantic Determinant.

$$\begin{tikzcd}
 & \prod_{r=1}^{s} \begin{bmatrix} p_r \\ n_r \end{bmatrix} \arrow[dl] \arrow[d] & & \\
\prod_{r=1}^{s} \begin{bmatrix} p_r \\ n \end{bmatrix} & \prod_{r=1}^{s} \begin{bmatrix} p \\ n_r \end{bmatrix} \arrow[d] \arrow[dr] & & \\
 & \prod_{r=1}^{s} \begin{bmatrix} 1 \\ n_r \end{bmatrix} \arrow[d] & \begin{bmatrix} p \\ n \end{bmatrix} \arrow[d] \arrow[dr] & \\
 & \begin{bmatrix} 1 \\ n_1 \end{bmatrix} \begin{bmatrix} 1 \\ n_2 \end{bmatrix} & \begin{bmatrix} p \\ 2 \end{bmatrix} \text{(G)} & \begin{bmatrix} 2 \\ n \end{bmatrix} \text{(H)}
\end{tikzcd}$$

(A) General Multistage Q.D.

(C) p-Homogeneous Multistage Q.D.

(B) n-homogeneous Multistage Q.D.

(D) Linear Multistage Q.D.

(E) General One Stage Q.D.

(F) Linear Two-Stage Q.D.

(G) = Binary One-Stage Quantic Determinant.
(H) = Quadratic One-Stage Quantic Determinant.

Of the above, the only cases known to the writer as having been previously studied are those designated (F), (G) and (H) as follows:

(F): E. Pascal, *I Determinanti*, Milan, 1897, page 138, names this theorem "Teorema di Kronecker," but the reference which he gives is incorrect. The exponents are $u_1 = n_2$ and $u_2 = n_1$.

(G): A theorem of Faà di Bruno deals with this case. The demonstration is given on pages 228-29 of his *Théorie des Formes Binaires*, Turin, 1876. The exponent u of the original determinant here becomes $p(p+1)/2$.

(H): These determinants are discussed by Hunyady, Journal für Mathematik (vol. 89 (1879), p. 58). H. Leitzmann, in his translation of Pascal's *I Determinanti*, credits the theorem to Scholtz in place of Hunyady, as in the original. The exponent u proves to be $n+1$.

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ADDENDUM:—(E): Sir Thomas Muir, "A General Theorem Giving Expression for Certain Powers of a Determinant." Report of the South African Association for the Advancement of Science, Vol. 1, April, 1903. This deals with what have been called above "General One-Stage Quantic Determinants" and designated by $\begin{bmatrix} p \\ n \end{bmatrix}$. Thus $\begin{bmatrix} p \\ 2 \end{bmatrix}$ (G) and $\begin{bmatrix} 2 \\ n \end{bmatrix}$ (H) are particular cases.

PERMISSIBLE STRESSES ON RIVETS IN TENSION

By C. R. Young, *Professor of Structural Engineering*,
and W. B. Dunbar, *Lecturer in Engineering Drawing*

So commonly are rivets employed in situations where they must resist tension, or a combination of tension and flexure, or a combination of tension, shear and flexure, that one might reasonably expect to find in standard specifications full guidance for proportioning connections of this character. A search of current specifications, however, discloses an almost complete silence on the subject. While 25 or 30 years ago specifications very commonly prohibited the use of rivets in tension in no uncertain words, the disposition in recent years has been to keep the peace by removing all references of every character to the subject.

Only a very few specifications make any attempt at all to prescribe working stresses for rivets in tension. One of these is the Standard Specification for Steel Structures for Buildings of the Canadian Engineering Standards Association, in which rivets in tension are allowed a working stress of 10,000 lb. per sq. in. On the other hand, the specifications for steel bridges issued by the same association, and in force as standards at the time of writing, make no reference whatever to the subject.

Prior Tension Tests

Unwillingness on the part of the framers of specifications for steel construction to deal definitely with the matter of tension in rivets is the more remarkable by reason of the fact that experimental evidence has long existed of the high tensile resistance of rivets reasonably well driven. This has already been discussed by one of the authors at some length elsewhere.*

In the year 1900, D. C. Tennant carried out some interesting tension tests in the laboratories of the University of Toronto. (Transactions of the Engineering Society of the University of Toronto, 1899-1900.) All rivets were of 5/8 in. diameter, and were driven by pneumatic hammer without any unusual effort being made to obtain favourable

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*Bulletin, American Institute of Steel Construction, 1927.
Canadian Engineer, Nov. 8, 1927.
Contract Record and Engineering Review, Nov. 2, 1927.
Canadian Machinery and Manufacturing News, Nov. 3 and 10, 1927.
Engineering and Contracting, Dec., 1927.

results. Holes were drilled 11/16 in. in diameter. The ultimate tensile strength, based on the area of the rivet before driving, varied from 58,700 to 80,000 lb. per sq. in., with an average of approximately 72,100 lb. per sq. in. In only one case out of 28 did the ultimate strength on this basis fall below 67,200 lb. per sq. in.

Incidental to an extensive investigation of riveted work undertaken by the Verein deutscher Brücken-und Eisenbaufabriken, there were two tests of riveted joints in tension reported in Der Eisenbau, in 1912. Rivets had a diameter of 0.826 in. The ultimate tensile stress on the rivets based on the area before driving averaged 43,500 lb. per sq. in. A considerable amount of flexure in addition to the tension was incidentally applied to the rivets, and quite naturally the values obtained were less than those obtained by Mr. Tennant, in whose specimens little or no flexure existed.

A useful series of tests on rivets subjected to a combination of tension and flexure was made at the Polytechnic Institute of Brooklyn, in 1915, as reported by C. R. Hill in "Engineering News," April 12, 1917. Both 3/4- and 7/8-in. rivets were employed and the driving was done by machine, except for the rivets in one specimen which were driven by pneumatic hammer. In these specimens, as in those for the German tests, there was a large amount of flexure along with the tension on the rivets. The average ultimate tensile stress on the rivets in lb. per sq. in. of area before driving ranged from 28,700 to 47,600 lb. per sq. in. The one specimen riveted by pneumatic hammer gave the lowest results, although not very much below the results obtained for the companion specimen riveted by machine.

Another record of high tensile value of rivets is found in the tests of Frank Burton, City Building Commissioner of Detroit, who stated in 1924 that 60 rivets driven by pneumatic hammer gave elastic limits from 34,000 to 39,000 lb. per sq. in. and ultimate strengths from 60,000 to 64,000 lb. per sq. in.

UNIVERSITY OF TORONTO TESTS

In order to supplement the existing experimental data on the subject, and to throw light, if possible, upon some phases of the problem not clarified by previous researches, the authors undertook, during the summers of 1927 and 1928, under the auspices of the School of Engineering Research, a series of tests on rivets (1) in pure tension, (2) subjected to a combination of tension and flexure, and (3) subjected to a combination of tension, shear and flexure. The Dominion Bridge Co. Ltd., kindly offered the facilities of its Toronto plant for the manufactuer of the rivets, the specimens in which they were employed, and the loading girder. Grateful acknowledgment of this courtesy is here made.

Two series of specimens were prepared. Series A comprised specimens designed to disclose the strength of rivets in (1) pure tension and (2) in combined tension and flexure, while series B (including BB) comprised specimens prepared for the purpose of showing the strength of rivets subjected to a combination of tension, shear and flexure. Experimental work on Series A was carried out in the summer of 1927, and a certain amount of preliminary work was done in connection with Series B. Difficulties growing out of the large deformation of the rivets of this series made necessary the deferring of the final work upon it until the summer of 1928. Certain specimens marked BB were reserved from the preliminary ones prepared in 1927, and after slight modification to make possible their employment in the altered loading girder were included in the B series for corroborative purposes.

QUALITY OF RIVET STEEL

In order that the characteristics of the rivets being tested might be fully known, two stock bars of each of the rivet sizes tested in Series A and BB, that is, 5/8, 3/4 and 7/8 in., were subjected both to the tensile test and to the bending test. The results of the former test are contained in Table 1, from which it is seen that the average yield point varied from 33,110 lb. per sq. in. for the 7/8-in. bars, to 38,750 lb. per sq. in. for the 3/4-in. bars. The corresponding average ultimate strengths varied from 45,560 to 55,275 lb. per sq. in. The elongation in 8 in. was in no individual case less than 21.5 per cent. nor higher than 30.3 per cent.

In the bending test, each specimen bent cold through 180° flat on itself without sign of cracking on the outside of the bent portion.

For Series B a single bar for each of the sizes, 5/8, 3/4 and 7/8 in. was similarly tested. The results in tension are also given in Table 1. From this it is seen that the yield point ranged from 32,810 to 45,280 lb. per sq. in., and the ultimate strength from 48,780 to 59,760 lb. per sq. in., the higher values in both cases being for the 3/4-in. stock. The elongation for the latter was only 18.5 per cent. The relatively high values for 3/4-in. rivets indicated in Fig. 16 may have been occasioned by this higher-strength material.

The bending test for the material of Series B rivets was entirely satisfactory.

MANUFACTURE OF TEST RIVETS

As a further check on the performance of the rivets under test, it was decided to observe carefully their temperatures as manufactured for use in the specimens. This was done by means of a Leeds and Northrup optical pyrometer, disappearing filament type. In Table 2 are indicated the minimum, maximum and average temperatures for the rivets of

TABLE 1—PROPERTIES OF STEEL BAR STOCK USED IN MANUFACTURE OF RIVETS FOR TENSION TESTS

Specimen No.	Diameter of Rough Bar, In.	Diameter of Turned Specimen, In.	Yield Point in Lb. per Sq. In.	Ultimate Strength in Lb. per Sq. In.	Percentage Elongation in 8 In.
		Series A and BB			
1	5/8	0.498	37,670	51,600	26.4
2	5/8	0.493	39,490	53,400	26.2
		Average	38,580	52,500	26.3
3	3/4	0.501	40,140	56,450	21.5
4	3/4	0.507	37,360	54,100	26.6
		Average	38,750	55,275	24.1
5	7/8	0.511	31,690	44,610	30.3
6	7/8	0.508	34,530	46,510	26.9
		Average	33,110	45,560	28.6
		Series B			
7	5/8	0.562	36,710	49,630	31.5
8	3/4	0.518	45,280	59,760	18.5
9	7/8	0.504	32,810	48,780	29.0

All bars passed the bend test satisfactorily.

TABLE 2—TEMPERATURES MAINTAINED IN THE MANUFACTURE OF RIVETS FOR TENSION TESTS

Diameter of Rivet, In.	Temperature of Furnace in Degrees F. Immediately Before Withdrawal of Rod			Temperature of Rod in Degrees F. at Mid-period of Operation of Machine		
	Min.	Max.	Average	Min.	Max.	Average
			Series A and BB			
5/8	1,985	2,022	1,997	1,793	1,862	1,830
3/4	2,004	2,125	2,051	1,793	1,868	1,827
7/8	2,148	2,175	2,160	1,945	2,049	2,018
			Series B			
5/8	1,901	2,258	2,076	1,570	1,823	1,706
3/4	2,085	2,157	2,108	1,615	1,687	1,654
7/8	1,975	1,985	1,978	1,570	1,715	1,648

each size immediately before the withdrawal of the rod from the furnace and in the same table the temperature of the rod at the mid-period of the manufacturing operation in the machine. This table discloses the fact that the rods were heated in the furnace to a temperature varying from 1,901 to 2,258° F., while the manufacturing operation was carried out between 1,570 and 2,049° F. These temperatures were such as the operator of the machine had been in the habit of maintaining regularly in the manufacture of rivets for employment in commercial structural work.

To simplify the manufacture, all rivets were made 5 in. long and were later sheared cold to the length requisite to form a proper head, having regard to the various grips required in the specimens.

DRIVING OF TEST RIVETS

It was deemed advisable to note further the temperature of the rivets as they were reheated for driving, and also the temperature upon the commencement of the actual operation of driving. The results of these observations are contained in Table 3. It is there shown for Series A, for example, that the temperature of the rivet in the heating furnace ranged from 1,715 to 2,085° F. for machine-driven rivets, and the temperature at the beginning of driving for these rivets ranged from 1,417° to 1,775° F. For rivets driven by pneumatic hammer the temperatures were somewhat higher, ranging in the furnace from 1,934 to 2,388° F., and at the beginning of driving from 1,715 to 2,004° F.

Examination of the table shows that for Series BB the temperatures were comparable to those for Series A, while the temperatures for Series B were lower than for the other two series.

On account of the special form of the specimens, and by reason of loss of time at the beginning of the operation of driving with the machine and delay due to the necessity for straightening the rivets, which under machine driving were often considerably bent on the first application of pressure, machine driving was less expeditious than driving with pneumatic hammer. Hence the driving temperatures were somewhat lower for the former than for the latter. Conceivably this may have given the hammer-driven rivets some advantage over those driven by machine.

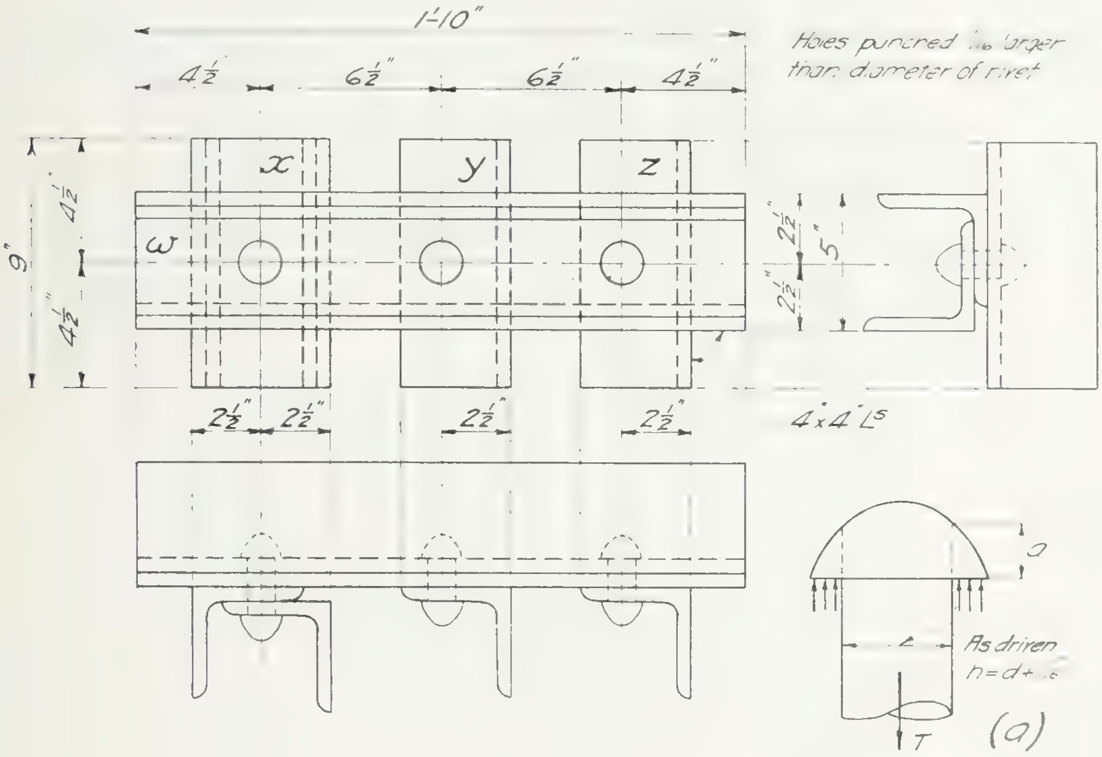
SERIES A

The specimens of Series A, shown in Fig. 1, were so planned that in each one the rivet passing through the crosspiece *x* could be subjected to pure tension, the rivet through the crosspiece *y* to a tension applied 1 1/8 in. from the centre of the rivet measured normal to the axis of the crosspiece, and the rivet through the crosspiece *z* to a tension similarly applied 2 1/4 in. from the rivet centre. By making 18 specimens, it

TABLE 3—TEMPERATURES MAINTAINED IN THE DRIVING OF TEST RIVETS

Diam. of Rivet	How Driven	Temperature of Furnace in Degrees F. Immediately Before Withdrawal of Rivet			Temperature of Rivet in Degrees F. at Beginning of Driving		
In.		Min.	Max.	Average	Min.	Max.	Average
				Series A			
5/8	Machine	1,715	2,085	1,872	1,417	1,733	1,534
3/4	Machine	1,811	1,965	1,889	1,428	1,751	1,637
7/8	Machine	1,787	1,985	1,920	1,510	1,775	1,693
	Grand average			1,894			1,621
5/8	Hammer	1,934	2,388	2,193	1,727	1,945	1,867
3/4	Hammer	2,085	2,258	2,205	1,715	1,970	1,891
7/8	Hammer	2,183	2,266	2,223	1,787	2,004	1,917
	Grand average			2,207			1,892
				Series B			
5/8	Machine	1,645	1,687	1,655	1,409	1,409	1,409
3/4	Machine	1,630	1,659	1,645	1,280	1,409	1,353
7/8	Machine	1,630	1,659	1,647	1,324	1,428	1,396
	Grand average			1,649			1,382
5/8	Hammer	1,775	1,868	1,845	1,502	1,630	1,572
3/4	Hammer	1,846	1,912	1,888	1,502	1,673	1,582
7/8	Hammer	1,823	1,955	1,921	1,536	1,727	1,637
	Grand average			1,885			1,597
				Series BB			
3/4	Machine	1,874	2,013	1,943	1,488	1,560	1,524
7/8	Machine	1,848	1,887	1,864	1,509	1,611	1,572
	Grand average			1,896			1,553
3/4	Hammer (1 specimen only)			2,319			2,031
7/8	Hammer	2,274	2,308	2,285	1,882	1,960	1,928
	Grand average			2,294			1,954

was possible to carry out each of the three tests mentioned, with three diameters of rivets, 5/8, 3/4 and 7/8 in., with three different rivet grips, and for each grip with a rivet driven by machine and with one driven by pneumatic hammer. Hence in Series A, 54 rivets in all were tested.



SPECIMENS OF SERIES "A"

Mark	Rivet Size	How Driven	Angles w and x	Angles y and z	No. Req'd
A 5/8 M1	5/8	Machine	4x4x1/4	4x4x3/8	[illegible]
A 5/8 M2	"	"	4x4x3/8	4x4x3/8	[illegible]
A 5/8 M3	"	"	4x4x1/2	4x4x3/8	1
A 3/4 M1	3/4	"	4x4x5/16	4x4x9/16	1
A 3/4 M2	"	"	4x4x1/2	4x4x9/16	1
A 3/4 M3	"	"	4x4x5/8	4x4x9/16	1
A 7/8 M1	7/8	"	4x4x3/8	4x4x3/4	1
A 7/8 M2	"	"	4x4x1/2	4x4x3/4	1
A 7/8 M3	"	"	4x4x3/4	4x4x3/4	1
A 5/8 H1	5/8	Pneumatic Ham.	4x4x1/4	4x4x3/8	1
A 5/8 H2	"	"	4x4x3/8	4x4x3/8	1
A 5/8 H3	"	"	4x4x1/2	4x4x3/8	1
A 3/4 H1	3/4	"	4x4x5/16	4x4x9/16	1
A 3/4 H2	"	"	4x4x1/2	4x4x9/16	1
A 3/4 H3	"	"	4x4x5/8	4x4x9/16	1
A 7/8 H1	7/8	"	4x4x3/8	4x4x3/4	1
A 7/8 H2	"	"	4x4x1/2	4x4x3/4	1
A 7/8 H3	"	"	4x4x3/4	4x4x3/4	1

FIG. 1—Specimens of Series A.

In order to reduce as much as possible the effect of dishing in the pure tension test, the crosspiece *x* as well as the longitudinal member *w* was made of a pair of angles arranged in channel form. In spite of this, however, dishing to the extent of 1/2 in. occurred in some of the thinner specimens, as indicated in Fig. 8. Variation in grip of the rivets was obtained by adopting a variety of thicknesses for the angles, as is indicated in the table of Fig. 1. All holes were punched 1/16 in. larger than the nominal diameter of the rivet. On account of the peculiar form of the specimens, the riveting was probably less uniform than would be the case for typical commercial work. Machine-driven rivets were driven with an air pressure of 100 lb. per sq. in., the force exerted by the machine being about 50 tons. Rivet heads were as indicated in Fig. 12.

METHOD OF TESTING, SERIES A

The crosspiece, x, consisting of a channel built up of two angles, was in a testing machine pushed away from the longitudinal element of the specimen, w, by means of an iverted U-shaped steel casting shown in Fig. 2, the plane of this U being parallel to the crosspiece x. The longitudinal member, w, was supported on the platen of the machine by means of another U-shaped casting of the same pattern placed with its plane parallel to the member w and with the prongs of the U pointing upward. The arrangements of the parts for a test in pure tension is indicated in Fig. 3.

The rivet connecting the crosspiece y to the longitudinal member w was tested by applying the load to the angle y at a distance of 1 1/8 in. from the gauge line of the angle y by means of knife edges attached to the ends of the prongs (see Fig. 2) of the inverted U-shaped casting. The angle z was correspondingly tested under an eccentricity of 2 1/4 in.

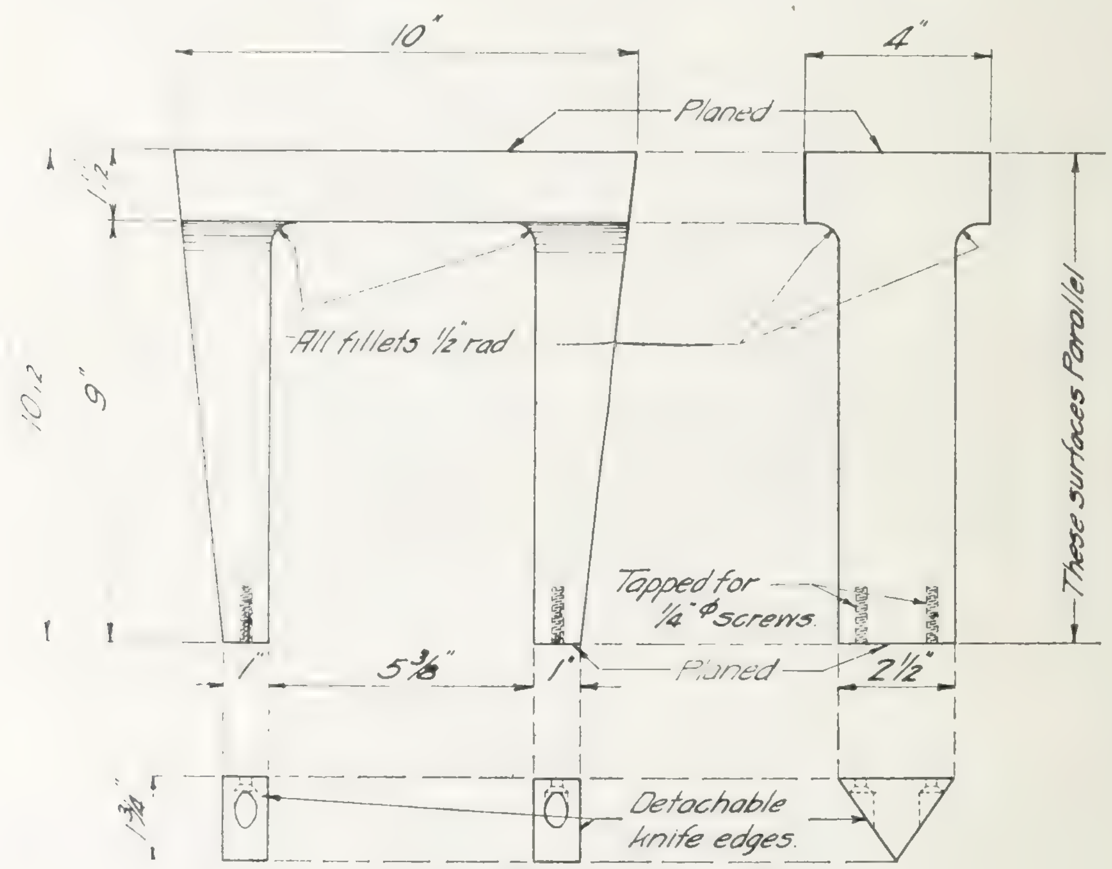


FIG. 2—Loading castings for Series A.

PURE TENSION TESTS

The manner of applying the loads to the rivets in pure tension has already been described. The use of the two U-shaped castings was found to be entirely satisfactory and the rivets through the crosspieces x were subjected as nearly as may be to a condition of pure tension.

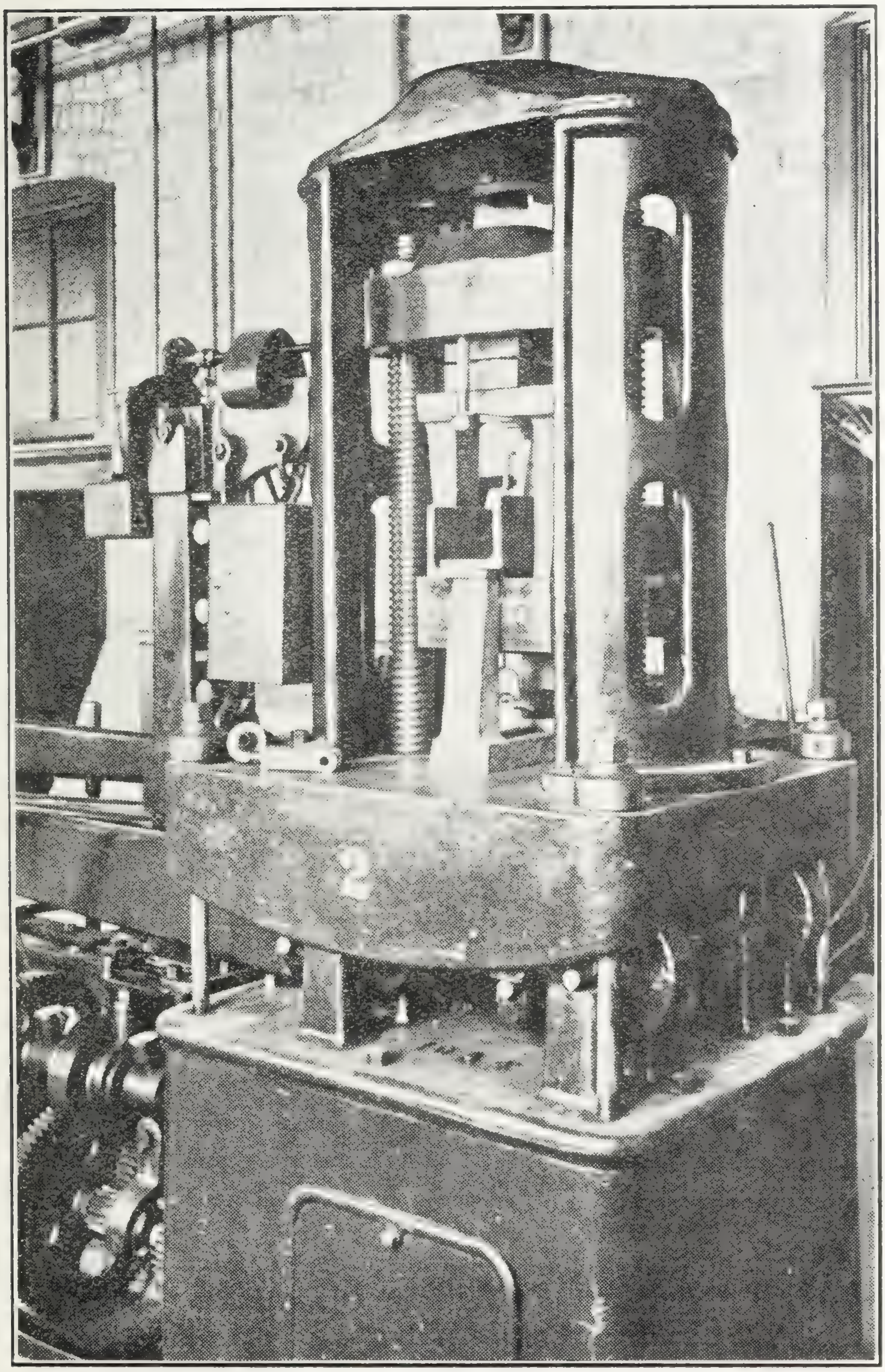


Fig. 3—Testing pure tension specimens, Series A.

Table 4—Results of Tests of Rivets in Pure Tension

Holes for Rivets Punched 1/16 in. larger than Nominal Diameter of Rivet.

Note—In the designation of the specimen, the fraction indicates the diameter of the rivet before driving; M indicates that the driving was by machine, and H that it was by pneumatic hammer. The figures 1, 2, 3 suggest the variation of grip, 1 being the smallest and 3 the largest.

Specimen	Nominal Grip of Rivet, In.	Total Maximum Load on Rivet, Lb.	Ultimate Unit Stress on Rivets in Lb. per Sq. In. On Area Before Driving	Ultimate Unit Stress on Rivets in Lb. per Sq. In. On Area After Driving
A 5/8 M1x	1	23,870	77,850	64,280
A 5/8 M2x	1 1/2	22,270	72,600	*60,000
A 5/8 M3x	2	22,370	72,920	60,250
A 5/8 H1x	1	25,450	83,000	68,550
A 5/8 H2x	1 1/2	22,740	74,150	61,300
A 5/8 H3x	2	20,200	65,850	54,420
A 3/4 M1x	1 1/4	37,790	85,500	†72,850
A 3/4 M2x	2	33,350	75,500	64,300
A 3/4 M3x	2 1/2	30,230	68,450	†58,300
A 3/4 H1x	1 1/4	34,620	78,400	66,750
A 3/4 H2x	2	31,380	71,000	60,450
A 3/4 H3x	2 1/2	30,430	68,900	58,650
A 7/8 M1x	1 1/2	41,990	69,850	60,800
A 7/8 M2x	2	39,930	66,450	57,850
A 7/8 M3x	3	35,530	59,100	51,470
A 7/8 H1x	1 1/2	40,345	67,150	58,440
A 7/8 H2x	2	36,820	61,270	53,380
A 7/8 H3x	3	33,140	55,160	48,000

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*Head sheared off. †Very eccentric head.

Table 4 contains the results of the tests of these rivets. The ultimate unit stress based on the area after driving, which is assumed to be the area of the hole, ranged from 48,000 lb. per sq. in. to 72,850 lb. per sq. in., and based on the area of the rivet before driving, from 55,160 to 85,500 lb. per sq. in. The variation of the ultimate strength on the basis of the area before driving may be readily seen from Fig. 9, in which those curves indicated by x appertain to the pure tension tests. From this diagram it is evident that an increase in the grip of a rivet lessens its ultimate tensile strength. In general, machine-driven rivets for the pure tension tests gave higher resistance for the various grips than did rivets driven by pneumatic hammer for the same grip, although the differences are not very striking. It is also evident that with an increase in the diameter of the rivet, the tensile strength in lb. per sq. in. of shaft decreases.

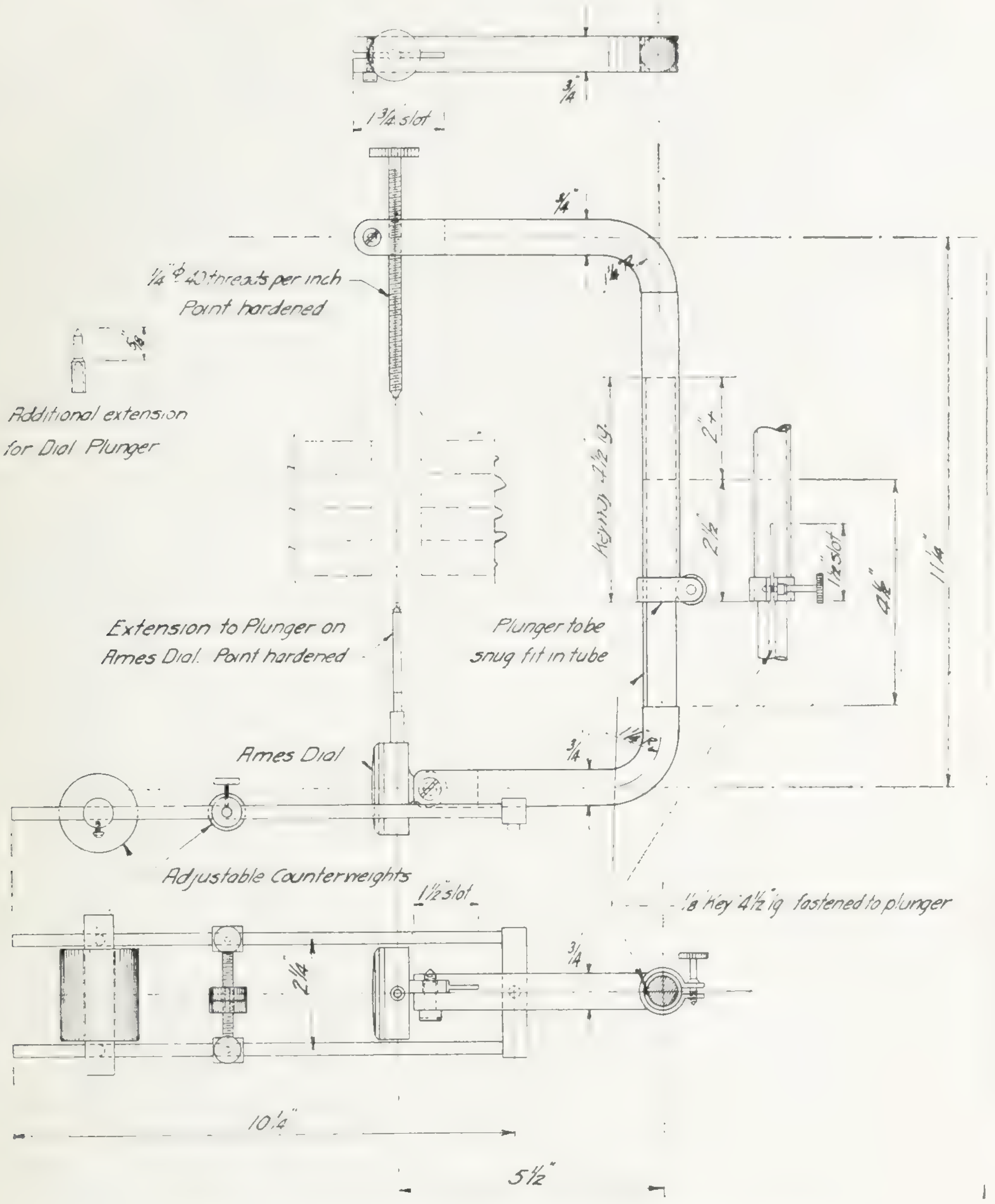

Fig. 4—Special extensometer, pure tension tests.

EXTENSION OF RIVETS

One feature of the study was the determination of the amount of extension of the rivet shaft between heads under gradually increasing centric tension. In order to measure these, it was found necessary to devise the special extensometer shown in Fig. 4, in which a standard Ames dial was incorporated. This instrument, which was found to work very well indeed, was placed in contact with the extremities of the heads of a rivet, and hence the extension read by the apparatus was the extension between these two extreme points. The extensions up to loading stages well above the yield point of the rivets were plotted in Figs. 5, 6 and 7, which refer respectively to rivets 5/8, 3/4 and 7/8-in. diameter. The diagrams show quite clearly that in a considerable number of cases no observable extension of the rivet occurred until a very large amount of load had been applied. To this striking characteristic of the curves reference will be made later.

With a view to ascertaining as nearly as might be the net extension of the rivet between the inside of the heads, an expression was derived for the extension of the two heads themselves. Assuming that the transfer of stress from the rivet shaft to a head proceeds by shear resistance on a circumferential area having as length the circumference πh of the rivet as driven, and as height the distance "a" as shown in Fig. 1(a), and assuming further that the vertical variation of shear stress is in accordance with the familiar parabolic law, it was determined that the total elongation of the two heads would be

$$\lambda = \frac{4Ta}{\pi h^2 E} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which T is the total tension and E is the modulus of elasticity. It was not deemed necessary to correct the diagrams in Figs. 5, 6 and 7 for the extension of the heads, as this quantity is comparatively small.

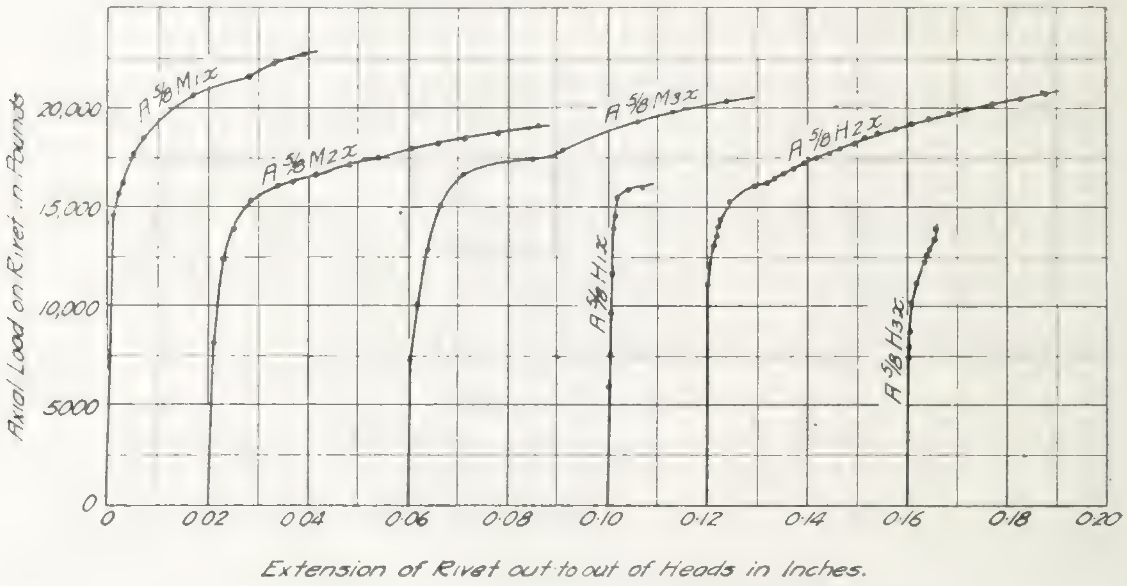

FIG. 5—Extension of 5/8-in. rivets in pure tension.

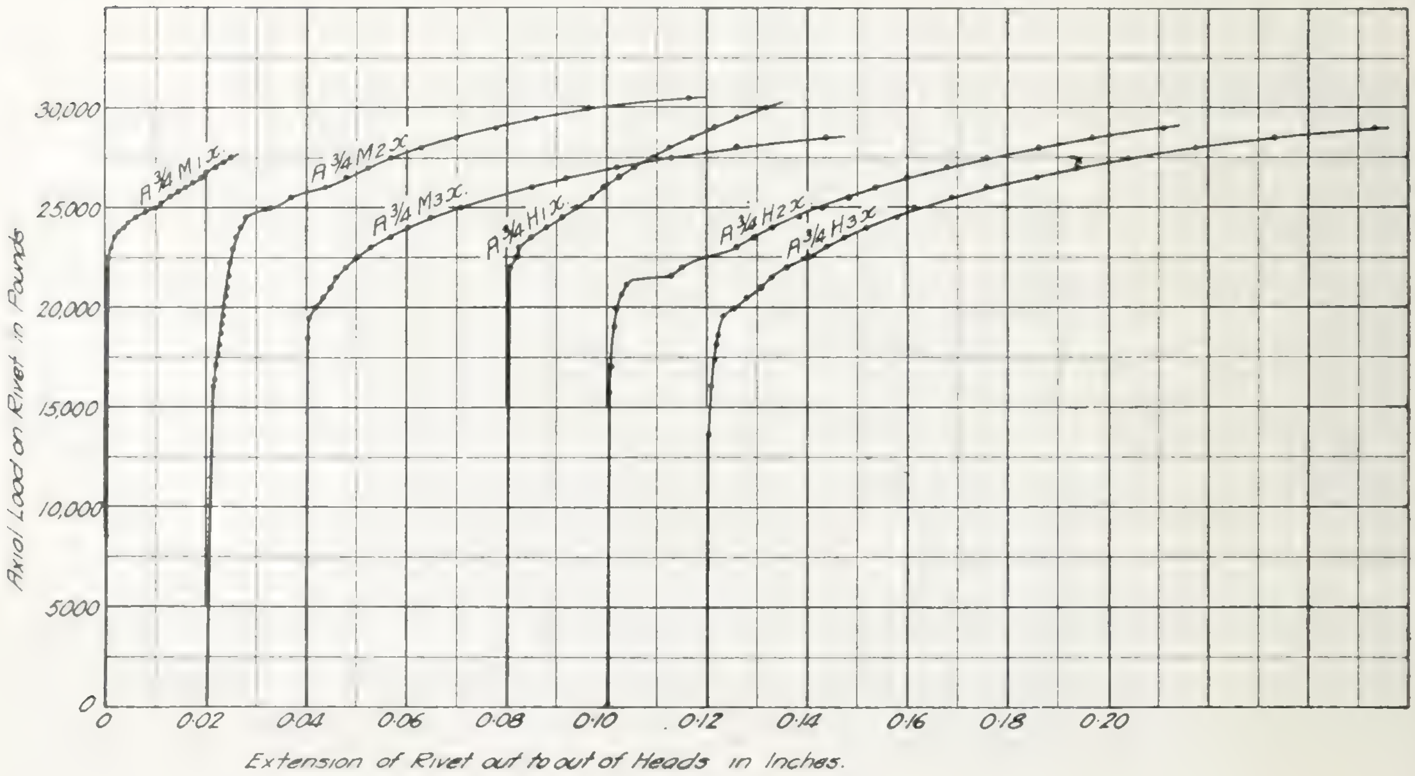

FIG. 6—Extension of 3/4-in. rivets in pure tension.

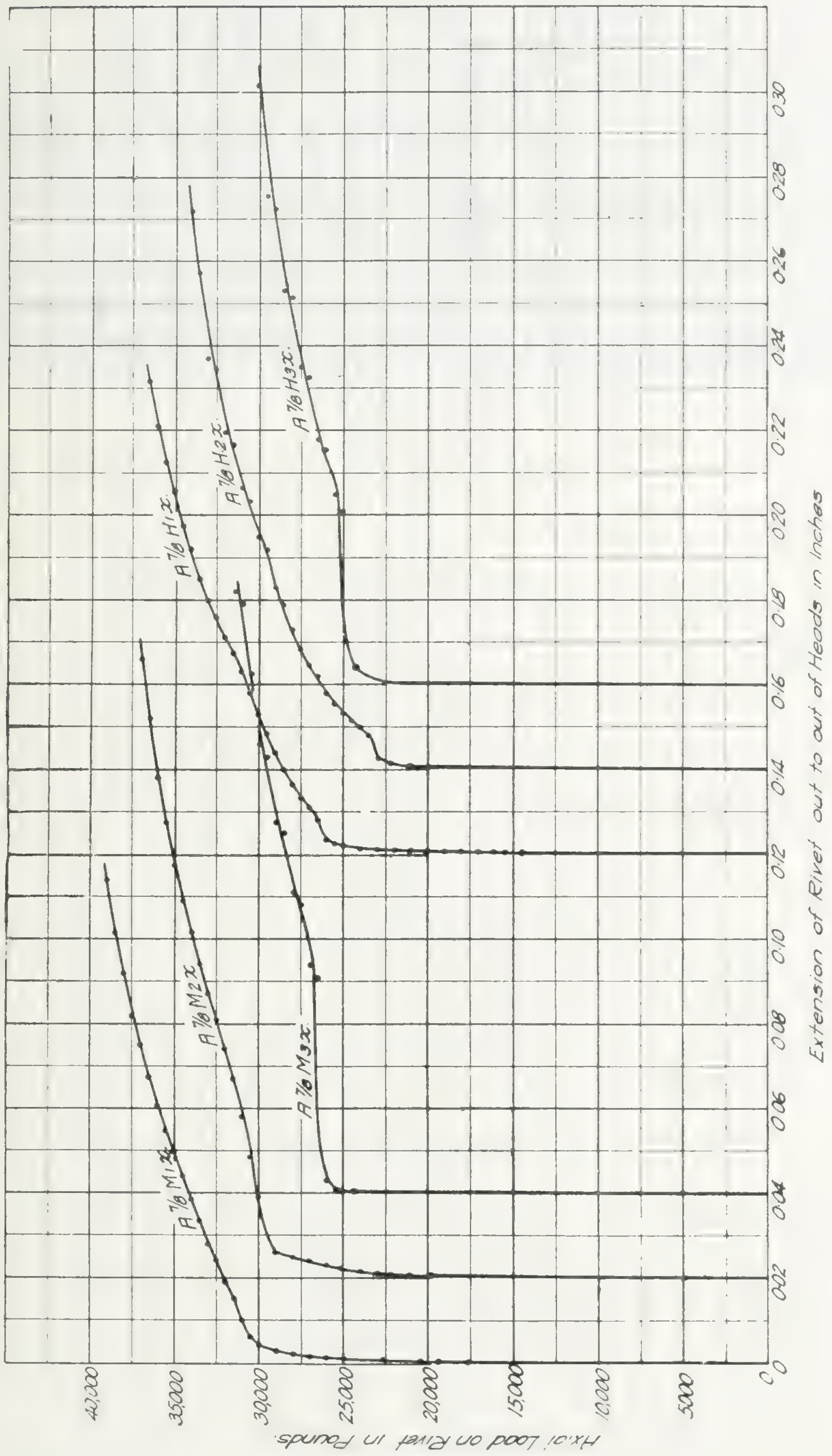

FIG. 7—Extension of 7/8-in. rivets in pure tension.

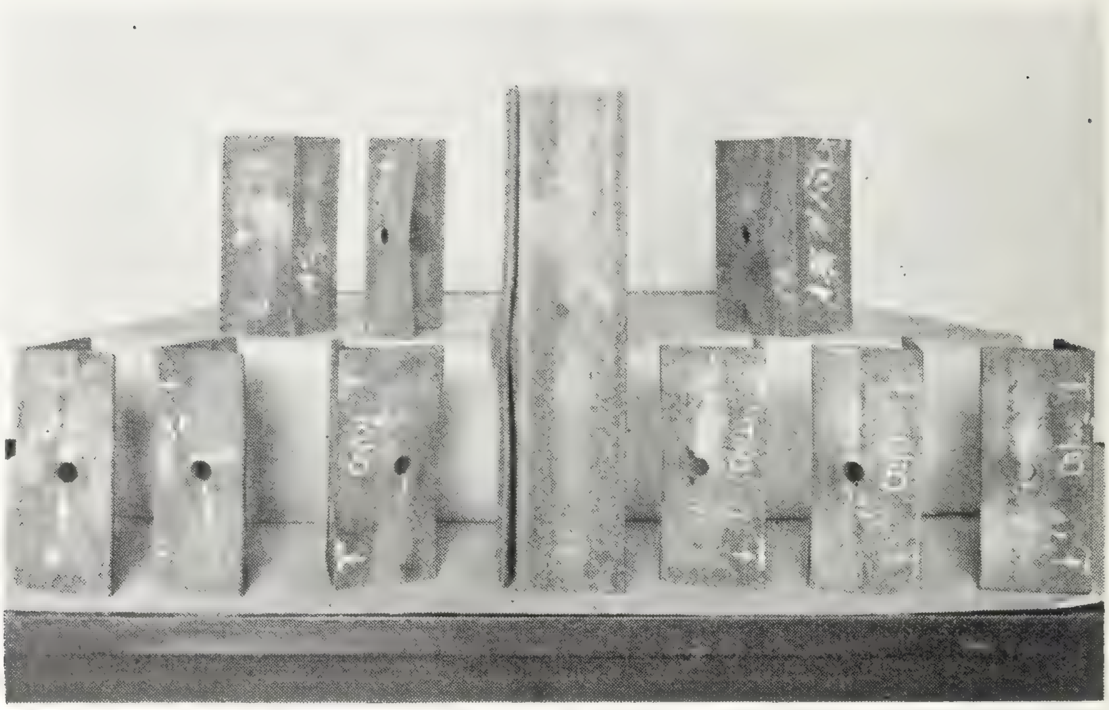

FIG. 8—Dishing effect in parts of thinner specimens, Series A.

COMBINATION OF TENSION AND FLEXURE

To ascertain the effect of a combination of tension and flexure on a rivet, the rivets connecting crosspieces *y* and *z* to the longitudinal member *w*, in Fig. 1, were subjected to loads applied respectively at 1 1/8 and 2 1/4 in. from the gauge lines of the transverse angles. The results obtained by testing the rivets in this manner are disclosed in Tables 5 and 6.

Naturally, a considerable falling off in average tensile stress developed on the rivet shaft is to be observed when the load is applied eccentrically. The general relation of the results may best be appreciated by an examination of Fig. 9. It is there seen that for specimens with an eccentricity of 1 1/8, indicated by the letter *y* in Fig. 9, there is a marked reduction of strength as compared with the specimens tested in pure tension. The doubling of the eccentricity, however, does not lead to a proportionate reduction of strength from that possessed by rivets in pure tension. Generally speaking, the machine-driven rivets for the specimens loaded at an eccentricity of 1 1/8 in. gave higher values than the rivets driven by pneumatic hammer, but where the rivets were tested under an eccentricity of 2 1/4 in., the hammer-driven rivets appeared to behave quite as well as those driven by machine. Evidently the behaviour of the angles and other conditions incidental to the specimens had more influence on the strength than the method of rivet driving. As in the case of pure tension, the larger the rivet diameter the less the strength in lb. per sq. in. of shaft.

TABLE 5—RESULTS OF TESTS OF RIVETS FOR COMBINATION OF TENSION AND FLEXURE (ECCENTRICITY, $e = 1\ 1/8$ in.).

Holes for Rivets Punched 1/16 in. larger than Nominal Diameter of Rivets.

Note—In the designation of the specimen, the fraction indicates the diameter of the rivet before driving; M indicates that the driving was by machine, and H that it was by pneumatic hammer. The figures 1, 2, 3 suggest the variation of grip, 1 being the smallest and 3 the largest.

Specimen	Nominal Grip of Rivet, In.	Total Maximum Load On Rivet, Lb.	Average Ultimate Unit Stress on Rivets in Lb. per Sq. In. On Area Before Driving	Average Ultimate Unit Stress on Rivets in Lb. per Sq. In. On Area After Driving
A 5/8 M1y	1 1/4	16,700	54,450	*45,000
A 5/8 M2y	1 1/2	18,530	60,400	49,950
A 5/8 M3y	1 3/4	15,130	49,320	40,780
A 5/8 H1y	1 1/4	16,090	52,450	43,360
A 5/8 H2y	1 1/2	14,050	45,800	37,860
A 5/8 H3y	1 3/4	11,890	38,780	32,020
A 3/4 M1y	1 3/4	20,490	46,380	39,510
A 3/4 M2y	2 1/8	17,230	39,010	33,220
A 3/4 M3y	2 3/8	19,185	43,450	†37,000
A 3/4 H1y	1 3/4	17,845	40,400	34,400
A 3/4 H2y	2 1/8	19,820	44,890	38,220
A 3/4 H3y	2 3/8	19,140	43,330	36,910
A 7/8 M1y	2 1/4	21,915	36,480	31,760
A 7/8 M2y	2 1/2	21,365	35,530	30,930
A 7/8 M3y	3	20,880	34,730	30,230
A 7/8 H1y	2 1/4	20,625	34,330	29,890
A 7/8 H2y	2 1/2	19,870	33,050	28,790
A 7/8 H3y	3	20,790	34,570	30,110

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*Angle failed. †Head sheared off.

INITIAL TENSION

One of the reasons adduced for avoiding the use of rivets in tension is that as a result of shrinkage of the rivet in cooling, a tensile stress is developed frequently in excess of the yield point of the rivet material. In support of the prohibitory attitude, there have been frequent references to certain investigations reported in the Zeitschrift des Vereines deutscher Ingenieure, November 23, 1912. The experiments there reported showed that the tensile stresses developed in rivets parallel to the shaft as a result of cooling subsequent to being driven by a hydraulic riveting machine amounted to about 31,000 lb. per sq. in. Many have held to the view that if any additional load in tension were applied to the rivet it would be on the verge of failure.

With this view the authors are not in accord. The initial tension in a rivet shaft may fairly be considered as generated in resistance to the increase of thickness that tends to take place in the plates or other parts connected together by the rivet following the thinning of these parts by the pressure of the riveting machine and by the shrinkage of the rivet in cooling.

For present purposes, these plates may be considered as replaced by two elastic collars or sleeves of aggregate length under zero stress of l (see Fig. 10). The tension which these virtual springs will produce in the rivet shaft will be directly proportional to the amount of contraction Δl from the unstressed length produced in them by the corresponding force ΣP arising from the pressure of the riveting machine and the shrinkage of the rivet in cooling.

TABLE 6—RESULTS OF TESTS OF RIVETS FOR COMBINATION OF TENSION AND FLEXURE
(ECCENTRICITY, $e = 2\ 1/4$ in.)

Holes for Rivets Punched 1/16 in. larger than Nominal Diameter of Rivet.

Note—In the designation of the specimen, the fraction indicates the diameter of the rivet before driving; M indicates that the driving was by machine, and H that it was by pneumatic hammer. The figures 1, 2, 3 suggest the variation of grip, 1 being the smallest and 3 the largest.

Specimen	Nominal Grip of Rivet, In.	Total Maximum Load on Rivet, Lb.	Average Ultimate Unit Stress on Rivets in Lb. per Sq. In.	
			On Area Before Driving	On Area After Driving
A 5/8 M1z	1 1/4	10,550	34,390	28,420
A 5/8 M2z	1 1/2	8,040	26,190	*21,650
A 5/8 M3z	1 3/4	10,220	33,320	27,540
A 5/8 H1z	1 1/4	10,530	34,320	28,370
A 5/8 H2z	1 1/2	11,400	37,170	*30,710
A 5/8 H3z	1 3/4	10,205	33,260	27,490
A 3/4 M1z	1 3/4	14,335	32,440	27,630
A 3/4 M2z	2 1/8	12,900	29,210	24,890
A 3/4 M3z	2 3/8	12,735	28,820	24,550
A 3/4 H1z	1 3/4	13,625	30,840	26,280
A 3/4 H2z	2 1/8	13,810	31,260	26,620
A 3/4 H3z	2 3/8	13,610	30,800	26,230
A 7/8 M1z	2 1/4	15,620	25,990	22,620
A 7/8 M2z	2 1/2	13,920	23,160	20,170
A 7/8 M3z	3	14,200	23,620	20,570
A 7/8 H1z	2 1/4	14,280	23,760	20,690
A 7/8 H2z	2 1/2	14,840	24,680	21,490
A 7/8 H3z	3	14,390	23,930	20,840

---

*Angle failed.

On the other hand, any reduction of aggregate length of the collars will bring a proportionate relief in tension on the rivet shaft. This will be apparent if one regards the two collars as two helical springs bearing against each other along the surface AA. Reduction of length of the springs by any means whatsoever will lessen the pressure transmitted by them in an axial direction, that is, normal to AA.

Consider now that a tension ΣP is applied to the rivet shaft by a series of small rods passing through the collars parallel to their length and on the centre of the annular cross-section of the collars, each rod being anchored to the collar through which it passes by a countersunk head at AA. This approximates the mechanism by which tension is applied to rivets in practical constructions.

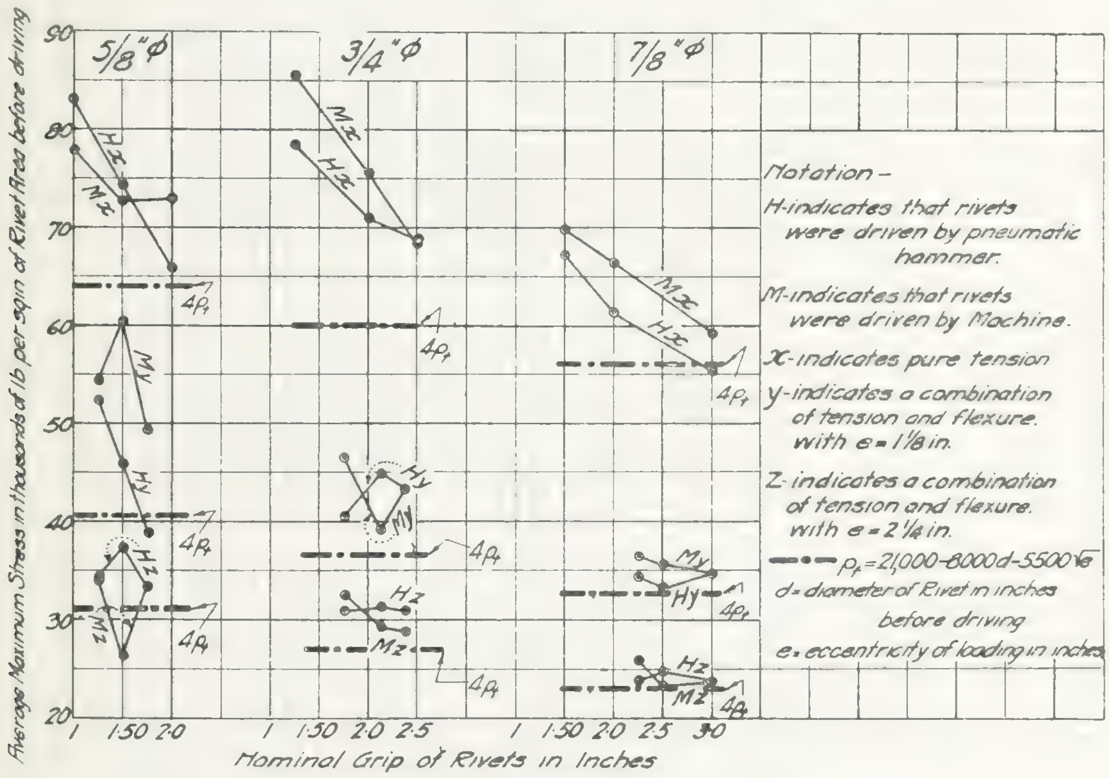


FIG. 9—Variation of ultimate tensile strength with grip, rivet diameter and eccentricity.

With the application of ΣP to the collars (or springs) their aggregate length will shorten by an amount Δl indicated in Fig. 10(a), and will then be l-$2\Delta l$. The pressure transmitted to one collar (spring) by the other will therefore be reduced by a corresponding amount, that is, by ΣP; or, in short, ΣP pounds less force is delivered by the collars to the rivet heads than was the case before the tension ΣP was applied by the rods. Consequently the imposition of a tension of ΣP has not altered the total tension in the rivet shaft. No increase of stress on the rivet is therefore possible until the load imposed has exceeded the initial tension existing in the rivet.

Experimental support of this view is available in the extension observations on the rivets subjected to pure tension in the University of Toronto tests. In Figs. 5, 6 and 7 it will be observed, as has already been pointed out, that in numbers of cases no deformation whatever was discernable until a very considerable load had been applied to the rivet. If one divides the total loads shown in the diagrams by the area of the rivet shaft as driven, it will be found that, in general, extensions of the rivet are not discernable until a stress has been reached comparable with the initial stress in rivets found experimentally by the German investigators.

A further circumstance of importance is that the rivet shafts in most of the tests reported in this paper developed an ultimate tensile strength in excess of the known ultimate tensile strength of the material in standard tension tests. Had the rivets been on the verge of failure due to high initial tension it would not have been possible to apply the large loads actually applied to the rivets before failure was produced.

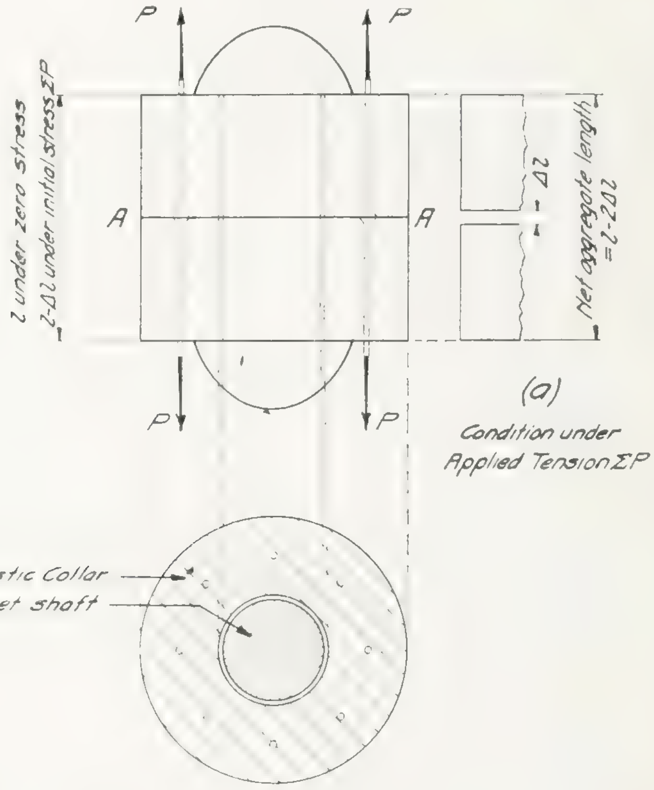

P
P
P
P
A
A
l under zero stress
l-Δl under initial stress ΣP
Δl
Net aggregate length =l-2Δl
(a)
Condition under Applied Tension ΣP
Elastic Collar
Rivet shaft

FIG. 10—Effect of applying tension to rivet under initial stress.

PERMISSIBLE STRESSES FOR TENSION WITH FLEXURE

Consideration of the diagrams contained in Fig. 9 leads to the conclusion that the working stress on rivets in tension, or in tension and flexure, should depend both on the diameter of the rivet and upon the eccentricity with which the tensile force is applied. Upon this basis a formula has been devised such that a factor of safety of approximately 4 is maintained with respect to the minimum values obtained in the University of Toronto tests. At the same time, the results obtained in other tests have been carefully considered in the devising of the formula.

For cases of pure tension, and for cases of a combination of tension

TABLE 7—COMPARISON OF PROPOSED FORMULA FOR PERMISSIBLE TENSILE STRESS ON RIVETS WITH RESULTS OF TESTS FOR RIVETS SUBJECTED TO TENSION OR TENSION AND FLEXURE

Formula, $p_t = 21{,}000 - 8{,}000\,d - 5{,}500\,\sqrt{e}$
in which p_t = permissible tensile stress on rivets;
d = diameter of rivet in inches, before driving;
e = eccentricity of loading on rivet in inches (to be taken as two-thirds the gauge for certain practical cases mentioned in text).

Name of Investigation	Type of Stress	Minimum Test Values Divided by Four, Lb. per Sq. In. — Diameter of Rivets Before Driving: 5/8 in.	3/4 in.	7/8 in.	Permissible Tensile Stress by Formula, Lb. per Sq.In.
Tennant	Pure tension, $e=0$	14,680	......	......	16,000
Verein deutscher B.-und E.	Tension and flexure, $e=1.47$	......	......	*10,880	7,750
Polytechnic Institute of Brooklyn	Tension and flexure, $e=1.16$	......	8,800	......	9,070
" " "	Tension and flexure, $e=1.16$	......	......	7,180	8,070
University of Toronto, 1927	Pure tension, $e=0$	16,460	......	......	16,000
" " "	Pure tension, $e=0$	......	17,110	......	15,000
" " "	Pure tension, $e=0$	......	......	13,790	14,000
" " "	Tension and flexure, $e=1.13$	9,690	......	......	10,200
" " "	Tension and flexure, $e=1.13$	......	9,750	......	9,200
" " "	Tension and flexure, $e=1.13$	......	......	8,260	8,200
" " "	Tension and flexure, $e=2.25$	6,550	......	......	7,750
" " "	Tension and flexure, $e=2.25$	......	7,200	......	6,750
" " "	Tension and flexure, $e=2.25$	......	......	5,790	5,750

*Diameter of rivet, 0.826 in.

and flexure, with angles of a thickness comparable to those used in the tests herein reported, the formula for permissible tensile stress is as follows:

$$p_t = 21{,}000 - 8{,}000d - 5{,}500\sqrt{e} \qquad \text{(2)}$$

in which
p_t = permissible tensile stress on rivets;
d = diameter of rivet in inches before driving;
e = eccentricity of loading on rivet in inches.

In the determination of the eccentricity e, it was assumed, as a result of consideration of the nature of the flexure of the leg containing the tension rivets, which may be called the outstanding leg, where the force is applied to the angle by a riveted connection to the other leg, that e should be about two-thirds the value of the gauge for the outstanding leg.

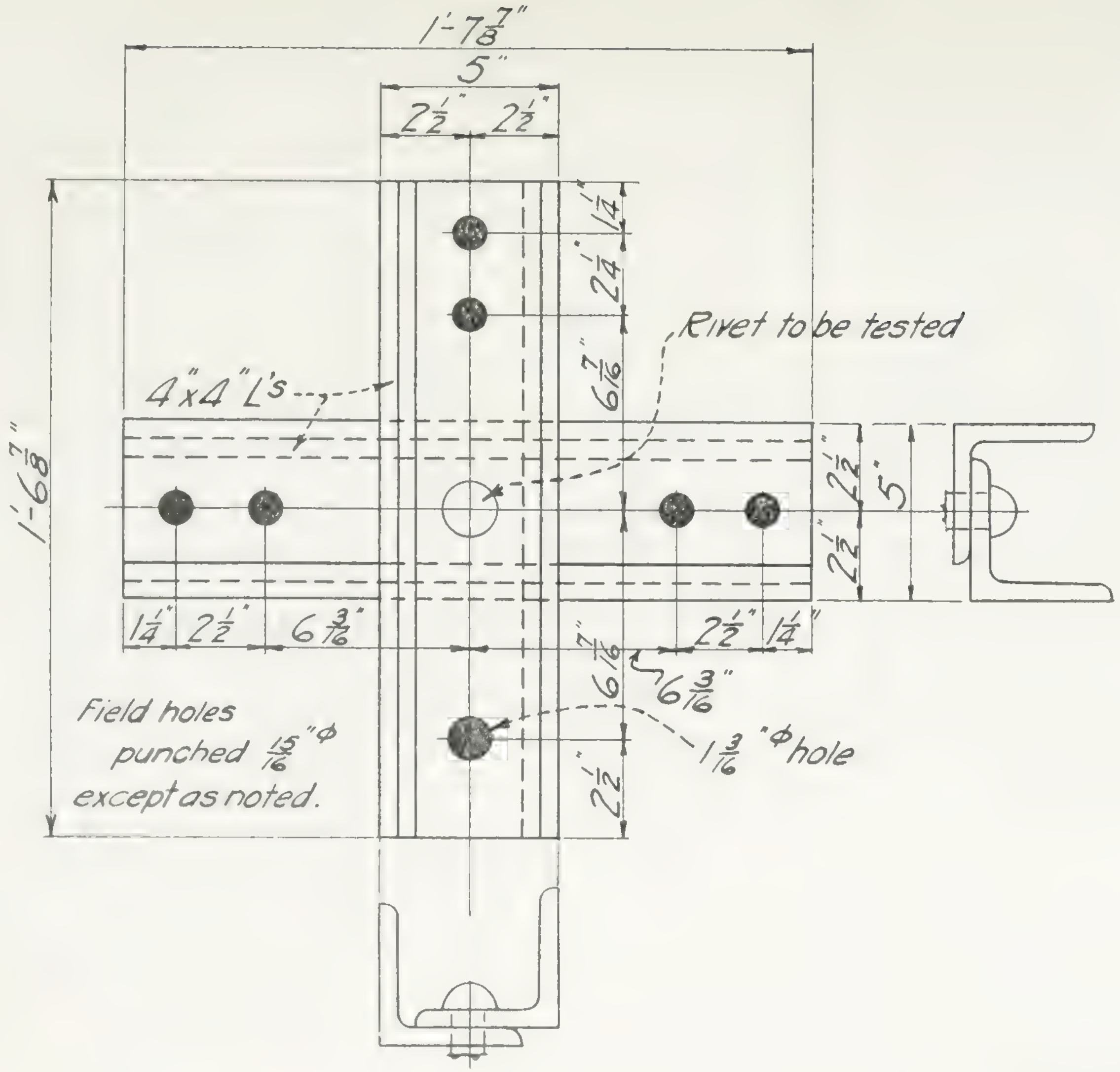
In Table 7 there is indicated the relation of the values given by this formula to the minimum test values for all the tests reported herein. The correspondence is as good as could normally be expected. Recognition has been given to the fact that the material for the 3/4-in. rivets was of a higher quality than that for the 5/8-in. and the 7/8-in. rivets. In order to appreciate visually the relation of this formula to the results obtained in the University of Toronto tests, the values given by the formula have been multiplied by 4 and the points corresponding to these values located under the respective rivet sizes in Fig. 9.

SERIES B AND BB

In carrying out the investigation of the rivets of Series B and BB, which were subjected to a combination of tension, shear and incidental flexure, the chief difficulty experienced was in devising a means of applying tension and shear to a rivet in a definite and known relation. However, based upon preliminary work done in 1927, a practicable form of specimen indicated as Series B and BB, shown in Fig. 11, was developed. These specimens were designed to be, and were successfully, tested in the special loading girder shown in Fig. 13.

The number of specimens tested in the second investigation was 45, of which 36 belonged to Series B and 9 to Series BB. The specimens of Series BB were some that were made for a trial series in 1927, and after certain modification to permit their being used in the loading girder as finally developed, were tested to supplement the results from Series B.

In the specimens, the test rivet grips two built-up channels placed back to back in rectangular cruciform arrangement. These channels were made of pairs of angles as shown in Fig. 11. For Series B, three sizes of rivets were used, viz., 5/8, 3/4 and 7/8-in., the rivets for one-half of the specimens being driven by machine and for the other half by pneumatic hammer. For each diameter of rivet and for each method of



SPECIMENS OF SERIES "B" AND "BB"

Mark	Rivet Size	How Driven	Size of L's	No. Req'd
B 5/8 M1	5/8"	Machine	4x4x1/4	2
B 5/8 M2	"	"	4x4x3/8	2
B 5/8 M3	"	"	4x4x1/2	2
B 3/4 M1	3/4	"	4x4x5/16"	2
B 3/4 M2	"	"	4x4x7/16	2
BB 3/4 M2	"	"	4x4x1/2	1
B 3/4 M3	"	"	4x4x9/16	2
BB 3/4 M3	"	"	4x4x5/8	1
B 7/8 M1	7/8	"	4x4x3/8	2
BB 7/8 M1	"	"	4x4x3/8	1
B 7/8 M2	"	"	4x4x1/2	2
BB 7/8 M2	"	"	4x4x1/2	1
B 7/8 M3	"	"	4x4x11/16	2
BB 7/8 M3	"	"	4x4x3/4	1
B 5/8 H1	5/8	Pneumatic Ham.	4x4x1/4	2
B 5/8 H2	"	"	4x4x3/8	2
B 5/8 H3	"	"	4x4x1/2	2
B 3/4 H1	3/4	"	4x4x5/16	2
B 3/4 H2	"	"	4x4x7/16	2
B 3/4 H3	"	"	4x4x9/16	2
BB 3/4 H3	"	"	4x4x5/8	1
B 7/8 H1	7/8	"	4x4x3/8	2
BB 7/8 H1	"	"	4x4x3/8	1
B 7/8 H2	"	"	4x4x1/2	2
BB 7/8 H2	"	"	4x4x1/2	1
B 7/8 H3	"	"	4x4x11/16	2
BB 7/8 H3	"	"	4x4x3/4	1

FIG. 11—Specimens of Series B and BB.

driving, test rivets having three different grips were employed, these grips being approximately 1.7, 2.3 and 3.1 times the diameter of the rivet before driving. For Series BB only 3/4 and 7/8-in. rivets were used, five of the specimens being machine-riveted and four of them being riveted by pneumatic hammer. A variation of grip existed also, as will be seen by consulting Fig. 11.

Holes for test rivets were punched 1/16 in. larger than the nominal diameter of the rivet. Rivets were of the dimensions indicated in Fig. 12.

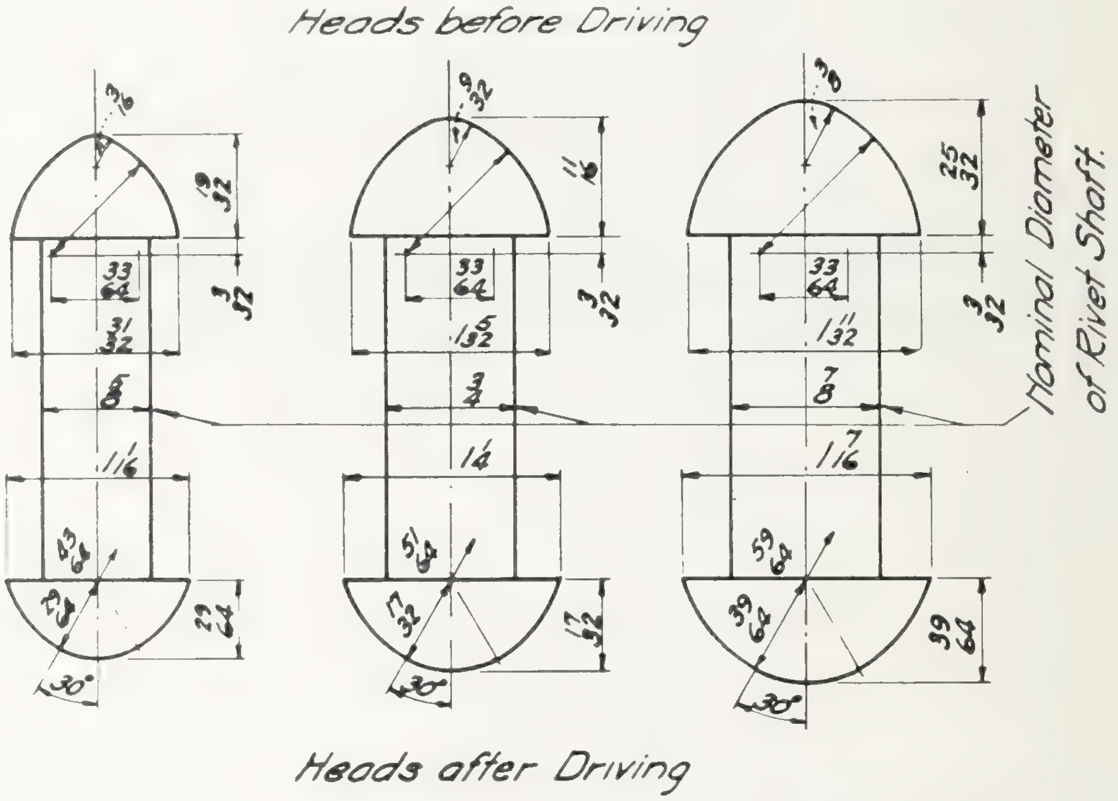


FIG. 12—Dimensions of rivets used in tests.

METHOD OF TESTING

In order to subject the test rivet to simultaneous tension and shear in definite known relation, the special loading girder indicated in Fig. 13 was designed.

There is, of course, a considerable amount of incidental flexure on any rivet subjected to a combination of tension and shear, but no attempt was made to evaluate it in the present investigation. As will be seen from Fig. 13, the loading girder was divided at about the third point into two sections, Nos. 1 and 2. In operation this girder was united at the top flange by a cruciform test specimen shown in Fig. 11, and at the bottom flange by 3-in. greased pin passing through a round hole in the pin plates of Section 2 and through a vertically slotted hole in the pin plates of Section 1. The test specimen was so placed that the horizontal member was bolted to the jaws of Section 1 of the loading girder and the vertical member to the jaws of Section 2.

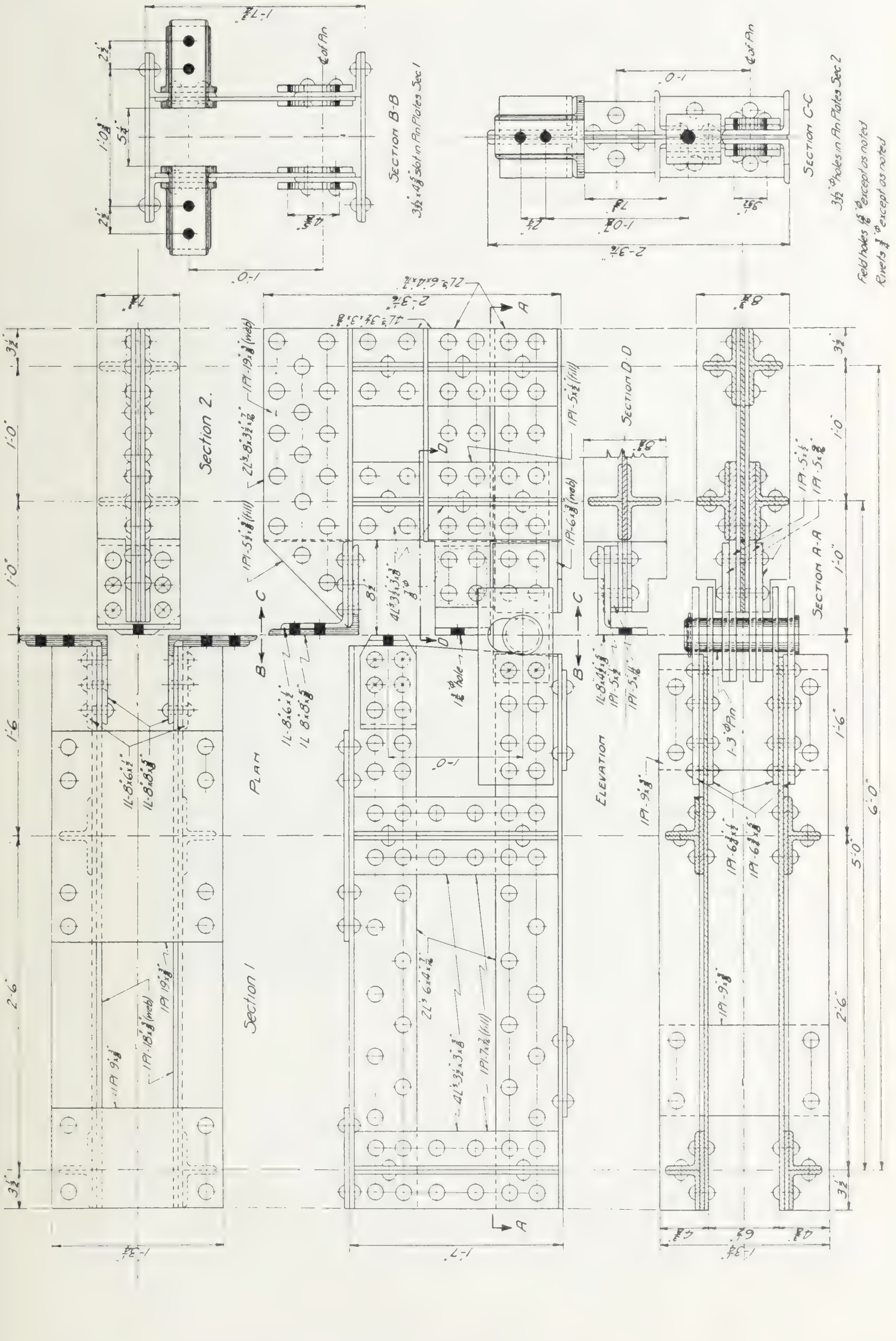

Section B-B
3½ x 4⅜ slot in Pin Plates Sec 1
Section C-C
3½ φ holes in Pin Plates Sec 2
Field holes 15/16 φ except as noted
Rivets ⅞ φ except as noted
Section 2.
Section 1
Plan
Elevation
Section D-D
Section A-A
1-3 φ Pin
1⅛ φ hole
2-3½
1-0
6-0
5-0
2-6
1-6

FIG. 13—Details of loading girder.

For operation, the loading girder was placed on the cross beam of a 200,000-lb. Riehlé Universal Testing Machine, as shown in Fig. 14. Loads were applied through a knife edge by the moving cross head of the machine at a point 2 ft. 6 in. from the end bearing of Section 1. Such superimposed load resulted in the jaws of Section 1 trying to force the horizontal member of the test specimen away from the vertical member, which was bolted to Section 2, and hence the test rivet was subjected to tension. At the same time a shear was set up at the junction of Sections 1 and 2.

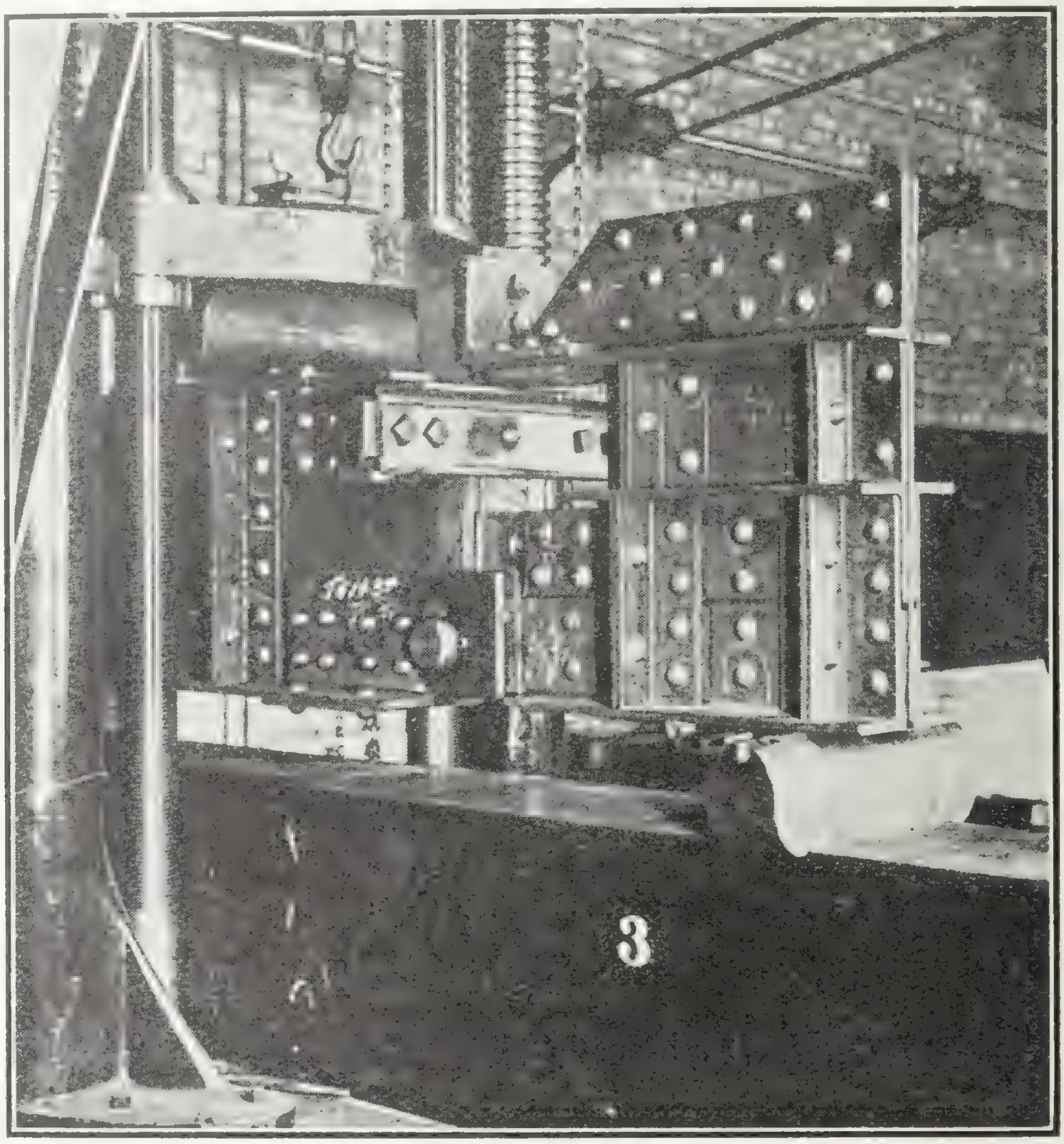


FIG. 14—Loading girder with test specimen in place.

Evidently the amount of shear actually borne by the test rivet would depend on the fraction of the total shear at the section that is resisted by sliding frictional resistance of the pin on the vertical plane sides of the slotted hole. Experimental investigation of the matter showed that the coefficient of this sliding friction for loads within the range of the present investigation averaged about 0.15. The shear absorbed by this friction would then be 0.15 C', where C' is the total compression on the side of the pin. As C' is exactly equal to the total rivet tension T', then, if the total shear at the girder joint is V, the shear borne by the rivet alone would be

$$V' = V - 0.15\ T' \quad \ldots\ldots\ldots\ldots (3)$$

Two types of loading were adopted, that is, (1) with the shear at the joint between the girder sections due to the *superimposed* load equal to the tension on the test rivet due to the same load, or $V = T$, and (2) with the shear at the joint due to *superimposed* load equal to one-half of the tension on the test rivet due to this load, or $V = 1/2\ T$.

The desired relation of the shear at the joint to the rivet tension for *super-imposed* load, and vice-versa, was easily obtained. Since the effective depth of the girder at the pin, as shown in Fig. 13, was 1 ft., the relation $V = T$ could be satisfied by placing the knife edge support for Section 2 at 1 ft. from the pin, and for the relation $V = 1/2\ T$ at 2 ft. from the pin.

The relation of the total shear at the joint arising from the combination of the superimposed load and the weight of the apparatus to the total tension on the test rivet would be slightly different from the ratios expressed immediately above, which were established for superimposed load only. This effect is a variable one, but at its maximum it does not alter the ratios of shear to tension by more than 1 3/4%. Neglecting this small factor, which may well be merged into the slightly variable coefficient of friction, the ratios of actual rivet shear to actual rivet tension for the $V = T$ loading would be, if T be replaced by T', the tension due to all causes,

$$V' = T' - 0.15\ T' = 0.85\ T' \quad \ldots\ldots\ldots\ldots (4)$$

and for the $V = 0.5\ T$ loading,

$$V' = 0.50\ T' - 0.15\ T' = 0.35\ T' \quad \ldots\ldots\ldots\ldots (5)$$

As a result of the preliminary work done in 1927, the difficulties of operation, which grew out of the large deformation of the rivets, were overcome. In the work as finally carried out, the loading girder functioned very satisfactorily. The pin was thoroughly greased and worked freely in the slot, although the lateral clearance was necessarily small. The jaws of Section 1 had to be well greased, as in certain cases due to the spreading of the angles the clearance was reduced to a very small amount. Special note has been made in Tables 8 and 9 and in Fig. 16 of the two cases where friction was evident.

The nature of the fractures of the test rivets may be seen in Fig. 15, which also shows the rivets of Series A tested in pure tension and in tension and flexure in 1927. In Series B a considerable number of eccentric heads appeared, but there was no apparent deficiency of strength arising from these. Failure in general was the ordinary transverse shear failure, more marked for the nominal $V = T$ loading than for the nominal $V = 1/2\ T$ loading.

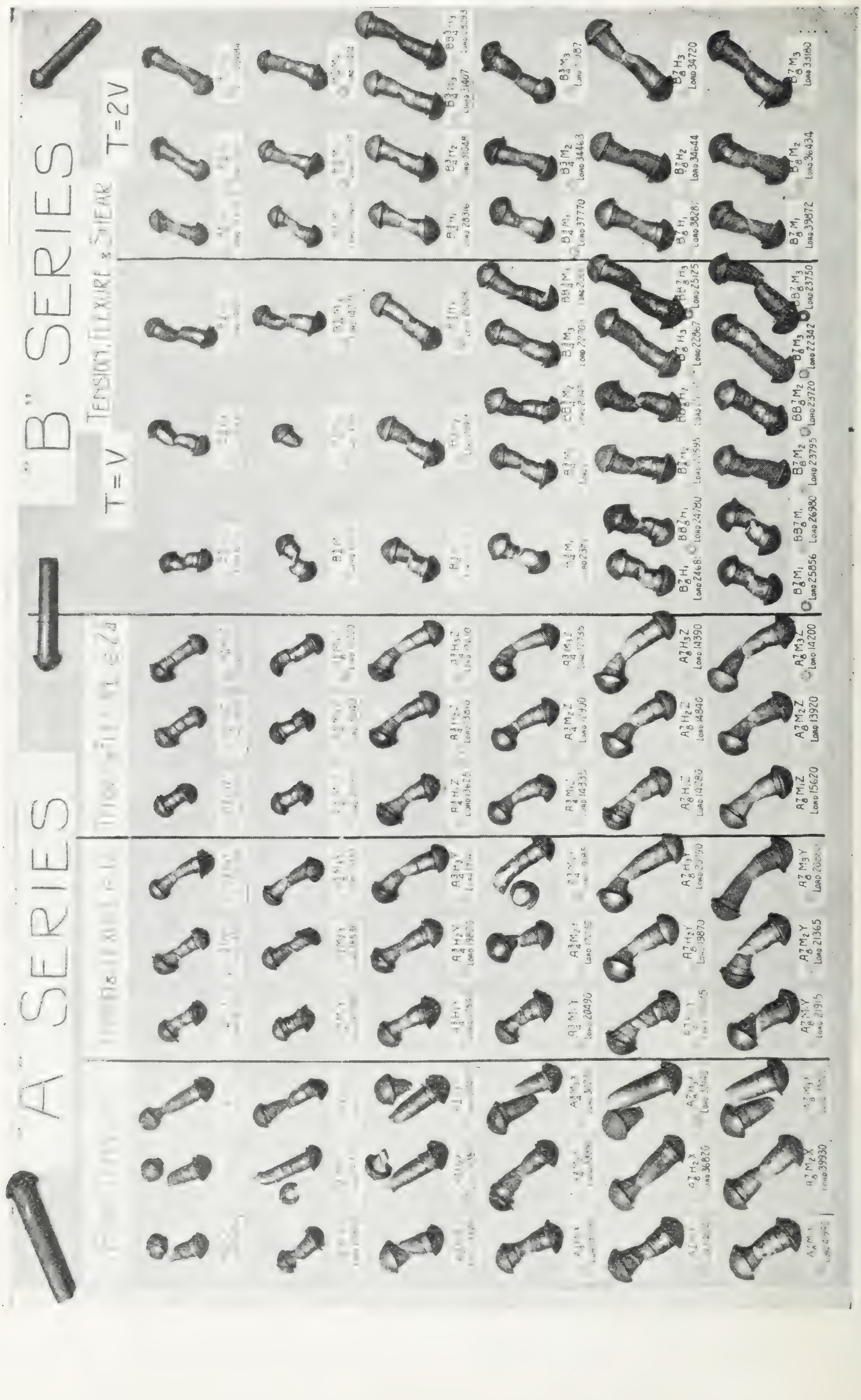


FIG. 15—Rivets of A, B and BB Series after failure.

Attention is directed to the interesting type of failure disclosed in specimens BB 7/8 H2, BB 7/8 H3, BB 7/8 M1 and BB 7/8 M3. In all cases the stress ran up before this type of failure, although the reason for it is not apparent.

TABLE 8—RESULTS OF TESTS OF RIVETS IN COMBINATION OF TENSION, SHEAR AND FLEXURE, WHERE $V = T$ FOR SUPERIMPOSED LOAD AND $V' = 0.85\ T'$ FOR TOTAL LOAD

Holes for Rivets Punched 1/16 in. Larger than Nominal Diameter of Rivets.

Note:—In the designation of the specimen, the fraction indicates the diameter of the rivet before driving; M indicates that the driving was by machine, and H that it was by pneumatic hammer. The figures 1, 2, 3 suggest the variation of grip, 1 being the smallest and 3 the largest.

Specimen	Nominal Grip, In.	Dead Load Tension, Lb.	Superimposed Load Tension, Lb.	Total Maximum Tension, Lb.	Average Ultimate Tensile Stress in Lb. per Sq. In.	
					On Area Before Driving	On Area After Driving
B 5/8 M1	1	316	15200	15516	50570	41800
B 5/8 M2	1 1/2	331	13975	14306	46630	38540
B 5/8 M3	2	345	13930	14275	46530	38460
B 5/8 H1	1	316	15800	16116	52530	43420
B 5/8 H2	1 1/2	331	14200	14531	47360	39146
B 5/8 H3	2	345	13675	14020	45700	37770
B 3/4 M1	1 1/4	322	23500	23822	53920	45940
B 3/4 M2	1 3/4	338	22550	22888	51810	44140
BB 3/4 M2	2	346	20125	20471	46340	39480
B 3/4 M3	2 1/4	353	22350	22703	51390	43790
BB 3/4 M3	2 1/2	361	19650	20011	45290	38590
B 3/4 H1	1 1/4	322	20300	20622	46680	39770
B 3/4 H2	1 3/4	338	20575	20913	47340	40330
B 3/4 H3	2 1/4	353	20175	20528	46460	39590
B 7/8 M1	1 1/2	331	25525	25856	43000	37460
BB 7/8 M1	1 1/2	331	26650	26981	44870	39090
B 7/8 M2	2	345	23450	23795	39570	34470
BB 7/8 M2	2	345	23375	23720	39450	34360
B 7/8 M3	2 3/4	367	21975	22342	37160	32370
BB 7/8 M3	3	375	23375	23750	39500	34410
B 7/8 H1	1 1/2	331	24350	24681	41050	35750
BB 7/8 H1	1 1/2	331	24450	24781	41210	35900
B 7/8 H2	2	345	22250	22595	37580	32730
BB 7/8 H2	2	345	24950	25295	42070	36640
B 7/8 H3	2 3/4	367	22500	22867	38030	33130
BB 7/8 H3	3	375	24750	25125	41780*	36400*

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*Evidences of friction in jaws of loading girder.

TABLE 9—RESULTS OF TESTS OF RIVETS IN COMBINATION OF TENSION, SHEAR AND FLEXURE, WHERE $V=1/2\ T$ FOR SUPERIMPOSED LOAD AND $V'=0.35\ T'$ FOR TOTAL LOAD.

Holes for Rivets Punched 1/16 in. Larger than Nominal Diameter of Rivet.

Note:—In the designation of the specimen, the fraction indicates the diameter of the rivet before driving; M indicates that the driving was by machine, and H that it was by pneumatic hammer. The figures 1, 2, 3 suggest the variation of grip, 1 being the smallest and 3 the largest.

Specimen	Nominal Grip, In.	Dead Load Tension, Lb.	Superimposed Load Tension, Lb.	Total Maximum Tension, Lb.	Average Ultimate Tensile Stress in Lb. per Sq. In.	
					On Area Before Driving	On Area After Driving
B 5/8 M1	1	885	23083	23968	78120	64570
B 5/8 M2	1 1/2	912	20625	21537	70200	58020
B 5/8 M3	2	934	19540	21474	69990	57850
B 5/8 H1	1	885	21750	22635	73780	60980
B 5/8 H2	1 1/2	912	20125	21037	68570	56670
B 5/8 H3	2	934	19750	20684	67420	55720
B 3/4 M1	1 1/4	896	36875	37771	85490*	72850*
B 3/4 M2	1 3/4	923	33540	34463	78010	66470
B 3/4 M3	2 1/4	947	31040	31987	72400	61690
B 3/4 H1	1 1/4	896	27420	28316	64090	54610
B 3/4 H2	1 3/4	923	30125	31048	70276	59880
B 3/4 H3	2 1/4	947	30460	31407	71090	60570
BB 3/4 H3	2 1/2	963	27330	28293	64040	54570
B 7/8 M1	1 1/2	912	38960	39872	66310	57760
B 7/8 M2	2	934	35500	36434	60590	52780
B 7/8 M3	2 3/4	971	32210	33181	55180	48070
B 7/8 H1	1 1/2	912	37375	38287	63670	55460
B 7/8 H2	2	934	33710	34644	57620	50190
B 7/8 H3	2 3/4	971	33750	34721	57740	50300

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*Evidences of friction in jaws of loading girder.

RESULTS, SERIES B AND BB

The results obtained for the nominal $V=T$ loading are given in Table 8 and shown graphically in Fig. 16. In order to show the relative importance of the tension on a rivet due to the dead load of the loading girder and the specimen, this dead load tension has been listed parallel to the superimposed load tension. The dead load shear on the rivet is not recorded in the table, but the maximum effect on the relation of the total tension to the total shear is negligible, as has already been pointed out.

For the loading in question, the ultimate tensile strength of the rivets based on the area before driving ranged from 37,160 to 53,920 lb. per sq. in. Based on the area after driving, the range was from 32,370 to 45,940 lb. per sq. in. The area estimated for the latter case was that of the holes, which were punched 1/16 in. larger than the nominal diameter of the rivet.

For the loading $V = 1/2\ T$, the results are given in Table 9 and in Fig. 16. Here again the dead load tension has been listed parallel to the superimposed load tension. Based on the area of the rivets before driving, the average ultimate tensile stress varied from 55,180 to 85,490 lb. per sq. in., while on the basis of the area after driving the range was from 48,070 to 72,850 lb. per sq. in. Attention, however, should be drawn to the fact that for the specimen B 3/4 M1, there was friction evident in the jaws of the loading girder and this probably accounts for the especially high values given by this specimen. If the results for it be disregarded, the maximum attained tensile stress for the loading in question would be, based on the area before driving, 78,120, and on the basis of the area after driving, 66,470 lb. per sq. in.

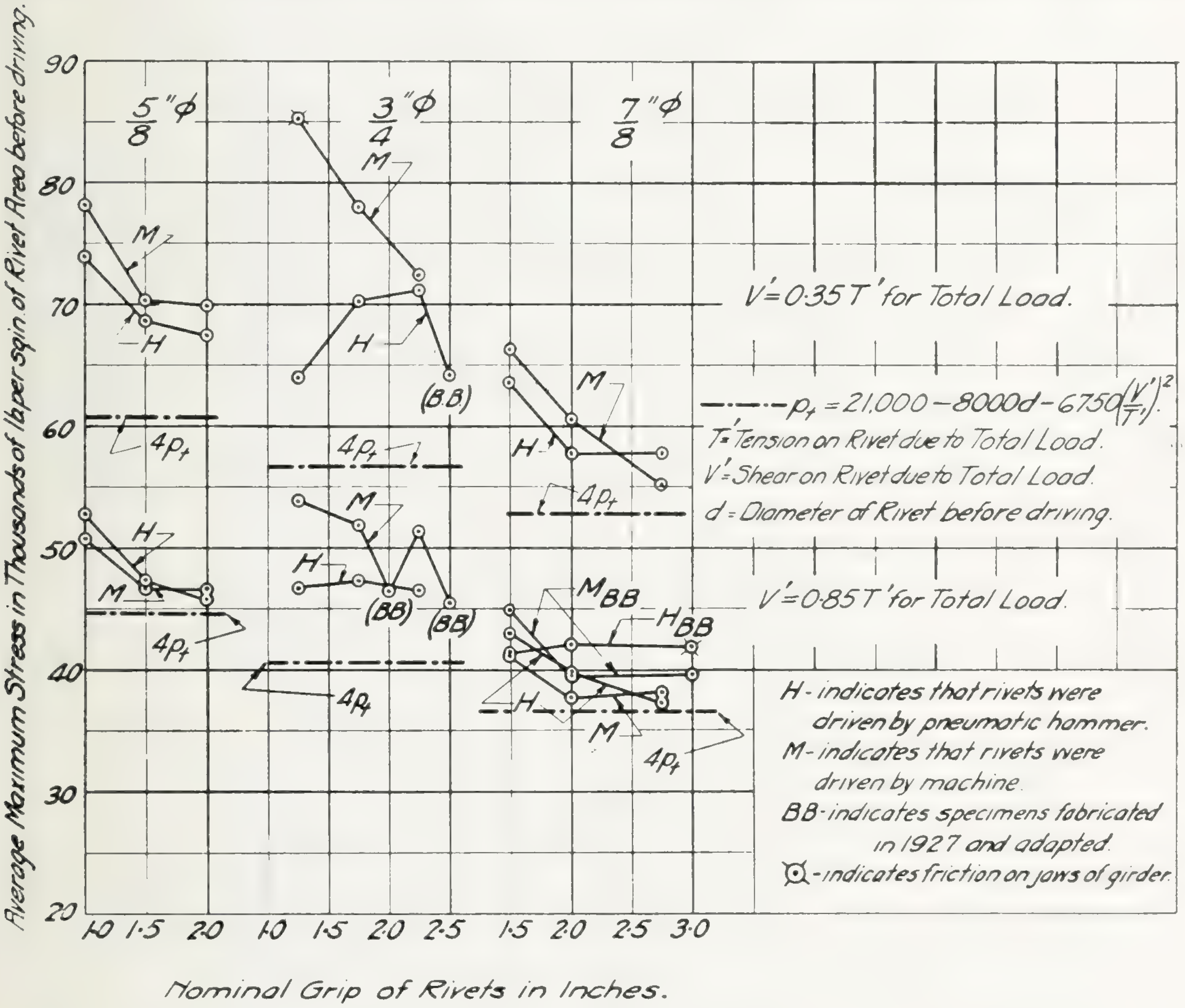

FIG. 16—Variation of ultimate strength with grip, rivet diameter and ratio V'/T'.

A study of the results plotted in Fig. 16 discloses the following facts:

(*a*) The average ultimate tensile strength in lb. per sq. in. of rivet before driving decreases as the diameter of the rivet increases.

(*b*) There is a tendency for the tensile strength of the rivets in lb. per sq. in. to diminish as the grip of the rivet increases, both for the $V=T$ loading and the $V=1/2\ T$ loading.

(*c*) Rivets driven by pneumatic hammer did not give such good results as those driven by machine for the $V=1/2\ T$ loading, but were nearly as good as machine-driven rivets for the $V=T$ loading.

WORKING STRESSES IN TENSION, SERIES B AND BB

From a study of the results it is evident that the permissible tensile stress on rivets subjected to tension, shear and incidental flexure, should vary in accordance with the relation of the tension in the rivet to the shear across it, and should at the same time vary with the diameter of the rivet. The variation of strength with grip is not sufficiently consistent to make it necessary for any formula for permissible stress to include the grip factor. Based on a factor of safety of 4 on the minimum values obtained in the tests here reported, the following tentative formula is suggested:

$$p_t = 21{,}000 - 8{,}000\,d - 6750\left(\frac{V'}{T'}\right)^2 \ldots\ldots\ldots\ldots\ldots (6)$$

in which p_t = permissible tensile stress on rivets in lb. per sq. in. of rivet before driving;

T' = total tension on rivet due to all causes;

V' = total shear on rivet due to all causes;

d = diameter of rivet in inches before driving.

This formula has been devised to take account of variations in the ratio of V' to T' from zero to 0.85, the latter being the highest value covered by the tests. It is of interest to note that then $V'=0$; this formula gives the same values as Eq. (2) when $e=$o.

In Fig. 16, which shows the ultimate tensile stress disclosed by the rivets, this formula, multiplied by 4, has been plotted for the loadings $V'=0.85\ T'$ and $V'=0.35\ T'$, as well as for the three diameters of rivets there shown.

A POROUS MEDIUM FOR FLOTATION CELLS

By F. C. Dyer, *Associate Professor of Mining Engineering*
and H. L. McClelland, *Demonstrator in Mining Engineering*

In connection with some other work, it was found necessary to use a porous medium which would withstand considerable abrasion, and at the same time, with low air pressure, deliver a flow of air as uniformly as possible over a surface covered with water.

Many materials were tried and discarded. Cotton fabrics, unless of considerable thickness, gave an uneven distribution of air. Some portions of the material gave large air bubbles, other portions small bubbles. Some portions delivered practically no air at all. Unavoidable variations of air delivery permitted the cotton to become wetted in part with great changes in air delivery. The trouble due to wetting was partially overcome by treating the cotton with waterproofing materials such as dilute rubber solutions, various oils, etc., care being taken to waterproof the threads only, without blinding the pores. But the other objections to the material still held, such as lack of rigidity, poor resistance to wear, so that this waterproofed material was discarded. None of the woven fabrics were sufficiently resistant to wear.

A piece of sheet rubber with holes punched through it by means of a sewing machine required too much pressure to force the air through the holes. Also the pressure stretched the rubber, giving uneven size to the holes, and distorted the surface. The rubber had excellent wearing properties but the other disadvantages were too great.

A porous silica brick gave a fairly uniform air flow, but was heavy, troublesome to fit in place and easily broken.

A search for tough materials suggested linoleum as a suitable substance. Its abrasive resistant qualities are well-known because of its use on concentratin tables. Further, linoleum, being waterproof, would not be affected by moisture conditions, but a porous linoleum was not to be had. Drilling holes in the numbers and small sizes required in a sheet of linoleum would be not only difficult but too costly to be considered. Punching the small holes required was found to be practically useless due to the resiliency of the linoleum closing up the holes when made, even if the extremely small punches necessary had been practicable.

An approximation to a porous linoleum was made by stripping the canvas backing from a sheet of linoleum, leaving the surface covered with a network of grooves where the burlap threads had been impressed into the material. The stripping of the back was simple, the canvas being glued to place. The stripped sheet was then cut into ribbons and these ribbons placed on edge, grooved face to smooth face, and pressed together with a little durofix cement to make a joint. The result was unexpectedly good but not as good as desired. The burlap threads being of uneven size and large, caused the pores to be large and uneven with consequent unevenness of bubble.

To cure this unevenness of size of hole an attempt was made to cut grooves of the right size and spacing in a piece of linoleum by means of a multiple saw. The resulting grooves were uniform and when the piece was cut up and reformed to give a perforated sheet the resulting air distribution was very good. The groove-cutting operation was slow and trouble was encountered from the closing of the holes by the cement used to fasten the strips together.

Finally a method of manufacture, and a material from which to manufacture, was found which gave a porous material with the desired properties. The material was the uncured mixture of cork and oils from which linoleum is made. The method of manufacture was to roll this material into strips with a roll provided with ridges to form the grooves. These strips were then arranged so that a grooved face was against a smooth face, forming a porous sheet the same thickness that the strips were wide. (Fig. 3.)

The material was given by the Dominion Oil Cloth Co. The raw material was cut up, granulated and heated with an old-fashioned crimping roll until soft and sticky, and then passed through a roll provided with longitudinal ridges for impressing grooves in the material. (Fig. 1.)

Subsequently the roll was changed for another with the ridges around the roll instead of longitudinally forming a sheet with grooves in it. (Fig. 2.) This sheet was then cut into strips with a guillotine, the cutting being across the grooves. This change was made when it was found that the soft sheet could be cut without closing the ends of the grooves. These strips were placed face to back under slight pressure and then cured in an oven for several days. No cement was required, the natural stickiness of the linoleum being sufficient to make a bond between the strips. The pressure required to roll the grooves in the linoleum is so great that the slight pressure required to place the strips in contact is not sufficient to close the holes formed by the grooves. The resulting sheet gave excellent results, in spite of the crudities due to the improvised laboratory apparatus with which it was made. Under commercial conditions the strength of the sheet and its smoothness would be considerably better.

The grooves in the strips were about 12 one-thousands of an inch each way and spaced in the row about 3/16th of an inch apart. The strips were about 3/8 of an inch wide and 1/8 of an inch thick. These dimensions, of course, should be changed to suit particular requirements. The resulting sheet was 3/8 of an inch thick with pores spaced in rows 1/8 of an inch apart and distanced 3/16 of an inch in the row.

The piece used as a smoke distributor was excellent. Placed in the bottom of a glass-sided flotation cell the results were all that could be desired. The bubbles were small. Every hole functioned. The bubbles were all of the same size, showing very even distribution of air and requiring less pressure of air than the blanket it displaced.

Air was blown through a piece of this material into saturated lime water for several days. Lime was deposited on the face of the blanket but none close to the pores, and there was not the slightest indication of liming up in the tubes themselves. This sheet being non-absorbent for water, the lime-bearing water could not penetrate the material to the walls of the tube, hence this material is free from the troubles experienced with absorbent materials, thus avoiding the necessity for increasing air pressure until a shutdown is required for acid treatment.

The material used in this particular case was made from cork and oils. Cheaper materials might have been used, such as wood pulp for cork, reducing the cost.

For special purposes this method of manufacture can be applied to other materials such as tar products (like congoleum), soft metals, glass, rubber, clay products or other material into which grooves can be rolled and subsequently set to give a harder material.

For a flotation blanket this material offers advantages over the ordinary woven blanket:

1. The material is strong and stiff, therefore it lies flat under considerable pressure.
2. Being highly resistant to wear it has a long life and seldom requires repairs.
3. It can be cut, sawed, planed, nailed or glued by any carpenter with ordinary tools. Repair or patching when necessary is easy.
4. Like other linoleum it is resistant to weak acids and alkali and is not affected by ordinary flotation pulps.
5. The material is impervious to and unaffected by water.
6. The material does not collect lime in the pores, therefore acid treatments are not necessary.
7. The holes maintain their size and do not lime up, therefore increase of air pressure is not necessary.
8. The air passages being straight and smooth, a minimum of air pressure is required.
9. All the pores function, and are of the same size, and uniformly distributed, therefore air distribution is uniform.
10. The holes may be of some predetermined size, therefore the bubbles may be small and uniform in size.

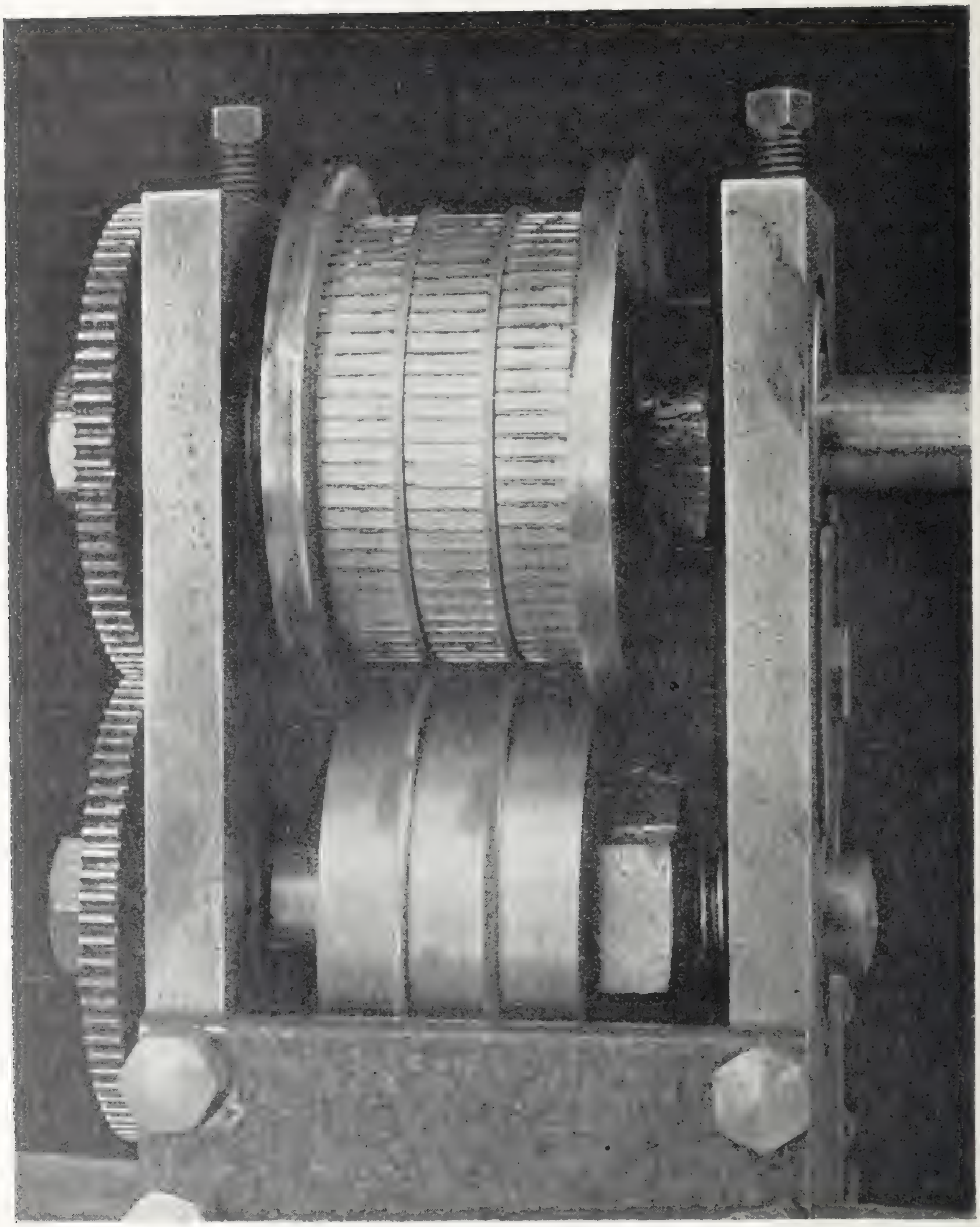

Fig. 1

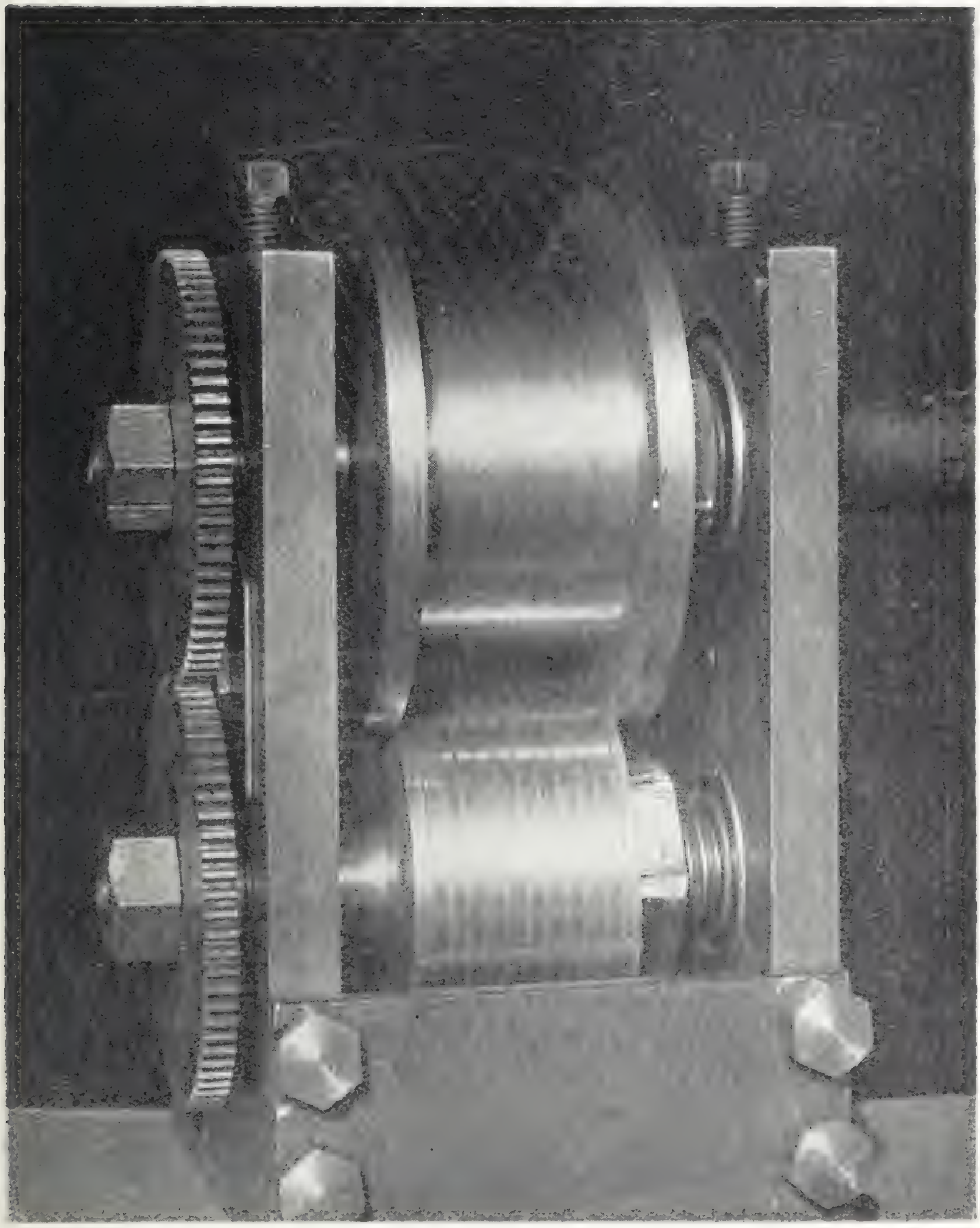

Fig. 2

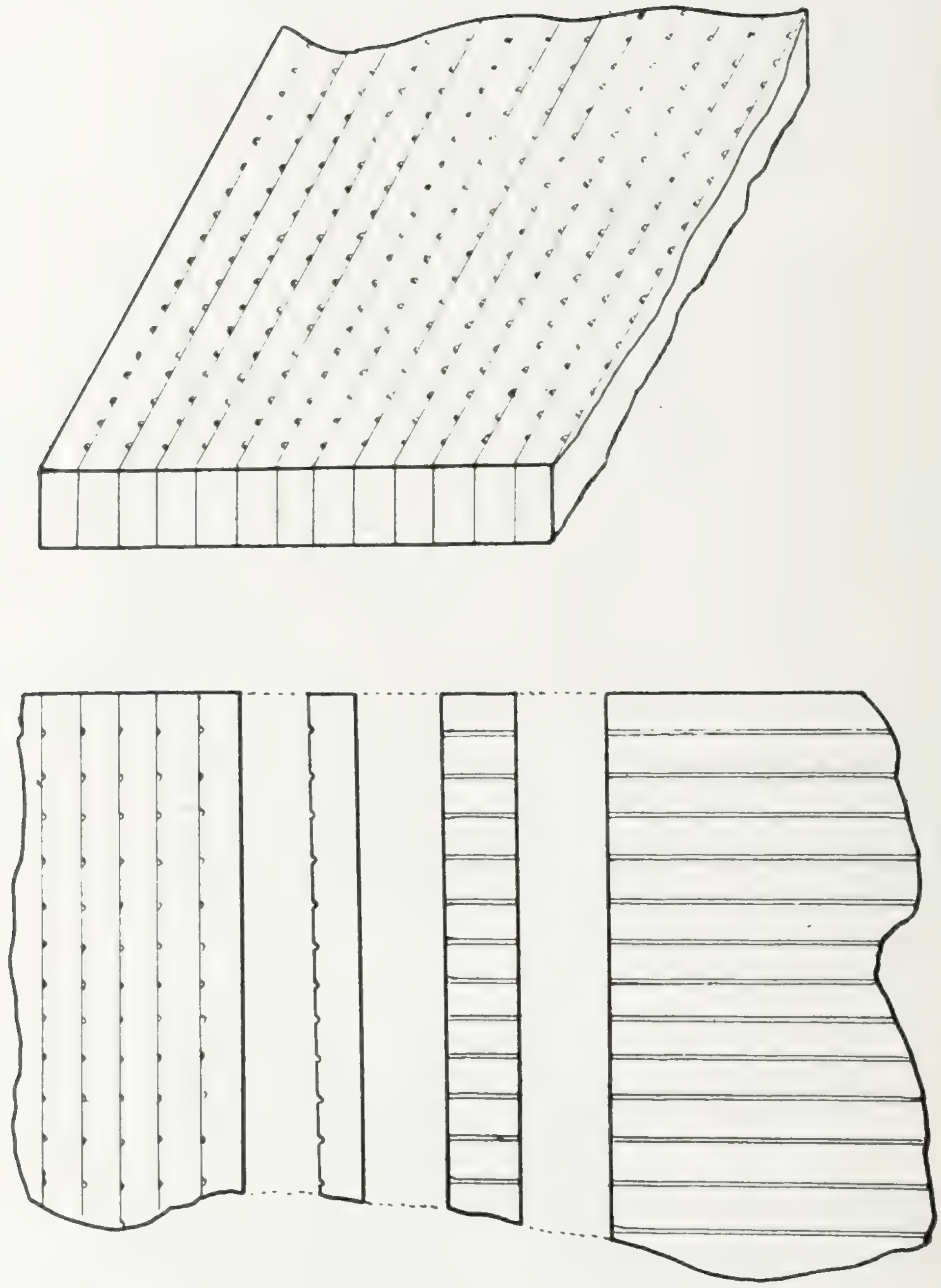

Fig. 3

REVERSED CLASSIFICATION BY CROWDED SETTLING

By F. C. Dyer,

Associate Professor of Mechanical Engineering

Hydraulic classification of mineral particles is commonly performed under conditions to which have been given the names Free Settling and Hindered Settling.

In hydraulic classification the particles are separated by currents of water operating against gravity. The particles themselves are sufficiently far apart not to interfere with each other, hence the term *Free* Settling. If the upward velocity of the current be sufficiently reduced the particles will still classify, but under conditions where the particles jostle each other. Part of the upward energy of the water is used in causing horizontal agitation. This condition is *Hindered* Settling.

If the upward velocity be reduced to zero while horizontal agitation is impressed in sufficient degree, classification will still take place. Under this condition the particles will be as crowded together as is consistent with the particles changing their relative positions. Classification under these conditions may be termed *Classification by Crowded Settling.*

Since there is no upward velocity required to give buoyancy to the particles an upward current of water is not necessary, and water may be eliminated, the necessary horizontal agitation being given by mechanical means.

Horizontal agitation may be applied to a mass of particles by mechanical means in several ways.

Let us consider one of the simplest, that of a mass of particles supported by an oscillating plane surface, such as is well known in the Wilfley Table, but without causing progression. It is essential that there be sufficient friction between the plane surface and the particles, and between the particles themselves, that the motion of the plane surface is transmitted from the surface to the particles and from one particle to another, and yet there must not be so much friction as to prevent one particle clipping over another, otherwise classification would be impossible.

In a bed of particles as in Fig. 1, the motion of the plane surface will be transmitted in part to the lower layer (a) of particles. This layer of particles will in part transmit the motion to a layer above (b), and this layer in turn will transmit part of its motion to the layer above (c), and so on until the motion transmitted in decreasing amounts becomes negligible. If the bed is very deep the lower particles may be held down by the weight of the upper layers so that there can be no slip. Therefore, classification by crowded settling can take place only in beds of particles sufficiently shallow to allow of slip between each layer of particles. Without slip classification would be impossible.

When the agitation is sufficient to cause slip, the compactness of the mass of particles is necessarily lessened, allowing particles sufficiently small to pass through the interstices between larger particles, and to sift downwards. As a result of this screening action the smaller particles will tend to accumulate in a layer at the bottom of the bed. If screening alone was the only operation taking place, the lowest layers would consist of larger balls with their interstices occupied by much smaller balls. Observation and theory show that this is not so.

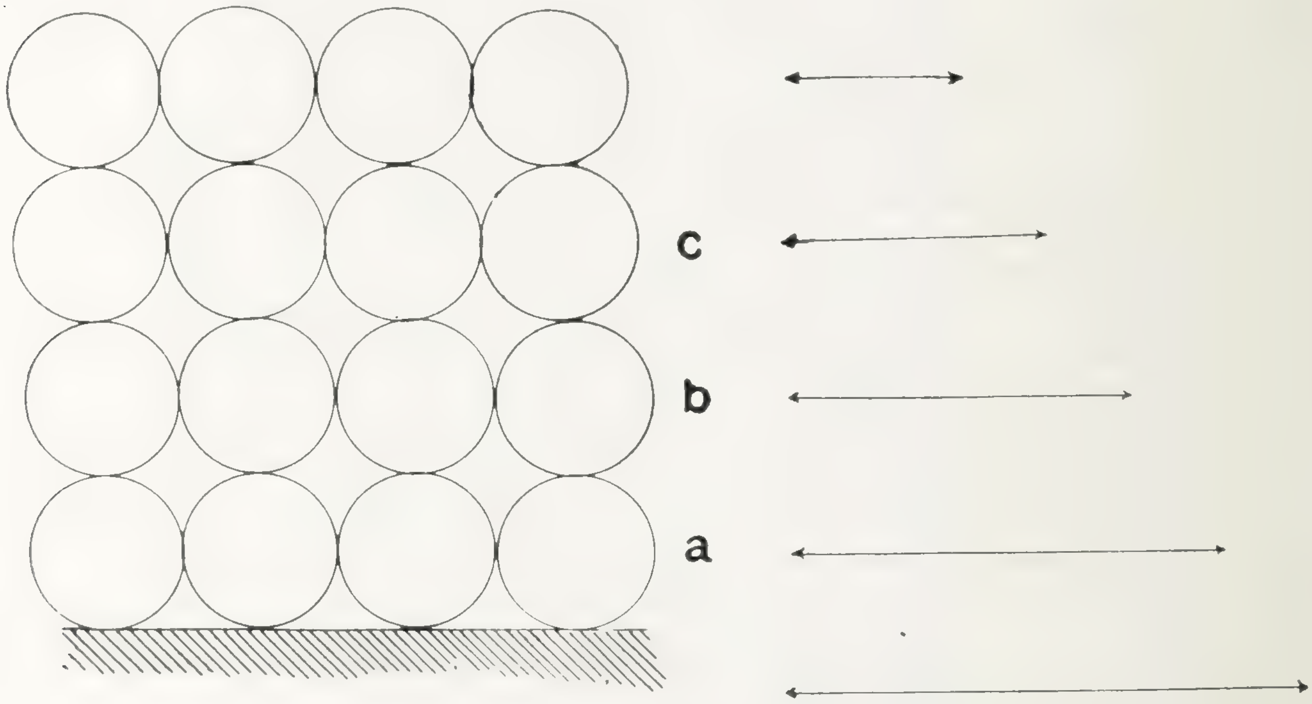

FIG. 1.

In a bed of particles consisting of particles of various sizes, downward sifting is not the only change that will take place. A true clasification takes place in that not only do small particles sift downwards, but larger particles are forced upwards.

It is easy to see that if the large particle is much larger than a smaller one, it may ride over it in the same way as a wheel rides over a stone in the road. This action may take place, however, with horizontal agitation if there is any difference at all between particle sizes if the particles are crowded together.

Referring to Fig. 2, A represents a large particle at the bottom of the bed and B, B_2 and B_3, smaller particles surrounding and in contact with the larger particle. These again will be surrounded by other particles. When the plane surface moves, friction transmits some of the motion to the particles, but in less degree because of the inertia of the particles causing slip. With a slowing up of the motion of the surface the particles will tend to keep moving due to their momentum. The small particle *B* cannot slip away from its position against *A* because of friction and the opposition of the surrounding particles, therefore the large particle *A* will tend because of its larger diameter and consequently greater momentum to ride over the smaller. Some of its horozintal energy (either of inertia or momentum) is converted to a vertical component at the point of contact C. Similarly at each contact of *A* and *B*, B_2, B_3, etc., there is a vertical component, some upward and some downward. Because of the progressive looseness of the bed of particles towards the top, the downward components of the upper small particles will be less than the upward components of the lower particles, the resultant of them all being a tendency to force the larger particle upward and the smaller particle downward.

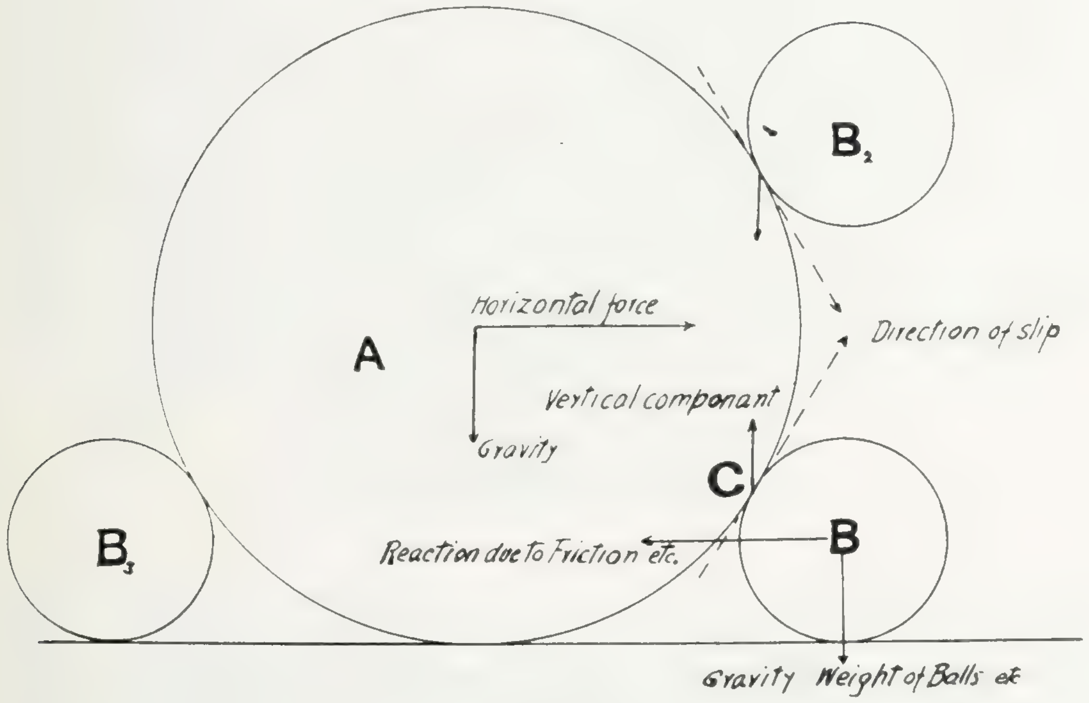

FIG. 2

The theoretical final arrangement will be a graded arrangement of particles from the largest at the top to the smallest at the bottom as in Fig. 3.

Fig. 4 is a photograph of a mixed mass of steel balls ranging from 1 inch downward to 1/8 inch before agitation.

Fig. 5 is the top of the mass and Fig. 6 the bottom of the same mass of balls after agitation.

Irregular shaped particles such as crushed rock will behave in the final resultant the same way as do balls. Flat particles may not classify in the same way because of the absence of sloping contacts on which the particles may slide. A wheel may ride over a block by a partial revolution, but it is not necessary that a particle revolve provided it can slip at the point of contact.

If the degree of agitation be insufficient the resultant upward force may not be great enough to raise the larger particle through the superincumbent mass, while if the degree of agitation is too great the bed of particles may be so loose that the small particles are pushed aside and the larger particle will sink. In these cases classification will be incomplete.

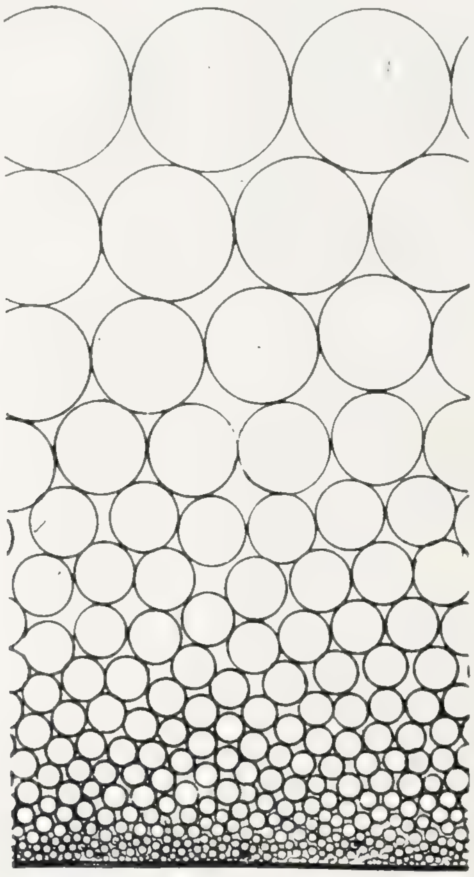

FIG. 3.

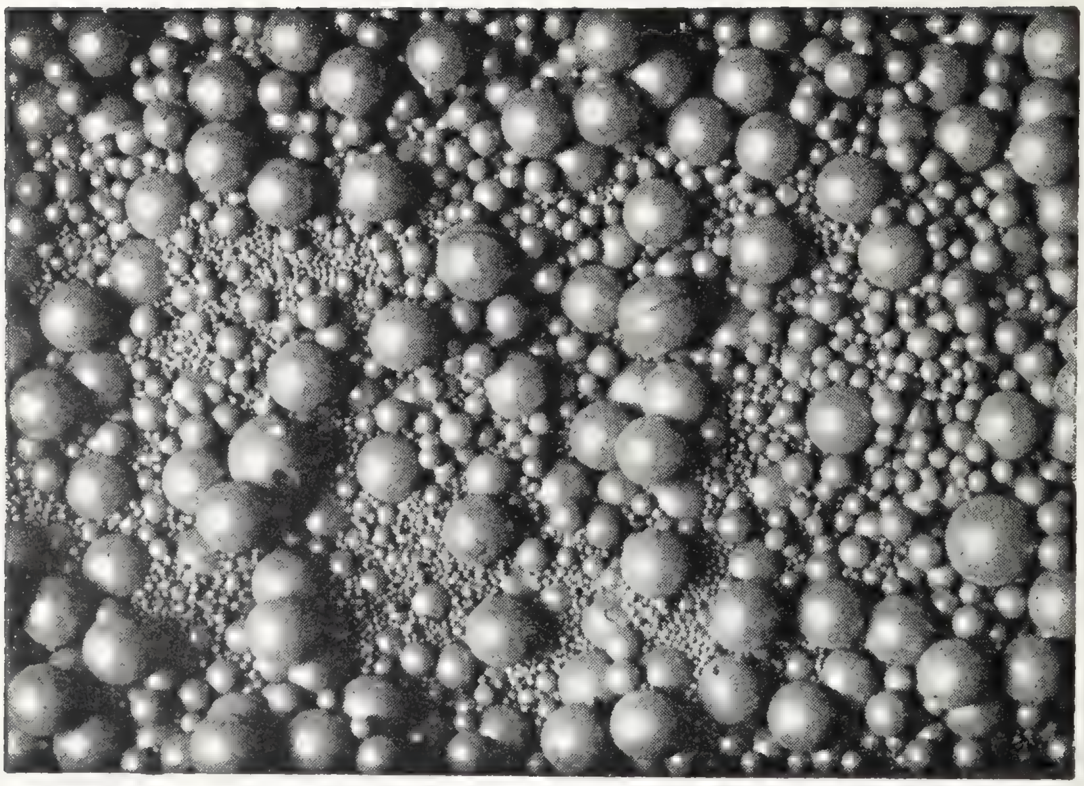

FIG. 4.

The violence of horizontal agitation required to produce classification by crowded settling is controlled by the masses of the individual particles. Agitation sufficient for the larger ones may be too much for the smaller so that for good classification there is an upper limit and a lower

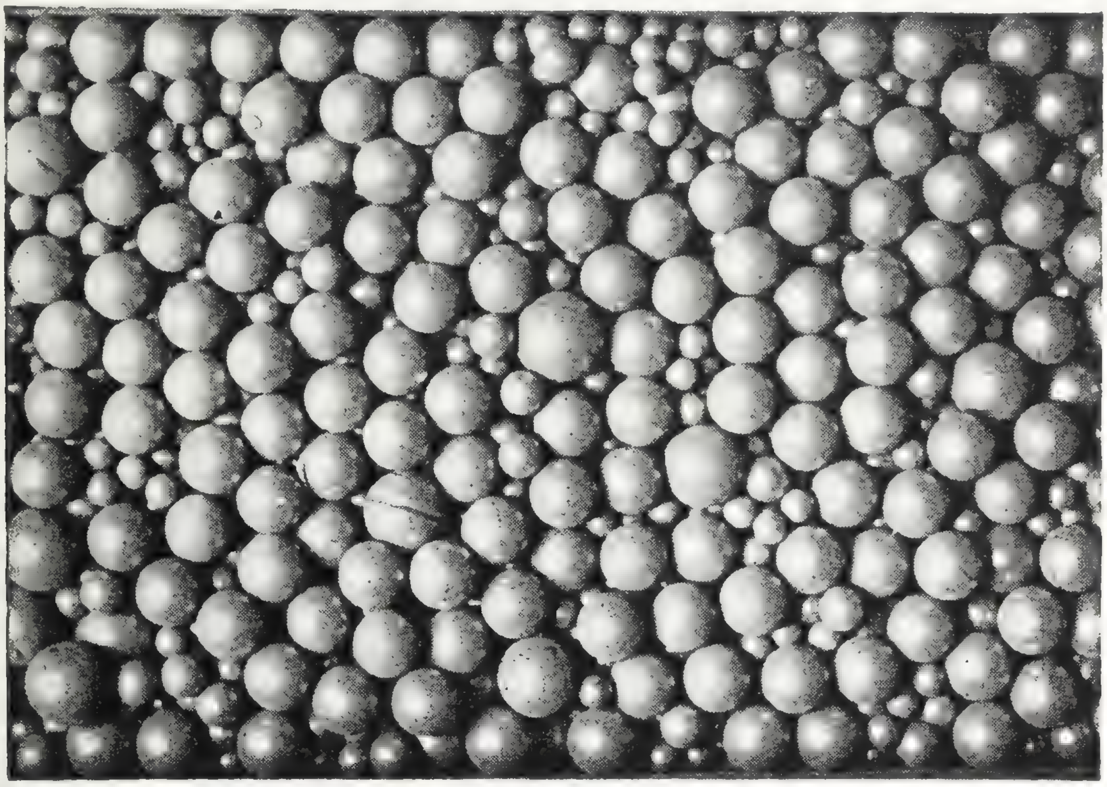

Fig. 5.

Fig. 6.

limit to the size of particles in a bed that can be classified by a given degree of agitation.

The presence of water by reducing the apparent mass of the larger particles will render them more easily raised. With the smaller particles the decreased apparent mass may make them more easily pushed aside. So that in a wet pulp less agitation may be required and at the same time the smaller particles may require to be removed.

If in the bed there are two sets of particles of differing specific gravity subjected to horizontal agitation each group will tend to arrange itself in order with the larger particle at the top and the smaller at the bottom. The particles of greater specific gravity will find their way to the lower part of the bed. The lighter material will arrange itself with the finest

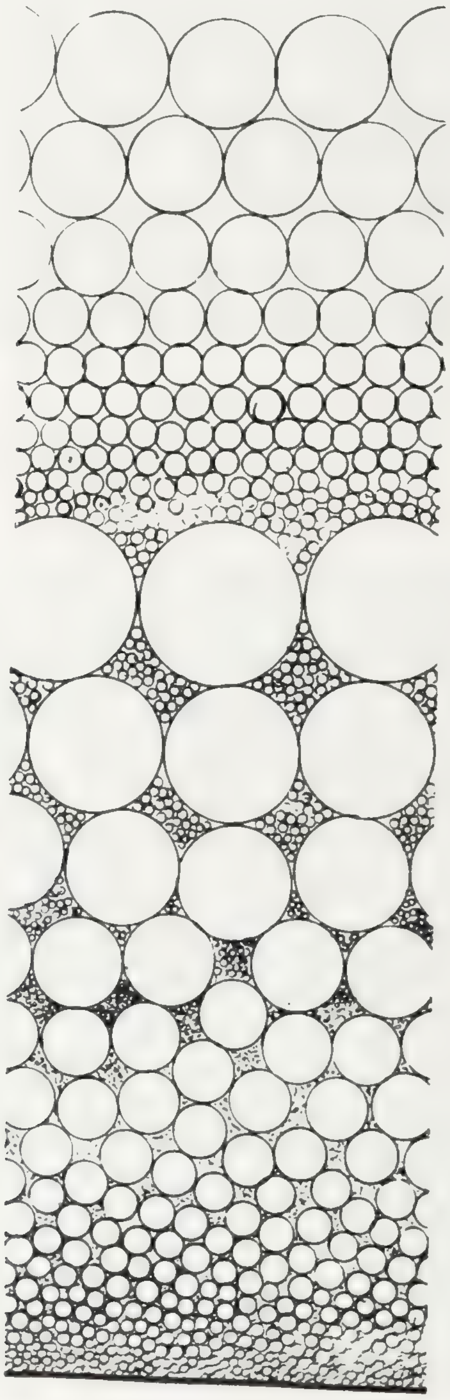

FIG. 7.

particles at the bottom, occupying, if sufficiently small, the interstices of the lower heavier layer, and above that, arranging its particles in order of increasing size toward the upper part of the bed. The arrangement will be as indicated in Fig. 7. Thus we have a classification both as to size and to specific gravity.

Figures 11 and 12 are photographs showing this reversed classification in a layer of balls.

The mixture consists of wooden balls and steel balls of several sizes. Fig. 11 shows the mixture before agitation. Fig. 12 shows the classification after agitation. The layers in turn have been picked off without disturbing the lower layers. The darker layers are wooden balls. The brighter ones are steel. The small wooden balls between the interstices of the large steel balls may be identified by the holes in them.

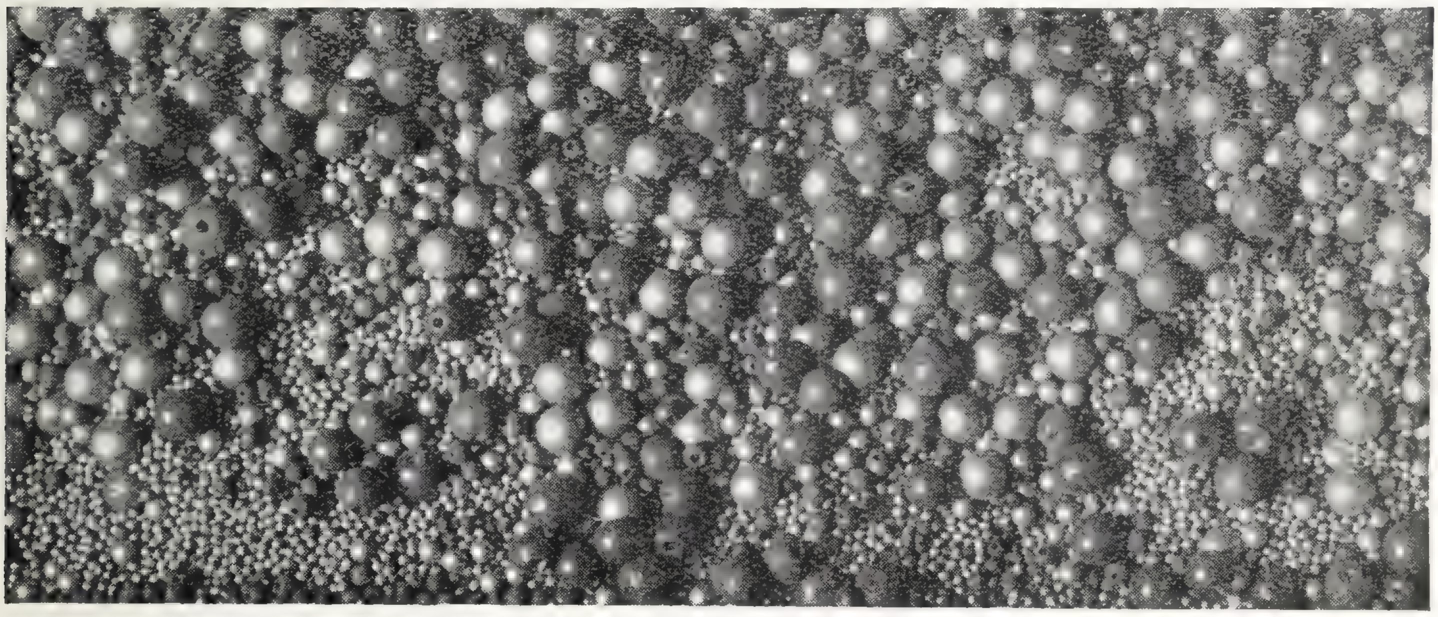

FIG. 11.

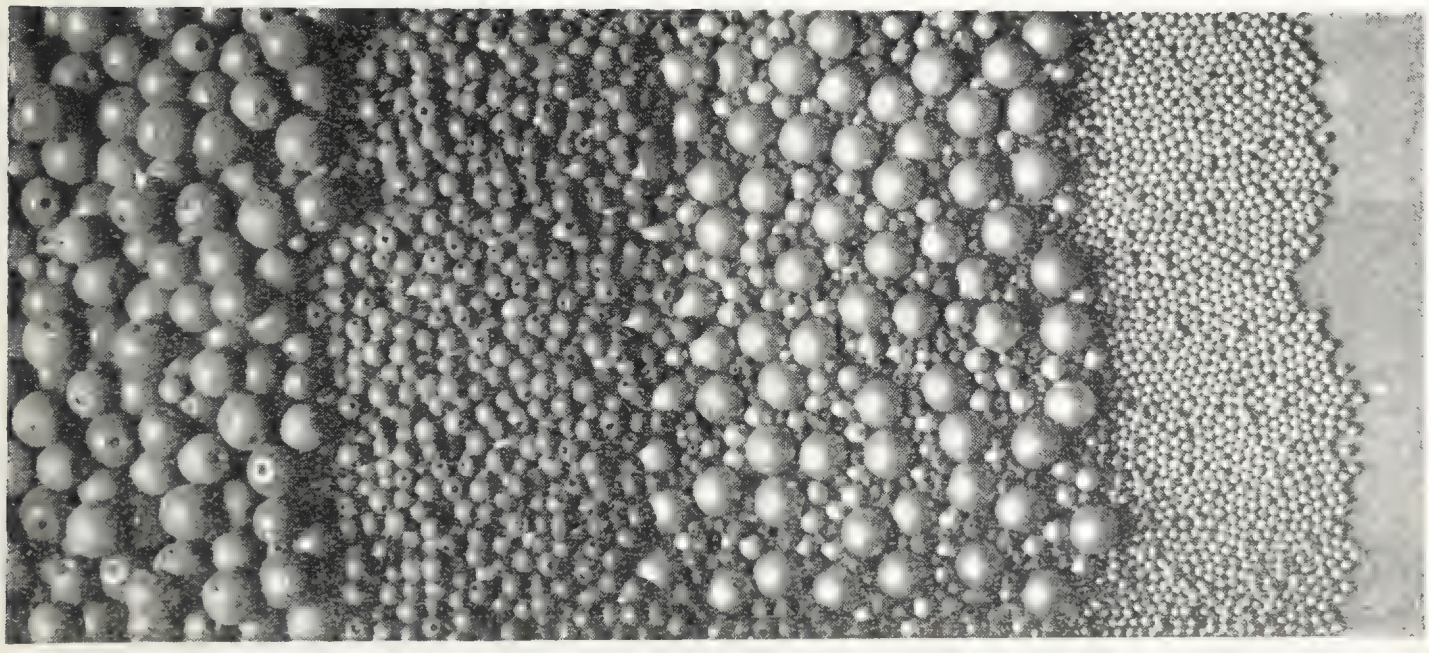

FIG. 12.

Comparing the diagrams of association ratios in Figs. 7 and 10 of Crowded Settling Classification with Fig. 8 of Free Settling Classification and with Fig. 9 of Hindered Settling Classification, there will be noticed this difference—that while with F.S. and H.S. classification, the larger particles of light material are associated with the smaller particles of heavier material, with classification by Crowded Settling it is the larger particles of heavier material that are associated with the smaller particles of lighter material. The classification is *reversed.*

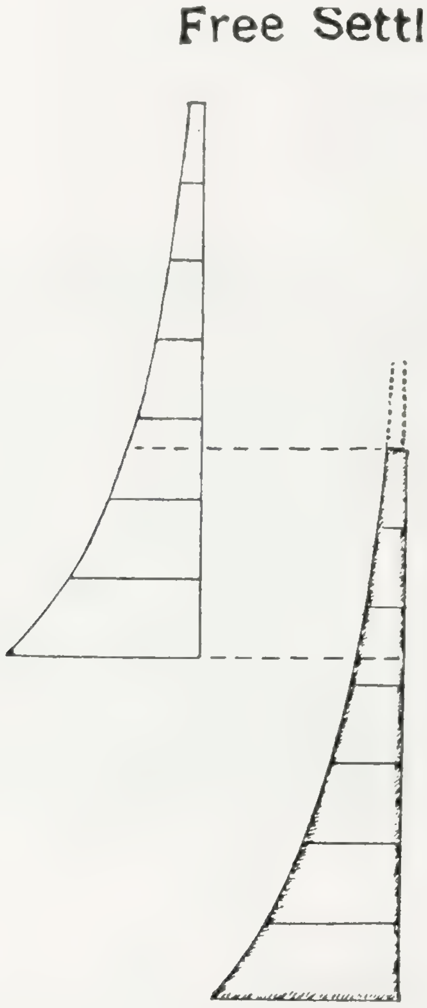

FIG. 8.

Unlike classification by Free Settling and Hindered Settling, which give approximately definite ratios to the diameters of consorting particles, in Reversed Classification by Crowded Settling there is no definite settling ratio, the diameters of consorting particles being dependent on the sizes and numbers of the particles of the two substances. The settling ratio is not a constant.

Classification completed it remains only to move the layers in different directions to make a separation.

Because for classification by crowded settling it is necessary that the particles be in contact, the two layers must also be in contact and there may be a mechanical difficulty in completely separating the two layers, giving rise to a middling in addition to the two clean layers.

If, as is frequently the case in mineral separations, the particles of the lighter material are small enough to go into the interstices of the lower layer then, though the top layer may be clean, the bottom layer will contain a middling.

The middlings arising from both these causes, however, are composed of particles of sizes that are the most widely separated by either free settling or hindered settling classification. Consequently a combination process comprising a water current classification and a classification by crowded settling is ideal for the separation of particles of differing size and specific gravity.

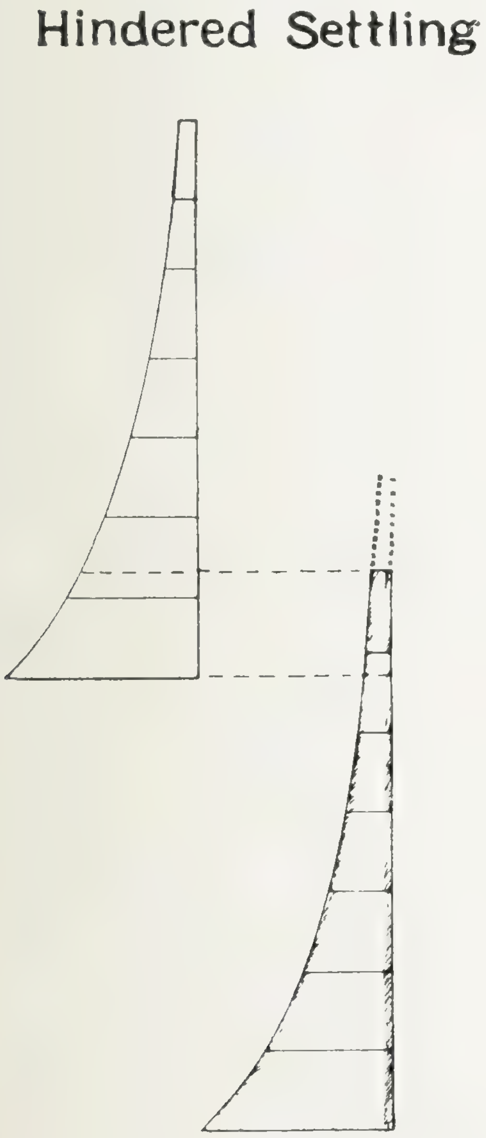

FIG. 9.

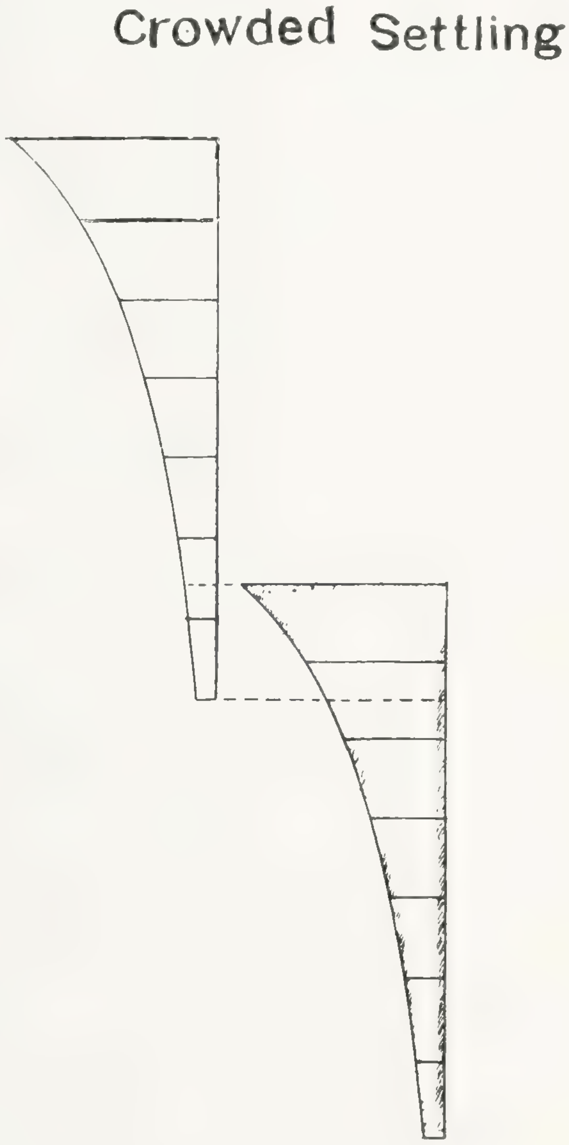

FIG. 10.

Reversed Classification by Crowded Settling may be summed up as follows:

(1) With a sufficient degree of horizontal agitation a layer of particles of varied sizes and the same specific gravity will arrange itself with the larger particles above and the smaller below.

(2) Considering any one material, the particles of greatest inertia will tend to go to the place of least motion.

(3) If there are particles of varied sizes and differing specific gravities, the one of greater specific gravity will be graded in size in the bottom layer and the one of lighter specific gravity will be graded in size in the upper layer.

(4) The larger particles of the material of greater specific gravity will associate with the smaller particles of the material of less specific gravity.

THE CHLORINATION OF METHANE

By M. C. Boswell, and R. R. McLaughlin

INTRODUCTION

Methane is the chief constituent of natural gas. "Wet" natural gas containing higher hydrocarbons which can be profitably removed is frequently encountered in boreholes; after the removal of these valuable constituents, the residual methane often goes to waste. Such an economic loss is now taking place in the province of Alberta, Canada, where a large quantity of light oil is being recovered from wet gas with only partial utilization of the residual methane.

A number of chemical methods have been suggested for the utilization of this methane. One of these is to convert the gas into compounds for which there is an industrial demand. In seeking for such a solution of the problem, at least three possible procedures are open:

(1) The conversion of methane into higher hydrocarbons such as the constituents of gasoline. This would be practically the reverse of the ordinary "cracking" process.

(2) The fractional oxidation of methane to methyl alcohol, formaldehyde or formic acid, which are, theoretically, intermediate stages in the oxidation of methane to carbon dioxide and water.

(3) The fractional chlorination of methane to produce methyl chloride methylene chloride, chloroform and carbon tetrachloride.

The investigation of this last method is the subject of this paper.

A good review of the literature on the chlorination of methane has been made by Jones, Allison and Meighan (4). That methane can be chlorinated to form all four products—methyl chloride, methylene chloride, chloroform and carbon tetrachloride—has been known for a very long time. As early as 1858 Berthelot (1) observed that mixtures of methane and chlorine reacted so vigorously as to result in the production of carbon and hydrogen chloride, unless the reaction was run very slowly. It was also found later that the chlorination of methane at a more moderate rate resulted in the production of mixtures of all four possible chlorinated products, which were difficult to separate. Moreover, the yields were not particularly good. Many investigations have been carried out for the purpose of increasing the yield of a single chlorinated product. The acceleration of the chlorination of methane by sunlight, and chlorination in the presence of catalysts have been the subjects of many researches. Some of these have been very successful in the production of carbon tetrachloride, notably those of the U.S. Bureau of Mines at Washington (4) in which yields in the neighborhood of 90% were obtained, using activated charcoals and "batchite" as catalysts and ratios of chlorine to methane from 5 to 1 to 2.5 to 1.

In a further study of this problem it seemed desirable to investigate the effect of certain factors influencing the reaction of chlorine on methane and, if possible, to determine the optimum conditions for the production of a single product, methyl chloride or carbon tetrachloride. Methyl chloride was first studied.

Previous unpublished work by the authors upon the mechanism of the Deacon reaction showed that when steam is passed over cupric chloride at 450° C. a reduction occurs, resulting in the formation of what may be termed a "reduction complex", oxygen and chlorine being evolved. The properties of this complex indicated that it might serve as a good catalyst for the chlorination of methane. Subsequent experiments confirmed this opinion. The preparation of the catalyst will be described later.

It seemed that the failure of earlier investigators to obtain good yields of methyl chloride was possibly due to the use of chlorine and methane alone, and that a sharp separation of the stages of the reaction might be accomplished by the dilution of the gases with an inert gas such as nitrogen. A further reason for investigating the effect of nitrogen on this chlorination process is to be found in the fact that where chlorine and methane alone are used in the catalytic chlorination to carbon tetrachloride, the reaction must be run slowly to avoid the separation of carbon, and even explosions. Pfeifer, Mauthner and Reitlinger (7) used nitrogen for the purpose of avoiding explosions, but did not investigate the influence of the concentration of nitrogen on the yield of single products. It was hoped that useful information might be secured by a study of nitrogen concentration and of the ratio of chlorine to methane.

In general, two methods of procedure may be adopted in order to study the effect of the various factors upon the yield of a compound produced by a given chemical reaction:

(1) The yield of the compound obtained can be determined by the actual isolation and weighing of it, or (2) the yield of the compound can be calculated from data obtained in an analytical procedure which has been tested and shown to be applicable. The latter method would not involve the isolation of the compound. According to the first method, a large amount of material must be transformed in order to minimize the experimental losses. This method is desirable in those cases where such losses are small. In cases where the losses of isolation are high the

second method is preferable. Methyl chloride, with a boiling point of −23.7°C. at atmospheric pressure, obviously falls into the latter class. For this reason it was decided to work out an analytical method suited to the chlorination of methane, by which the yield of methyl chloride could be determined, and which would at the same time furnish information regarding the nature of the other reactions accompanying the formation of methyl chloride.

This investigation, then, is divided into three parts:

(1) The development of the analytical method.
(2) The study of the chlorination of methane on an analytical scale.
(3) The study of the chlorination of methane on a larger scale, as indicated by (2), with the collection and measurement of the products.

DEVELOPMENT OF THE ANALYTICAL METHOD

There are four possible reactions in the chlorination of methane:

$$(1)\ CH_4 + Cl_2 = CH_3Cl + HCl$$
$$(2)\ CH_4 + 2Cl_2 = CH_2Cl_2 + 2HCl$$
$$(3)\ CH_4 + 3Cl_2 = CHCl_3 + 3HCl$$
$$(4)\ CH_4 + 4Cl_2 = CCl_4 + 4HCl$$

In any chlorination experiment involving the passage of definite volumes of methane and chlorine into the reaction tube, the data obtainable by a complete analytical procedure consist of: (1) the volumes of methane and chlorine taking part in the reaction; (2) the chlorine in the hydrogen chloride evolved, and (3) the chlorine in the other chlorinated products. Such data will enable one to calculate the yield of methyl chloride if that be the only product formed, or, if higher chlorinated products be simultaneously formed, to compute the minimum yield of methyl chloride. In the present experiments, the chlorine was always assumed to have formed the greatest possible amount of methylene chloride, in order to avoid the possibility of over-estimating the yield of methyl chloride. If, in reality, some chloroform or carbon tetrachloride was produced, the true yield of methyl chloride was correspondingly higher than that indicated by the experimental results given.

Much difficulty was experienced and many months of time consumed in developing a successful analytical method. The chief difficulties were: (1) the preparation of pure methyl chloride; (2) the working out of a method which would permit the determination of the free unused chlorine and the hydrogen chloride formed in the reaction but which would not interfere with the remainder of the necessary determinations; and (3) the quantitative hydrolysis of the organic chlorination products by a method which would be without effect upon the unused methane, and which would not give rise to any products interfering with the subsequent determination of methane by combustion to carbon dioxide and water. The method of hydrolysis, moreover, must be of such a nature as to permit the hydrogen chloride formed to be accurately determined.

The Preparation of Pure Methyl Chloride.

Pure methyl chloride was required for the development of the analytical procedure. Considerable difficulty was experienced in the preparation of this gas, owing to its appreciable solubility in water (four volumes at 0°C.), its action on mercury, and the necessity for the complete exclusion of air. The following method was finally adopted:

Cold, dry hydrogen chloride was passed into a flask containing anhydrous zinc chloride and methyl alcohol until the latter was saturated. The flask was then gently heated in a water-bath and the methyl chloride evolved was passed successively through three gas washers, the first two containing fuming sulphuric acid and the third distilled water. The methyl chloride was allowed to escape for an hour and a half, in order to free completely the apparatus from air, and was then collected in a gas-holder filled with air-free water. When a sufficient quantity had been collected it was shaken up with the water of the gas-holder and allowed to stand, in order to dissolve any trace of methyl alcohol present. Samples of gas were removed just prior to the experiments and measured in gas burettes over mercury. Contact with the mercury was limited to half an hour, since it had been found that contact for two weeks resulted in a reduction of the chlorine content by 30%.

Analysis of the Gases from the Catalyst Tube.

The chief problem in analysis of the effluent gases proved to be the quantitative hydrolysis of the methyl chloride. Complete hydrolysis of the methyl chloride and absorption of the hydrogen chloride produced by hydrolysis was found to take place on passing the gas over calcium hydroxide at 650°C., but unfortunately the decomposition products vitiated the subsequent methane determination. McKee (6) has found that some dimethyl ether is usually formed at 350°C. but that this can be prevented by the use of moist methyl chloride. This was tried and found satisfactory.

The gases from the catalyst tube were passed (i) through two gas washers containing 10% solutions of potassium iodide, to remove free chlorine and hydrochloric acid; (ii) through water at 95°C. to humidify them preparatory to hydrolysis; (iii) over chloride-free calcium hydroxide at 350°C., with the formation of methyl alcohol and calcium chloride; (iv) to a gas-holder containing water, in which they were allowed to remain a few hours for the absorption of the methyl alcohol; (v) through concentrated sulphuric acid, to dry them; (vi) over iodine pentoxide at 125°C., to oxidize carbon monoxide, the iodine being removed by passage over heated copper gauze, according to Boswell's modification (2) of the method of Levy (5); (vii) through potassium hydroxide solution and sulphuric acid for the removal of carbon dioxide and water; and, finally, (viii) through a combustion tube containing copper oxide, followed by sulphuric acid and potassium hydroxide, for the determination of the methane. After the experiment was completed the potassium iodide solution was first titrated with sodium thiosulphate to determine the unused chlorine, then with sodium hydroxide, to determine the hydrogen chloride evolved. The chlorine absorbed by the lime was determined by solution in nitric acid and precipitation by means of silver nitrate. The method of analysis, therefore, provided for the determination of the volumes of chlorine and methane unused, the volume of hydrogen chloride formed and the weight of chlorine in the chlorinated products other than hydrogen chloride. The accuracy of the method was demonstrated by the passage of mixtures of known volumes of the gases.

THE CHLORINATION OF METHANE ON AN ANALYTICAL SCALE

The Generation of Methane.

Methane was generated by the action of water on aluminium carbide. The gas was purified by passing it through fuming sulphuric acid and water. It was analyzed by the usual volumetric methods and also gravimetrically by combustion over copper oxide, with the following results: methane 92%; hydrogen 8%.

Preparation of the Catalyst.

The catalyst was prepared by impregnating 18 gm. of pumice, which had been passed through a 12-mesh sieve with a solution of 10 gm. of cupric chloride ($CuCl_2 . 2H_2O$) in 15 cc. of water. After being dried at 110°C., it was placed in a hard glass tube 17 mm. in diameter, and was heated in an electric furnace at 450°C. for nine hours, while moist nitrogen was passed over it. Previous experiments had shown that by this treatment the cupric chloride is reduced about three quarters of the way to the cuprous condition, with the evolution of chlorine and oxygen.

Apparatus and Procedure.

The apparatus shown in Fig. 1 was used, with only minor changes, throughout the whole series of experiments.

The nitrogen used in diluting the methane and chlorine was passed successively through J, where it was heated to 750°C. for the removal of oxygen and the oxidation of any organic material; through K, to remove any carbon dioxide; through L, to moisten it (beginning with experiment 19); and then through the train of apparatus. The gas washers P and P_2 were filled with a 10% solution of potassium iodide, Q and Q_2 were half-filled with distilled water, R and R_2 were filled with chloride-free lime and V with water. The nitrogen was allowed to bubble out through T until required. The burette E was filled with dry methane and left under a pressure of a few centimetres of mercury when filled to the 100 cc. mark. Chlorine from A was passed through B and bubbled up through

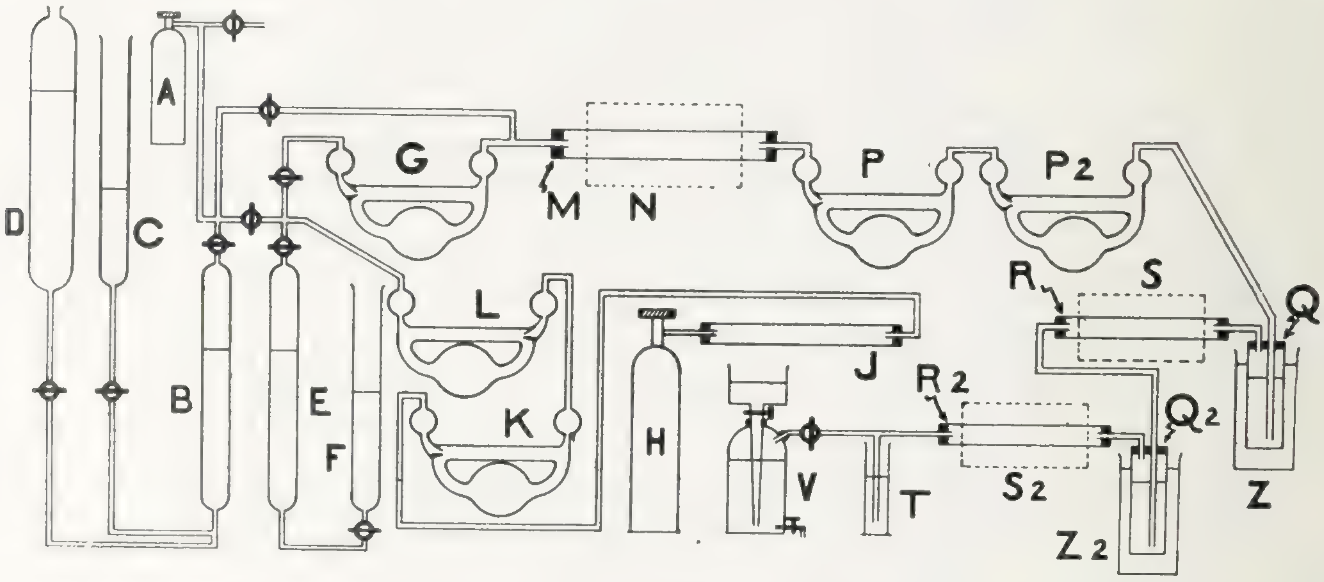

FIG. 1. APPARATUS FOR THE CHLORINATION OF METHANE ON AN ANALYTICAL SCALE.

A: Chlorine cylinder. B: Water-jacketed gas burette. D: 1000 cc. separatory funnel, placed about 6 ft. above B. C: Levelling tube. E: Mercury-filled, water-jacketed gas burette. F: Levelling tube at a higher level. G: Gas washer containing conc. H_2SO_4. *H: Nitrogen cylinder. J: Silica tube containing Cu and CuO heated to 750° C. K: Gas washer containing 1:1 KOH solution. L: Gas washer containing water. M: Tube containing the catalyst. N: Electric furnace. P,* P_2*: Gas washers containing KI solution. Q,* Q_2*: Humidifiers. Z,* Z_2*: Water baths to maintain Q and* Q_2 *at 95° C. R,* R_2*: Tubes containing lime. S,* S_2*: Electric furnaces. T: Water manometer. V: 10-litre gasholder.*

the water in D until required. The temperature of M was raised to 450°C., of R and R_2 to 350°C., and of the water baths Z and Z_2 to 95°C. Then the chlorine was shut off and water was allowed to flow from D into B until it reached the 100 cc. mark, leaving this volume of chlorine under pressure. The pinch-cocks at the top of the burettes E and B were then opened to the atmosphere for an instant, leaving in them 100 cc. of methane and chlorine, respectively, at atmospheric pressure and room temperature, the remainder of the apparatus being filled with nitrogen. The passage of methane and chlorine was then commenced, the rates being adjusted by means of pinch-cocks in the tubing supplying mercury and water, respectively, to the burettes. The methane-chlorine-nitrogen mixture could be dried, if desired, by passage through G. The passage of chlorine through the water in D, prior to the experiment, made certain that none of the measured sample of chlorine would dissolve in the water of the gas burette. As soon as the passage of methane and chlorine was commenced, the exit gases were collected in the gas-holder, atmospheric pressure being maintained in the system as indicated by the manometer T. The passage of nitrogen was continued for one and one-half hours after all the methane and chlorine had been passed, in order to ensure the collection of all the products of the reaction. While the nitrogen was still passing, the few drops of water which usually collected in the cold end of the reaction tube were driven over into P by gentle heating. The contents of P and P_2 were analyzed and after standing for two hours, the unused methane in the gas in the gas-holder was determined, as already described.

Effect of Moisture in the Gases.

In the preliminary experiments the reacting gases were thoroughly dried before entering the catalyst tube. The result obtained when the catalyst was freshly prepared (*i.e.*, in the reduced condition) was the absorption of a large volume of chlorine by the catalyst (experiment 17, Table I). Before proceeding to experiment 18, the catalyst was thoroughly saturated with this gas, and any excess of chlorine was swept out with nitrogen. In this experiment considerably more chlorine appeared in the products than was passed, showing that it had been given off by the catalyst. In order to avoid these difficulties the gases were saturated with moisture before entering the catalytic tube, with the result that the catalyst was maintained in the partially reduced and *constant* condition very favourable to the reaction. This constancy in condition of the catalyst is obviously highly desirable in quantitative experiments.

Effect of Reaction Temperature.

The most favourable temperature was believed to be 450°C., but experiments were also carried out at 420° and 475°C. By comparing the results of experiments 19-21 (Table I) it will be seen that the yield of methyl chloride was greatest at 450°C. A similar, but more marked result is seen in experiments 22 and 23 where lower ratios of chlorine to methane were used.

The Methane-Chlorine Ratio.

The volumes of methane and chlorine in the reacting mixtures were approximately equal in experiments 17-21, but that of chlorine was

TABLE I

Chlorination of Methane to Methyl Chloride on an Analytical Scale

Variables	Preliminary		Temperature			Chlorine and Temperature				Nitrogen		
Experiment No.	17	18	19	20	21	22	23	24	26	27	28	29
Temperature, in degrees C.	450	450	450	420	475	450	420	450	450	450	450	450
Time, in minutes	15	15	15	15	15	15	15	15	15	15	15	20
Gases passed:												
CH_4, cc.	88.0	86.7	86.5	85.6	86.3	85.5	85.6	84.6	86.2	86.2	85.9	86.1
Cl_2, cc.	122.9	94.6	93.7	92.7	93.3	45.9	46.5	30.5	31.1	30.4	29.8	30.4
Ratio, N_2:CH_4										3	6	10
Condition of gases	Dry	Dry	Moist	Moist	Moist	Moist	Moist	Moist	Moist	Moist	Moist	Moist
Determined:												
Gases unused, CH_4, cc.	61.8	26.6	23.6	34.8	29.0	38.3	53.3	43.4	43.1	76.0	58.2	59.7
Cl_2, cc.	0.0	21.2	21.8	0.0	2.8	0.1	trace	2.6		0.0	0.0	0.0
Cl_2 converted to HCl, cc.		61.5	20.0		28.3	5.3	1.3	3.8		7.4	2.2	7.4
Cl_2 in other products, cc.	18.3	53.2	54.8	45.0	52.7	35.1	26.8	21.3	20.7	16.0	20.0	12.3
H_2 in exit gases, cc.		24.7		30.4	27.2	16.0	14.1	22.7	14.6	3.1	0.0	0.0
Calculated: (*in vapor phase*)												
CH_4 used, cc.	26.2	60.1	62.9	50.8	57.3	47.2	32.3	41.2	43.1	10.2	27.7	26.4
CH_3Cl produced, cc.	15.8	13.8	16.2	11.6	9.3	24.2	11.0	39.8	41.4	0.0	15.4	24.6
CH_2Cl_2 produced, cc.	10.4	46.3	46.7	39.2	48.0	23.0	21.3	1.4	0.0	0.0	12.3	0.0
$CHCl_3$ produced, cc.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	8.8	0.0	0.0
CCl_4 produced, cc.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.4	0.0	0.0
Ratio, CH_3Cl:CH_2Cl_2	1.5	0.29	0.34	0.29	0.19	1.05	0.53	29			1.25	
Percentage yield of CH_3Cl, based on CH_4 used	60	23	26	23	16	51	34	97	100	0	56	100

reduced by 50% in the two succeeding experiments by mixing with it in the burette an equal volume of nitrogen. In the remaining experiments recorded in Table I, the methane-chlorine ratio was approximately three to one. It will be observed that, other things being equal, the yields of methyl chloride improved as the proportion of chlorine was reduced. In experiment 17, where a large volume of chlorine was passed, the good yield is doubtless to be accounted for by the fact that much chlorine was absorbed by the catalyst, none being found in the exit gases.

Nitrogen-Methane Ratio.

In the earlier experiments the volume of nitrogen was not measured, as its importance was at that time not fully realized. Later, however, a series of experiments was carried out to determine the effect of dilution with a large volume of nitrogen, upon the yield of methyl chloride. It will be noticed from Table I that the yield increased from zero to 100% as the nitrogen-methane ratio increased from 3:1 to 10:1. In experiment 29 (as also in No. 26) the yield is recorded as 100% because the evidence of complete conversion to methyl chloride was conclusive; the excess of methane used over methyl chloride produced is accounted for in both cases by the solubility of methane in water. For the same reason, it is believed that all the results recorded as "methane used" are slightly too high.

Effect of Hydrogen in the Methane.

As already stated, the methane used in this series of experiments contained 8% of hydrogen. In determining the yield, based upon the methane used, allowance was made for this dilution of the methane. So also, in calculating the methane-chlorine ratio, the volume of chlorine theoretically required to unite with the hydrogen of the methane to form hydrogen chloride was deducted, since this reaction was known to take place rapidly under the conditions of the experiments.

In spite of this reaction tendency, a considerable percentage of hydrogen was found in the residual gases. Further, the results given in Table I show that in most of the experiments much less than half the volume of the chlorine passed was contained in the hydrogen chloride evolved, and much more than half in the other chlorinated products.

It may be that the hydrogen came from the hydrogen chloride unaccounted for, the equivalent chlorine producing the observed excess of chlorinated products. On the other hand, it may have been derived from the action of lime and steam on the methyl chloride, but experiments indicated that this reaction did not take place. No explanation of these anomalies is offered.

THE CHLORINATION OF METHANE ON A LARGER SCALE

Apparatus and Procedure.

Having determined on an analytical scale the conditions under which methane can be chlorinated to produce only methyl chloride, with complete utilization of the chlorine, the next step was to carry out the chlorination on a scale sufficiently large to permit the collection and measurement of the products. Accordingly, an apparatus designed to produce 25 gm. of methyl chloride per experiment was constructed and set up.

As before, the methane was generated by means of aluminium carbide. In this case, however, ice was used instead of water, in order to delay the reaction. In a small autoclave were placed 288 gm. of powdered aluminium carbide and 432 gm. of crushed ice, and the cover of the autoclave was immediately bolted on. When the pressure rose to about 150 lb. per sq. in. the valve was opened for a moment to blow off most of the small quantity of residual air. Upon closing the valve the pressure quickly rose to 6000 lb. per sq. in., and then fell off to 4500 lb. per sq. in. on cooling. This provided 60 litres of methane under pressure. Analysis —made after experiment 43 had been completed—showed the gas to contain 18% of hydrogen. The effect of this hydrogen will be discussed later.

The catalyst was prepared as before, 173 gm. of $CuCl_22H_2O$ being used. The gases were moistened in order to maintain the catalyst in the proper condition. The reaction chamber in which the catalyst was placed consisted of a silica tube 60 cm. long and 5 cm. inside diameter, wound with resistance wire, and water-cooled at both ends. The effective volume was 17 times as great as in the previous experiments, making possible the passage of methane at the rate of 6000 cc. per hour.

The apparatus used in this series of experiments is shown in Fig. 2. The gas holders A, A_2 and A_3 were filled with methane, nitrogen was turned on at the desired rate, and chlorine was supplied at constant pressure by allowing a little to escape continuously through E. The gases from the reaction tube were passed through soda-lime and (in some of the experiments) calcium chloride in the tower S. The chlorinated products were condensed by passing them into large test tubes surrounded by liquid air in Dewar flasks.

The experiments required about six hours each. When the methane and chlorine had all been passed, the system was swept out with nitrogen for about 15 minutes. The test tubes containing the liquefied products were then disconnected from the soda-lime jar and, after closing the exit tube of T_2 and opening the entrance tube of T, the liquid air was removed from around the second tube. The small amount of condensate was thereby distilled back into the first tube which was then connected to the mercury manometer and to a calibrated Carius tube drawn down to a capillary, as shown in Fig. 2. The liquid air surrounding the tube was removed and a mixture of carbon dioxide snow and ether (−79°C.) was substituted for it. By this procedure any condensed methane was boiled off (b.p. −164°C.). The quantity was always small. A little methyl chloride was lost in this way, so that the weights recorded (Table II) are slightly below the actual yields. The system was then evacuated through Y and the pinch-cock at Y was closed. The methyl chloride in the test tube was then distilled over and condensed in the Carius tube which was surrounded by liquid air. When distillation was complete, the Carius tube was disconnected from the remainder of the

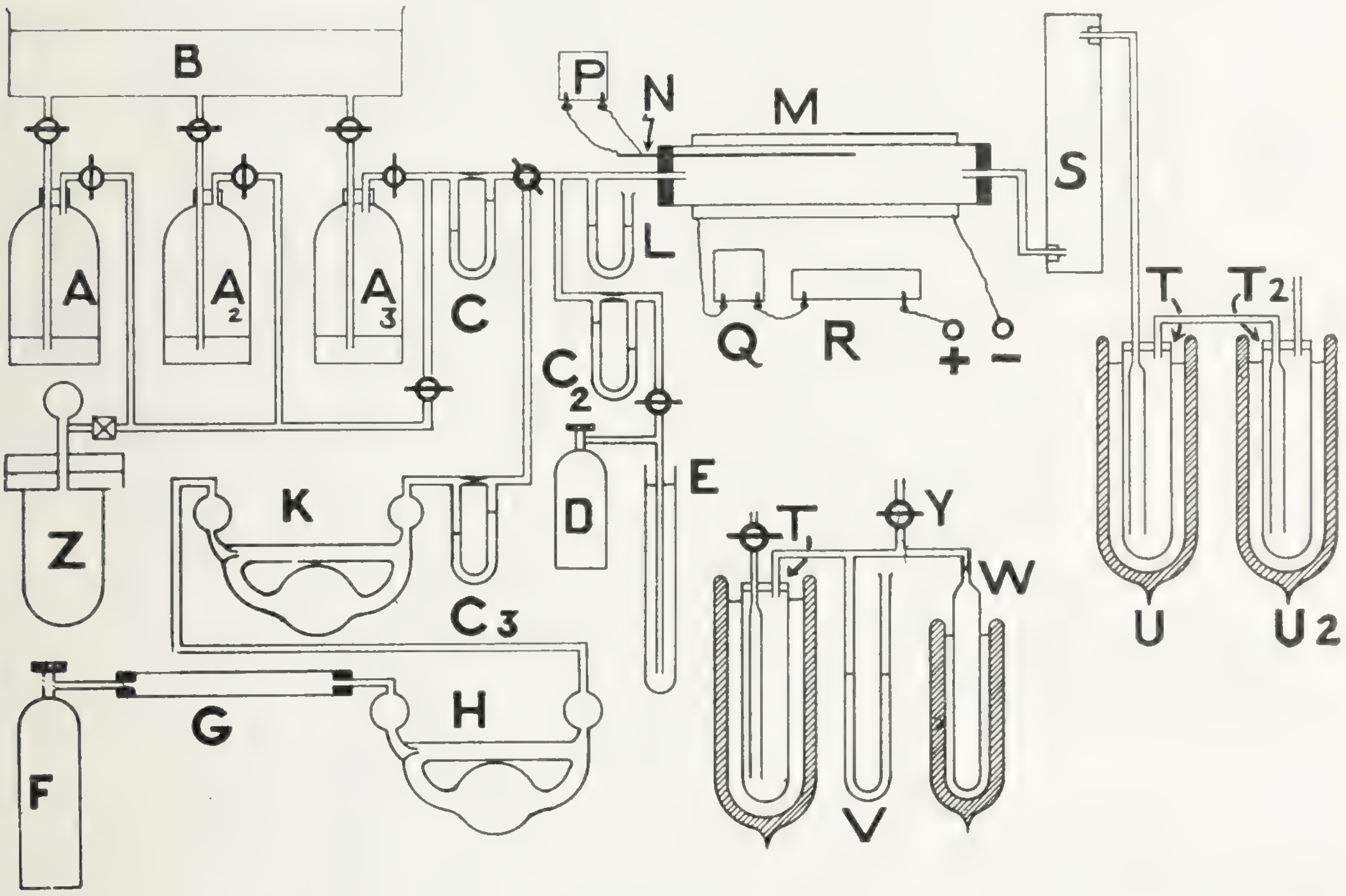

FIG. 2. APPARATUS FOR THE LARGER-SCALE CHLORINATION OF METHANE. *A, A_2, A_3: 15-litre gas holders, calibrated on the outside. B: Large water reservoir. C, C_2, C_3: Capillary flow-meters. D: Chlorine cylinder. E: Tube about 2 ft. long containing water, with delivery tube running nearly to the bottom. F: Nitrogen cylinder. G: Silica tube containing Cu and CuO at 750° C. H: Gas washer containing 1:1 KOH. K: Gas washer containing water. L: Water manometer. M: Reaction tube, wound with resistance wire. N: Thermocouple. P: Millivoltmeter. Q: Ammeter. R: Variable resistance. S: Tower containing soda-lime. T, T_2: Large test tubes with wide delivery tube leading to within about an inch of the bottom. U, U_2: Dewar flasks. V: Mercury manometer, capable of registering pressures from 15 mm. to 2 atmospheres. W: Carius tube drawn to capillary at the top. Z: Autoclave.*

apparatus, and, while still immersed in liquid air, the tip was sealed by a blow-torch. It was subsequently warmed to 0°C., the volume of methyl chloride was measured at this temperature, and its weight calculated. Any residue remaining in the test tube was fractionally distilled for the separation and determination of methylene chloride, chloroform, and carbon tetrachloride.

TABLE II

CHLORINATION OF METHANE TO METHYL CHLORIDE ON A LARGER SCALE

Experiment No.	35	37	38	40	43	46	47	48	49	50
Special purpose of Experiment		Better collection of products	Test of collecting system			Use of CH_4 freed from H_2	To confirm experiment No. 46	Effect of increased ratio of CH_4:Cl_2	To confirm experiment No. 48	To det. effect of faster passsage of CH_4 and Cl_2
Time, hr.	6.17	5.25	6.33	7.00	5.33	5.68	5.63	6.00	5.67	2.47
Gases passed:										
CH_4, cc.	36150	30400	33275	44500	32350	32580	32000	32680	34400	35400
H_2 in CH_4, %	18	18	18	18	18	1	0	1.4	0	0
Net CH_4, cc.	29643	24928	27285	36490	26527	32255	32000	32222	34400	35400
Total Cl_2, cc.	12350	8950	10700	13300	9800	10457	10365	4740	4480	4065
Net Cl_2 (total less vol. required to unite with H_2 of CH_4 as HCl), cc.	5843	3488	4710	5290	3977	10132	10365	4280	4480	4065
Effective ratio, CH_4:Cl_2	5.1	7.1	5.8	6.9	6.7	3.2	3.1	7.5	7.7	8.7
Ratio, N_2:CH_4	10	10	10	10	10	10	10	10	10	5
Produced:										
CH_3Cl, gm.	8.6	6.8	6.9	11.4	7.6	10.9	11.4	7.6	7.4	6.9
CH_2Cl_2, gm.	Trace	Trace	0.0	2.5	Lost	5.0	5.5	1.0	1.2	0.5
$CHCl_3$, gm.	0.0	0.0	0.0	0.0	2.0	0.2	0.5	0.0	0.0	0.0
CCl_4, gm.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Chlorinated products apparently lost, %						25	23	9	12	18.5
Percentage yield of CH_3Cl:										
Indicated	31	34	29	38	36					
Effective based on net Cl_2 passed	65	87	65	96	84	48	48	79	74	75

Efficiency of Collection Apparatus.

In experiment 35, the indicated yield of methyl chloride was found to be only 31% (Table II). For this reason the efficiency of the collecting apparatus was carefully tested (Experiments 37 and 38). In the former experiment the exit gases were bubbled through liquid air; in the latter the gases leaving the test tubes were passed over chloride-free calcium hydroxide at 600°C. for the hydrolysis of any escaping methyl chloride, the lime being subsequently treated, as in the experiments on an analytical scale, for the determination of chlorides. The calibration of the Carius tube was also checked by weighing the methyl chloride collected. This was done by weighing the sealed Carius tube, cooling in liquid air, breaking off the tip carefully, allowing the methyl chloride to evaporate on warming the Carius tube to room temperature, and weighing tube and tip again. These experiments zhowed that no appreciable amount of methyl chloride was being lost in the regular procedure.

Another experiment (No. 39) was carried out to determine whether free chlorine was escaping from the reaction tube, since that, although unlikely, would account for the low yield of methyl chloride obtained. The result was negative.

Effect of Hydrogen upon the Reaction.

Experiments 35-43 (Table II) were performed in the belief that, as in the first series of experiments, the methane contained 8% of hydrogen. The indicated yields of methyl chloride, however, were so out of harmony with previous results as to point to a relatively greater amount of hydrogen, which would combine with part of the available chlorine. This conjecture was confirmed by analysis, which showed a hydrogen content of 18%. This amount of hydrogen in the methane would, with a methane-chlorine ratio of three to one, combine with 54% of the chlorine present. In subsequent experiments the hydrogen was therefore removed by passage over heated copper oxide at 275°C., since Burrell and Oberfell (3) have shown that this results in the selective oxidation of hydrogen in the presence of methane. The gas was passed through a 1:1 potassium hydroxide solution before being collected.

The yields of experiments 35-43 were then recalculated on the basis of the net chlorine present. The results showed that some of the yields were of the high order anticipated, as may be seen by reference to Table II.

Experiments 46 and 47 were then carried out with the purified methane. The indicated yield of methyl chloride increased considerably, but at the same time more than a third of the chlorine appeared as methylene chloride and chloroform. Further, there was an indicated loss of about 25% of the products of chlorination.

Methane-Chlorine Ratio.

It was then realized that, in these two experiments, the ratio of methane to chlorine was too low, and the effective ratios of these two gases in the earlier experiments were accordingly calculated. The results, shown in Table II, revealed the fact that in experiments 46 and 47 the effective ratios of methane to chlorine were less than half those previously used. The reduced mass action of the smaller quantity of hydrogen chloride present had evidently permitted the formation of the higher chlorinated products.

In experiments 48 and 49 the methane-chlorine ratios were therefore raised to 7.5 and 7.7, resulting in the very satisfactory yields of 79% and 74% respectively, with only a small proportion of methylene chloride in either case. Further, the percentage of chlorinated products apparently lost decreased to 9 and 12. As the boiling point of methylene chloride (42°C.) is considerably higher than that of methyl chloride (−23.7°C.) it may be assumed that the losses consisted chiefly of the latter, and that the total yields in these two experiments were actually in the neighbourhood of 88% and 85%, respectively.

Rate of Passage of Gases.

In this series of experiments the rate of passage of the gases had been kept down to a point found by the experiments on an analytical scale to be within the limit necessary for satisfactory chlorination. In experiment 50, however, the rates of passage of methane and chlorine were doubled to determine what effect this might have upon the efficiency of the catalyst. The rate of passage of the nitrogen was not changed, and the nitrogen-methane ratio was therefore reduced by half. The yield of methyl chloride condensed was 75%, which was practically the same as at the slower rate of passage, in spite of the increased loss of chlorinated products (18.5%). If this loss represents methyl chloride the gross yield was over 90%.

OPTIMUM CONDITIONS FOR CHLORINATION TO METHYL CHLORIDE

The conditions recommended for the chlorination of methane exclusively to methyl chloride are as follows:

1. Nitrogen-methane ratio, ten to one. This can be reduced somewhat, under favourable conditions, without appreciably lowering the yield of methyl chloride.
2. Methane-chlorine ratio, seven or eight to one. If methane containing 8% of hydrogen be used this can be reduced to three to one.
3. Condition of gases, moist.
4. Temperature of reaction, 450°C.
5. Catalyst, partly reduced cupric chloride on pumice.

Under these conditions it has been found possible to chlorinate methane to methyl chloride with a yield based on chlorine passed, of 75-80% or higher. A small quantity of methylene chloride is also formed.

CHLORINATION OF METHANE TO CARBON TETRACHLORIDE

An attempt was now made so to control the chlorination of methane as to produce carbon tetrachloride exclusively. The apparatus, shown in Fig. 2, was exactly the same as that formerly used. Fresh catalyst was prepared in the same manner as before. Carbon dioxide snow in ether was used as the condensing medium, in place of liquid air. The results of this series of experiments are shown in Table III.

TABLE III

CHLORINATION OF METHANE TO CARBON TETRACHLORIDE

Experiment No.	51	52	53
Time, hr.	6.0	7.0	6.5
Gases passed:			
CH_4, cc.	3,270	3,720	6,280
Cl_2, cc.	12,000	14,000	26,000
N_2, cc.	none	none	47,000
Produced:			
CCl_4, gm.	18.6	21.8	34.5
$CHCl_3$, gm.	none	none	3.0
Higher-boiling liquid, gm.	trace	1.5	1.0
Yield of CCl_4, %	90.3	90.8	80.2

In experiment 51 methane and chlorine were passed in approximately theoretical proportions (1:4) without nitrogen. Nitrogen was, however, passed through the apparatus while the reaction tube was being heated up, and again at the end of the experiment, to sweep out the products. The methane was freed from hydrogen before use. The carbon tetrachloride obtained represented a yield of 90%. Its purity was tested by fractional distillation. There was no indication of chloroform or other low-boiling compounds. A slight residue of liquid boiling over 100°C. was obtained.

Experiment 52 was a duplicate of the above. The yield of carbon tetrachloride (90.8%) was slightly higher than before, as was also that of the higher-boiling liquid (b.p. about 120° C.).

A third chlorination experiment (No. 53) was then performed to determine the effect of doubling the rate of passage of the gases. As soon as these gases entered the reaction tube an explosion took place, a flame shooting back in the apparatus and carbon being deposited. The passage of the gases was continued, but when the explosions were found to occur at frequent intervals nitrogen was introduced. This decreased the frequency of the explosions, but they did not cease until the volume of nitrogen was seven times that of the methane. The data recorded for experiment 53 in Table III apply to the experiment as continued from this point.

The three grams reported as chloroform (b.p. 61.2°C.) consisted of all the distillate between 65° and 74° C., hence it no doubt contained considerable carbon tetrachloride (b.p. 76.7°C.). The yield of the latter, shown as 80.2%, was therefore probably somewhat higher. This yield, though not as good as at the slower rate of passage, was nevertheless considered very satisfactory.

THE CHLORINATION OF ETHANE TO ETHYL CHLORIDE

The chlorination of ethane is also of considerable interest, since this gas is an important constituent of some natural gas. For example, the natural gas used by the city of Pittsburgh, Pa., contains 10% of ethane, and that of one well in the Turner Valley oil and gas field, in southern Alberta, as much as 66%.

Ethane may be regarded as a potential source of ethyl alcohol, the first step in the production of the alcohol being chlorination to ethyl chloride, and the second, hydrolysis. Conditions suitable for the chlorination of methane to methyl chloride having been found, a preliminary study of the chlorination of ethane was made.

It was found that the procedure used in the chlorination of methane on an analytical scale would have to be considerably modified for the chlorination of ethane in order to make possible a calculation of the yield. Only a few experiments were performed, and no accurate data regarding these are available, but the yield of ethyl chloride, calculated from the ethane and chlorine used, was at least 75%. It is the intention of the authors to investigate further the chlorination of ethane on a scale sufficiently large to permit the accurate measurement of the ethyl chloride formed.

COMMERCIAL CONSIDERATIONS

The results of the experiments outlined above are presented as essential laboratory data upon which larger-scale experiments may be based. Much further work will be required to determine whether the methods used will be applicable on a commercial scale. Certain economic considerations may nevertheless be of interest.

The necessity for great dilution of chlorine with nitrogen and methane in the production of methyl chloride might, at first sight, be considered a grave objection to any commercial process based upon these laboratory methods. However, this is not necessarily the case for, after the removal of the hydrogen chloride and methyl chloride produced, the excess methane and diluent nitrogen can be recirculated through the apparatus, after the necessary addition of chlorine and methane. Thus, neglecting mechanical losses, it would be necessary to supply nitrogen to the system only at the outset.

If the hydrogen chloride formed were removed by lime, calcium chloride, of low commercial value, would be produced. However, if the hydrogen chloride were passed through a suitable water washer, it might be possible to recover most of it as a solution, without any appreciable loss of methyl chloride.

A possible change in the process on a commercial scale would be the substitution of methane for the diluent nitrogen.

If the chlorination of methane to carbon tetrachloride the tendency of the reacting gases to explode as their rate of passage is increased can be overcome by dilution with nitrogen, although this results in the production of some chloroform. The explosions are caused by the heat of reaction; it might therefore be possible so to regulate the speed of the undiluted reacting gases that the temperature of the catalyst would just be maintained at 450°C., without any external heating. The apparatus could also be so modified, that the gases would not mix until they entered the reaction chamber, thus effectively preventing serious explosions, whatever the rate of passage.

In the case of those natural gases which have a content of helium, the removal of methane as a chlorination product leaves a residue of helium. In the method described above for making methyl chloride the helium would accumulate in the circulating nitrogen. This latter could be drawn off from time to time for the recovery of the helium. It is obvious that this helium-nitrogen mixture would have, as the process proceeded and successive withdrawals of gas made, a gradually increasing content of helium.

SUMMARY

A small-scale method was first developed in which the degree of chlorination of methane to methyl chloride could be determined by analysis of the resulting gases. The optimum conditions so determined were then applied on a scale which permitted the isolation and measurement of the products. A yield of 80% or better was obtained when using as a catalyst partially-reduced cupric chloride and passing moist nitrogen, methane and chlorine in the ratios of 70:7:1 at 450°C. It was found that the proportion of chlorine could be more than doubled when 8% of hydrogen was present in the methane. Under such conditions the chlorine was completely utilized and only methyl chloride and hydrogen chloride were formed. The yield of isolated methyl chloride obtained was nearly 80% and this could be increased by operating on a larger scale. The same catalyst was successfully used in the chlorination of methane to carbon tetrachloride. A yield of 90% was obtained, with complete utilization of the chlorine. The chlorination of ethane to ethyl chloride, with a yield of at least 75% was also shown to be possible.

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REFERENCES

1. BERTHELOT, M. Synthèse de l'esprit de bois. Ann. chim. phys. (3) 52:97-103. 1858.
2. BOSWELL, M. C. The direct determination of oxygen in organic compounds. J. Am. Chem. Soc. 35:284-290. 1913.
3. BURRELL, G. A. AND OBERFELL, G. G. The use of copper oxide for fractionation combustion of hydrogen and carbon monoxide in gas mixtures. J. Ind. Eng. Chem. 8:228-231. 1916.
4. JONES, G. W., ALLISON, V. C. and MEIGHAN, M. H. The chlorination of natural gas. U.S. Bur. Mines Tech. Paper 255. 1921.
5. LEVY, L. A. The rapid estimation of carbon monoxide. J. Soc. Chem. Ind. 30:1437-1440. 1911.
6. MCKEE, R. H. Conversion of methyl chloride to methanol.—II. J. Ind. Eng. Chem. 15:788-795. 1923.
7. PFEIFER, J., MAUTHNER, F. and REITLINGER, O. J. Chlorination of methane. J. prakt. Chem. 99:239-242. 1919.

A STUDY OF SOME FACTORS INFLUENCING THE ACTIVITY OF ALUMINIUM AND FERRIC CHLORIDE IN THE FRIEDEL AND CRAFTS REACTION[1]

By M. C. Boswell and R. R. McLaughlin

It is well known that aluminium oxide, which is remarkably stable towards hydrogen, even at high temperatures, has a large capacity for absorbing water. Dilworth (1), in this laboratory, has shown that the catalytic activity of aluminium oxide in dehydration actions, as well as its capacity for adsorption, depends upon its water content. Likewise, it was thought that since aluminium chloride is reduced with difficulty in hydrogen, its activity in the splitting off of hydrogen chloride in the Friedel and Crafts reaction might be influenced by the presence or absence of adsorbed hydrogen chloride. In fact, experiments described in this paper show that aluminium chloride, prepared by the action of hydrogen chloride on aluminium, contains adsorbed hydrogen chloride to an extent which may equal 9 cc. per gram; also that the presence of this adsorbed hydrogen chloride is apparently somewhat unfavourable to catalytic activity in the Friedel and Crafts reaction, since its removal by sublimation in nitrogen raises the activity. The amount of hydrogen chloride adsorbed during the preparation of aluminium chloride from the metal and hydrogen chloride is very considerably greater than that which aluminium chloride is capable of adsorbing by sublimation in the gas after first being freed from the hydrogen chloride taken up during preparation.

Quantitative comparisons of its catalytic activity in the Friedel and Crafts reaction showed that aluminium chloride made by the action of chlorine is somewhat less active than that made by the action of hydrogen chloride on the metal, and that ferric chloride, especially if partially reduced, is very much less active than aluminium chloride. The comparisons revealed, further, the striking fact that, although ferric chloride alone has an activity not much more than one-third that of aluminium chloride, an approximately equimolecular mixture of the two materials has a catalytic activity somewhat greater than pure aluminium chloride itself.

EXPERIMENTAL

Adsorption of hydrogen chloride by aluminium chloride.

3.5 gm. aluminium was converted into the chloride by heating in a current of dry hydrogen chloride. Free hydrogen chloride was swept out of the tube by a stream of dry nitrogen at room temperature. The aluminium chloride was then sublimed in a current of nitrogen from one part of the tube to another, the gases issuing from the tube being passed through standard alkali solution. Care was taken that no aluminium chloride escaped during sublimation. The sublimation was repeated until no further hydrogen chloride was evolved. Titration of the standard alkali solution indicated that the hydrogen chloride evolved in two experiments was:—(1) 147 cc., (2) 98cc., corresponding to 9 cc. and 6 cc. respectively, per gram of aluminium chloride.

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In the first experiment, after removal of adsorbed hydrogen chloride in the way described, dry hydrogen chloride was passed over the material while it was sublimed back and forth several times; dry nitrogen was then passed at room temperature in order to displace free hydrogen chloride from the tube, and the adsorbed hydrogen chloride was determined as before by removal during sublimation. It was found that only 35.5 cc. hydrogen chloride had been adsorbed by the aluminium chloride, as compared with 147 cc. adsorbed during its preparation.

That there was no source of error due to sublimation of aluminium chloride from the tube was shown by acidifying the alkali solution, used to collect the hydrogen chloride, after each experiment, evaporating to dryness and testing for aluminium.

It was established by a separate experiment that the mere passage of dry nitrogen at room temperature did not remove any adsorbed hydrogen chloride from aluminium chloride. A fresh sample of aluminium chloride was prepared as before. When all the aluminium had been transformed into chloride, the tube, now full of hydrogen chloride, was cooled to room temperature and both ends closed with pinch cocks; the temperature and pressure were noted. A stream of nitrogen at room temperature was then passed through the tube, and the total hydrogen chloride displaced determined as described previously. The hydrogen chloride so displaced comprised that present in the tube, and any which had been adsorbed and was driven off by nitrogen at room temperature. The tube was again closed by pinch cocks at room temperature and pressure. The nitrogen in the tube was then expelled by means of a stream of carbon dioxide and collected in a nitrometer over a strong caustic alkali solution to determine the volume of the tube. This was also equal to the volume of free hydrogen chloride displaced in the first operation. The volume of hydrogen chloride displaced in the first operation was 264.8 cc. and that of nitrogen in the second operation 265.3 cc., showing that no adsorbed hydrogen chloride was evolved from aluminium chloride on passing nitrogen at room temperature for 1.5 hours.

Influence of adsorbed hydrogen chloride on catalytic activity in the Friedel and Crafts reaction.

The method chosen for a quantitative comparison of catalytic activity in the Friedel and Crafts reaction was a modification of that used by Rubidge and Qua (2), the amount of hydrogen chloride evolved by the action of benzene and chloroform being measured. A quantity of the ground catalyst was weighed in a tared, dry, 500 cc. flask; the later was placed in a water bath at room temperature and connected with an upright condenser from which a delivery tube led into a standard alkali solution. A mixture of dry benzene and dry chloroform was added in the proportion of 200-g. benzene and 40-g. chloroform for 30-g aluminium chloride. When the initial rapid evolution of hydrogen chloride had fallen off, the temperature of the bath was raised to 75-80°C., causing the evolution of hydrogen chloride again to become vigorous. The bath was kept at 80°C. for one hour and was then cooled to room temperature overnight. The excess alkali was titrated. Throughout the experiment nitrogen was passed through the reaction flask at a constant rate.

Concordant results were obtained by this procedure as shown in Table I. The results tabulated here were obtained in three experiments run side by side with different portions of the same batch of aluminium chloride.

TABLE I

MEASUREMENT OF ACTIVITY IN TERMS OF HYDROGEN CHLORIDE EVOLVED

Experiment No.	$AlCl_3$ used in grams	HCl evolved in cc. at N.T.P.	HCl per gram of $AlCl_3$, in cc. at N.T.P.
1	21.63	12,868	595
2	21.05	12,451	592
3	20.18	11,826	586

Two samples of aluminium chloride were then prepared by the action of dry hydrogen chloride on aluminium, and the adsorbed hydrogen chloride on one of them was expelled by sublimation in nitrogen in the manner previously described. The catalytic activity of the samples was measured by the procedure just described. The results are given in Table II.

TABLE II

COMPARISON OF THE CATALYTIC ACTIVITY OF $AlCl_3$ WITH AND WITHOUT ADSORBED HCl

Sample	$AlCl_3$ used in grams	HCl evolved in cc. at N.T.P.	HCl per gram of $AlCl_3$, in cc. at N.T.P.
$AlCl_3$ with adsorbed HCl	17,402	9,877	568
$AlCl_3$ without adsorbed HCl	15,352	9,380	611

Comparison of the activity of various catalysts.

Following the procedure described in the preceding section, the activity of the following catalysts was compared: (1) Anhydrous aluminium chloride made by the action of hydrogen chloride on aluminium; (2) Anhydrous aluminium chloride made by the action of chlorine on aluminium; (3) Anhydrous ferric chloride made by the action of chlorine on iron; (4) The partial reduction product obtained by subliming ferric chloride several times in hydrogen at 350°C.; (5) A mixture of (2) and (3); (6) A mixture of (2) and (4). The proportion of the benzene-chloroform mixture applied to the gram-mols of catalyst used was constant. The results obtained are summed up in Table III.

TABLE III

COMPARATIVE ACTIVITY OF CATALYSTS IN THE FRIEDEL AND CRAFTS REACTION

Experiment No.	Catalyst	Amount of catalyst in mols	Amount of benzene-chloroform mixture in cc.	HCl evolved in 90 hrs. cc. at N.T.P.	HCl evolved in 90 hrs. per 0.140 mols catalyst in cc. at N.T.P.	Relative activity of catalyst in terms of HCl evolved per gram-mol of catalyst
1	$AlCl_3$ (from Al and HCl)	0.085	97	4318	7123	98.6
2	$AlCl_3$ (from Al and Cl_2)	0.140	160	6653	6653	92.1
3	$FeCl_3$ (from Fe and Cl_2)	0.134	157	2625	2742	38.0
4	$FeCl_3$ (partially reduced)	0.116	136	1093	1324	18.3
5	Mixture of 0.079 mols (2) and 0.081 mols (3)	0.160	184	8236	7223	100
6	Mixture of 0.068 mols (2) and 0.072 mols (4)	0.140	152	6230	6230	86.3

The mixture (Experiment 5) of aluminium and ferric chlorides, as may readily be calculated, produced 55% more hydrogen chloride than would have been expected from the components acting separately. Similarly, the mixture (Experiment 6) of aluminium chloride and partially reduced ferric chloride yielded a quantity of hydrogen chloride 59% greater than that calculated from the separate activities of the components.

SUMMARY

In the preparation of aluminium chloride by the action of hydrogen chloride on the metal, hydrogen chloride is adsorbed and can be recovered to the extent of about 9 cc. per gram. After sublimation in nitrogen and re-sublimation in hydrogen chloride, however, the amount of adsorption is smaller and irregular. The adsorbed gas is not removed by a stream of nitrogen at room temperature. The activities in the Friedel and Crafts reaction of various preparations of aluminium chloride and of ferric chloride, and of mixtures of these, were determined. The order of decreasing activity was found to be as follows: a mixture of aluminium chloride and ferric chloride, aluminium chloride made by the action of hydrogen chloride on aluminium, aluminium chloride made by the action of chlorine on aluminium, a mixture of aluminium chloride and partially reduced ferric chloride, ferric chloride, and partially reduced ferric chloride. The most striking result of the measurements is that although ferric chloride alone has an activity of only about one-third that of aluminium chloride, an approximately equimolecular mixture of the two has an activity somewhat greater than that of pure aluminium chloride.

REFERENCES

1. BOSWELL, M. C. and DILWORTH, H. M. The mechanism of catalysis by aluminium oxide. J. Phys. Chem. 29:1489-1506. 1925.
2. RUBIDGE, C. R. and QUA, N. C. Friedel and Crafts reaction—the preparation of o-benzoylbenzoic acid and benzophenone. J. Am. Chem. Soc. 36:732-737. 1914.

THE REMOVAL OF THIOPHENE FROM BENZENE AND A NEW METHOD FOR THE PREPARATION OF THIOPHENE-FREE BENZENE ON A LABORATORY SCALE

By E. G. R. Ardagh, *Associate Professor of Chemical Engineering*
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The discovery of thiophene in coal-tar benzene by Victor Meyer in 1883 is an oft-told tale.[1, 2, 3] This same famous investigator also introduced the method of removing the thiophene by agitation with concentrated sulphuric acid, a method employed for many years on a large scale for the manufacture of thiophene-free benzene.[2]

Since Victor Meyer's time a number of additional methods for removing thiophene (as shown by the list following) have been discovered, the principal objection to his method being the loss of benzene, which may amount to as much as 12%,[20] and is not likely to be less than 5%.[20, 34]

(1) Agitation with concentrated sulphuric acid to convert the thiophene into its sulphonic acid, which is soluble in concentrated sulphuric acid.

(2) Treatment with bromine, followed by caustic soda (also by Victor Meyer).[26]

(3) Heating with aluminium chloride to decompose the thiophene.[29, 30, 31, 38]

(4) Chlorination of the thiophene with chlorine water or gaseous chlorine.[32, 33, 34]

(5) Condensation by treatment with formaldehyde, acetaldehyde, or phthalic anhydride, together with a condensing agent.[28, 35]

(6) Treatment with alkaline permanganate.[36] Disputed by Russig.[28]

(7) Mixing with an inert gas and subjecting to high temperature (1000°F.).[37]

(8) Submitting to high pressure and temperature in an autoclave.[40]

(9) Submitting the vapour mixed with hydrogen to high temperature and pressure in the presence of a catalyst, such as molybdenum trioxide or zinc oxide.[41]

(10) Passing the vapour mixed with air over a catalyst, such as vanadium pentoxide.[42]

(11) Bringing in contact with "neutral iron hydroxide."[43]

(12) Treatment with dry hydrogen iodide vapour.[44]

(13) Freezing the benzene followed by centrifuging or pressing.[28]

Thiophene is a liquid with a faint odour resembling that of benzene.

It boils at 80.2° and freezes at −37.1°. Like benzene, it forms sulphonic acids, nitro-compounds, halides, ketones, alcohols, etc., hence the difficulty of their separation.

The demand for thiophene-free benzene exists for the following reasons:—(1) Dye manufacturers require pure benzene and pure toluene to produce aniline and toluidine free from a yellow tint, which later on would give trouble.[5, 20] (2) Thiophene is thought to be the cause of the darkening of aniline exposed to light. (3) Benzene containing sulphur compounds causes corrosion when used as a fuel for internal-combustion engines. (4) Thiophene gives trouble in Auger's process for manufacturing chlorobenzene by forming a coating on the iron filings.[24] (5) Specially pure chemicals prepared on a small scale require thiophene-free benzene.

With respect to the darkening of aniline by light, however, our experiments show that even when thiophene-free benzene is used, and the nitrobenzene is reduced by tin in place of iron, the aniline slowly darkens when kept in the sunshine. Possibly traces of other sulphur compounds, *e.g.*, carbon disulphide, may also cause darkening.

For the detection of thiophene in benzene the indophenine test is the best known.[1, 6, 28] Several modifications of it are to be found in the literature.[8, 10, 11, 13] Schwalbe developed a quantitative colorimetric method based on this test.[17] We found the following method of making the test very satisfactory, and were able to detect in this way as little as one part of thiophene in 125,000 pts. of benzene.

A trace of isatin is placed in a test-tube, about 1 cc. of the benzene to be tested is added, followed by 2 cc. of concentrated sulphuric acid. After mixing thoroughly, the mixture is kept. In the presence of thiophene a deep blue colour soon develops. With diminishing amounts of thiophene the colour becomes pale green, and finally greenish-yellow as the thiophene nears the lower limit.

In our preliminary work, carried out with the assistance of Mr. E. D. Jackson and Mr. R. E. Bertram, we set out to check up some of the unsuccessful attempts made in the past to remove thiophene from benzene. The following summary gives a brief outline of this portion of our work.

(1) Passing the benzene-thiophene vapour mixed with hydrogen over reduced nickel gave a product free from thiophene, but about 30 cc. was the most that could be collected before the catalyst failed to remove all the thiophene. This poisoning of the nickel catalyst by thiophene has already been noted by Kubota and Yoshikawa,[45] also by Kubota and Kiyoshi.[46] Varying in temperature and attempting to protect the nickel by placing a column of reduced copper ahead of it failed to prevent either poisoning of the nickel or hydrogenation of the benzene.

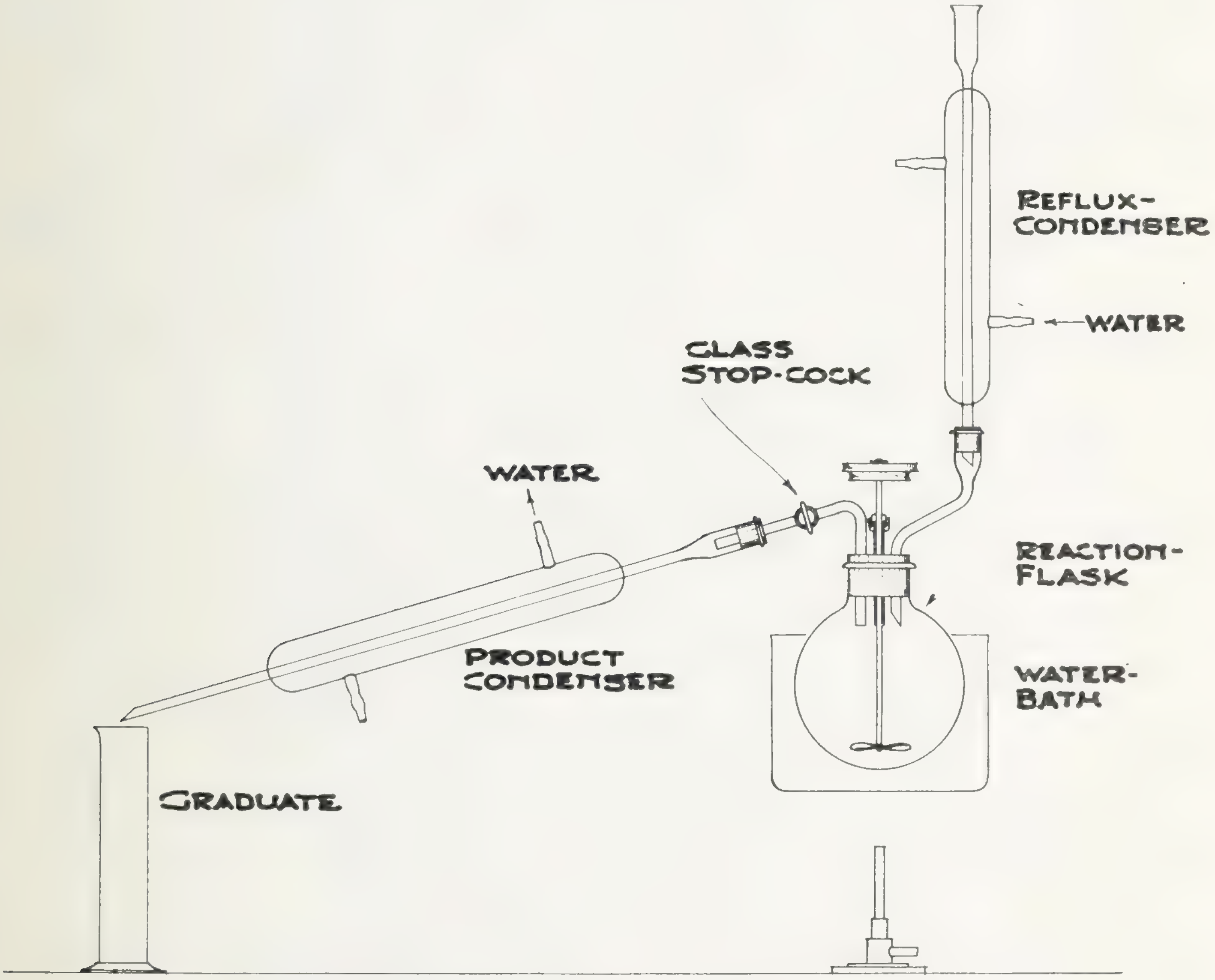
All of the following attempts also failed to remove the thiophene:—

(2) Passing the vapour mixed with air over copper oxide at various temperatures.

(3) Passing the vapour mixed with hydrogen over reduced copper at various temperatures.

(5) Passing the vapour mixed with nitrogen through a glass tube heated at 760°.

(5) Passing the vapour through bauxite heated at 320°.



APPARATUS FOR REMOVING THIOPHENE FROM BENZENE

(6) Passing the vapour through pure alumina (made from alum and ammonia and dried at 115°), heated at 320°.

(7) Passing the vapour through a mixture of copper oxide and lime made by adding an excess of lime to copper sulphate solution.

(8) Heating the liquid at 200° in a sealed glass tube containing a roll of copper gauze.

(9) The same as (8), except that mercury had been deposited electrolytically on about one-third of the copper.

(10) Agitating the liquid at room temperature with saturated lead acetate solution made strongly alkaline with caustic soda.[38, 39]

(11) The same as (10), but with the addition of flowers of sulphur.[38, 39]

(12) Agitation with 3% solution of hydrogen peroxide.[4]

(13) Freezing a portion of the benzene and pouring off the liquid. Repeating the freezing and decanting three or four times.

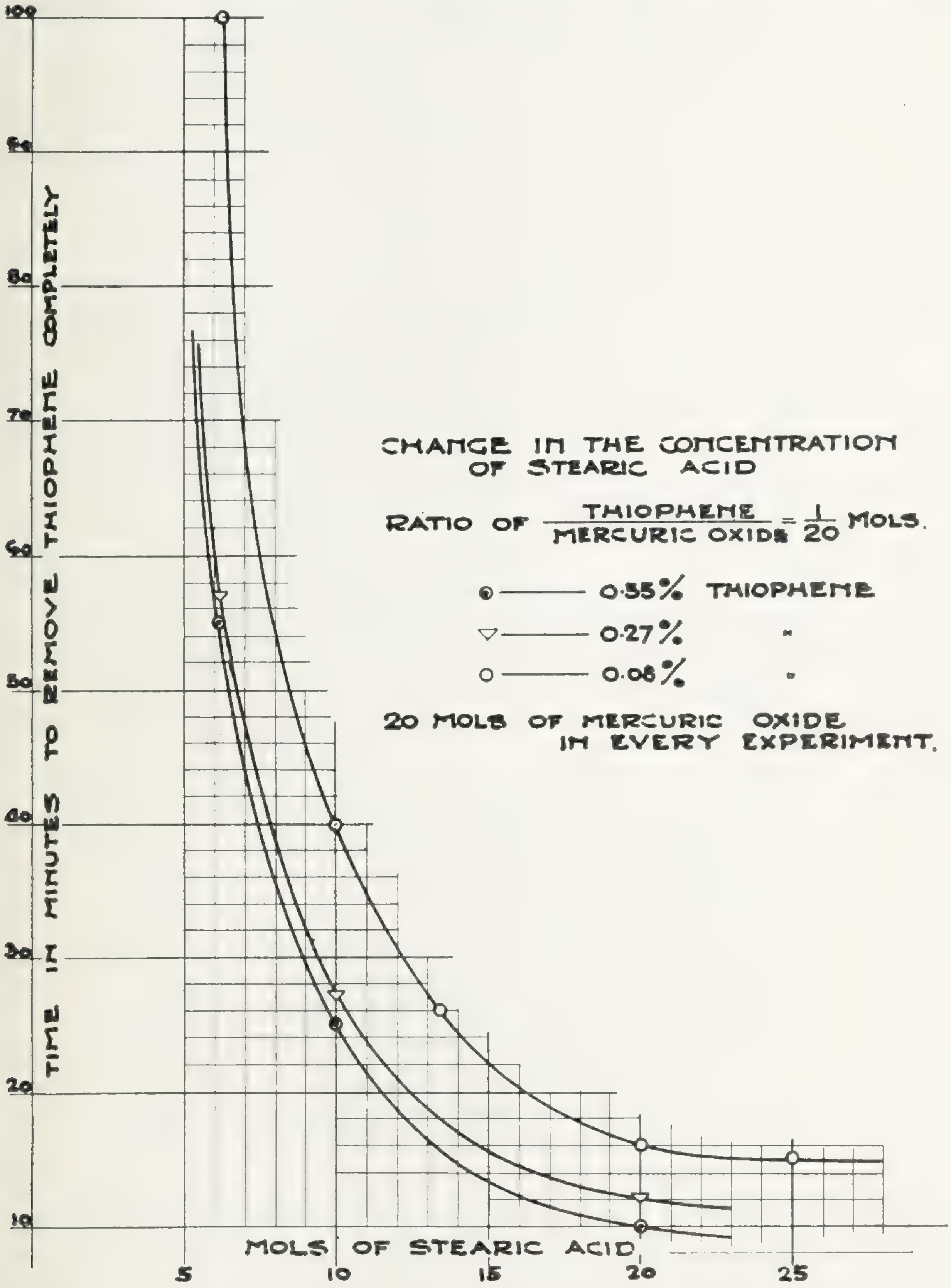
One well-known firm of chemical manufacturers claims to make thiophene-free benzene by method (13). No doubt a process of stirring while freezing in order to produce small crystals, followed by centrifuging and washing the crystals with thiophene-free benzene, would form the basis of a successful method, but it seems probable that a subsequent treatment, *e.g.*, with a small percentage of concentrated sulphuric acid, would always be necessary to remove the last trace of thiophene.

It was Denigès[14, 15] who discovered the reaction between mercury salts and thiophene which results in the formation of compounds insoluble in the most usual solvents. Denigès developed this discovery into a gravimetric method for determining thiophene in benzene. He used mercuric sulphate containing free sulphuric acid, which gives with thiophene a precipitate to which he assigned the formula $(HgSO_4,HgO)_2, C_4H_4S,H_2O$. When dried at 110—115° the precipitate loses the combined water, and if weighed at this stage contains 7.58% of thiophene. If methyl alcohol (free from acetone) is used, as recommended subsequently by Denigès,[15] the mercuric sulphate must be present in large excess, otherwise a precipitate containing 10.33% of thiophene is formed. To this precipitate containing 10.33% of thiophene Denigès assigned the formula $SO_4(HgO)_2.Hg.C_4H_4S$. Later, Dimroth,[16] using the acetate, assigned the formula $C_4H_2S(HgOAc).HgOH$ to his precipitate. Paolini, [18, 19] also using the acetate, finds Dimroth's formula incorrect, and assigns instead the formula $C_4H_4S(HgOAc)_4$. Several other investigators[23, 24] have added to our knowledge in this field.

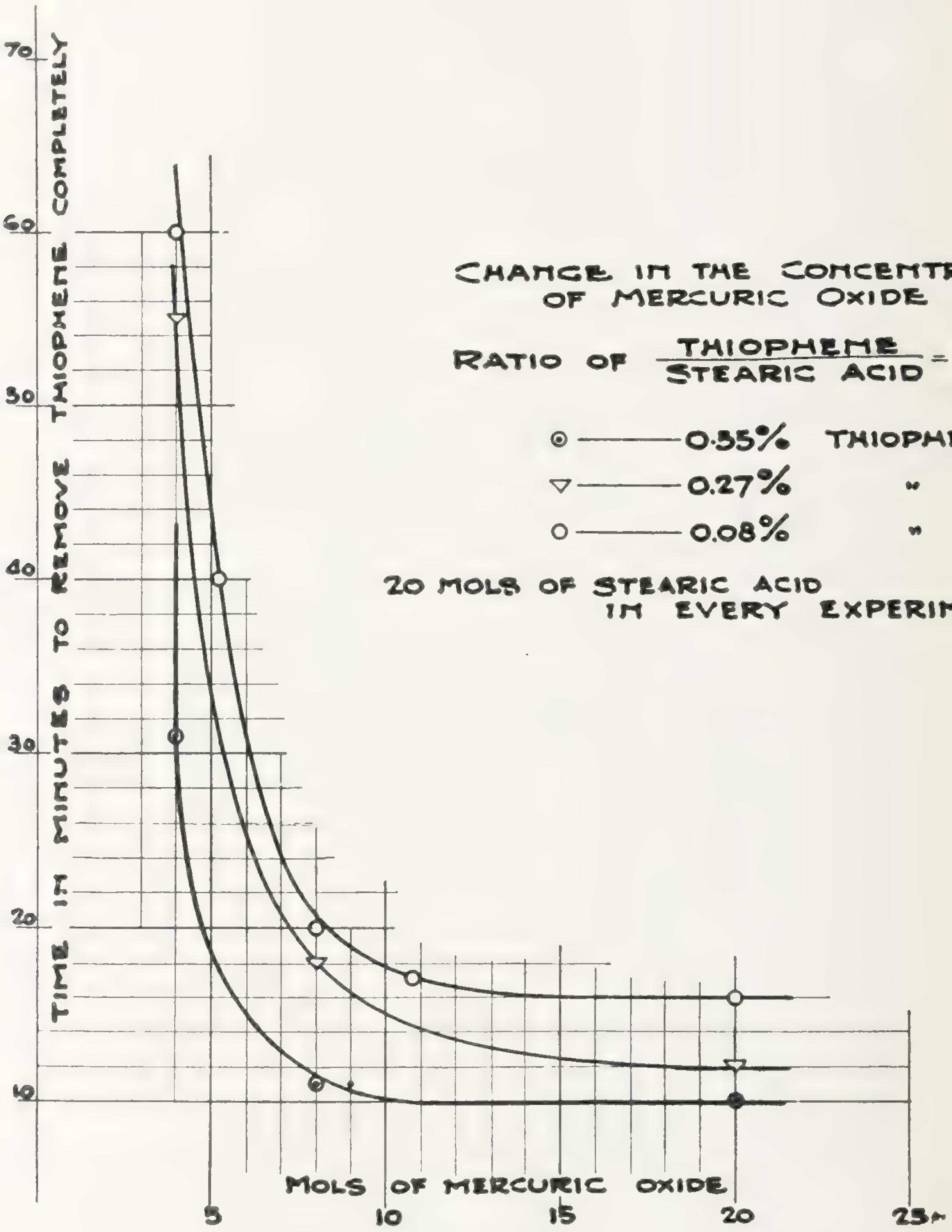
It occurred to the authors that the mercury salt-thiophene reactions might be used as a basis for the development of a simple method by which a thiophene-free benzene could be made in one operation on a laboratory scale. In this attempt they have succeeded, as the following results indicate.

In place of using the sulphate or acetate, the stearate was selected in order to avoid the carrying over into the distillate of a volatile acid, and in the second place because mercuric stearate comprises a molecule one end of which is soluble in water, the other in benzene. From the analogy of Twitchell's process, such a compound ought to acclerate the reaction between the mercury salt and the thiophene dissolved in the benzene.

Preliminary experiments by Mr. R. E. Bertram revealed the fact that a distillate of benzene free from thiophene can be obtained simply by adding red mercuric oxide and stearic acid to the benzene, heating on a water-bath for some time, and then distilling the benzene without removing the mercury compound. The mercuric oxide-stearic acid-thiophene compound is sufficiently soluble in benzene to give the indophenine test, but since on distilling the benzene the distillate is found to be thiophene-free, the compound at the boiling point of benzene is evidently not decomposed and is not carried over in the benzene vapour. When the mercuric oxide and stearic acid are heated together to 170—180° to form the basic mercury stearate and the product is added to the benzene, we found, to our surprise, that the thiophene is not removed from the benzene.



When large amounts of mercuric oxide and stearic acid are used, considerable foaming takes place, especially when the mercuric oxide is present in excess. To reduce the amount of foam and to minimize the danger of carrying the foam into the condenser, we used a two-litre flask for 100 cc. of benzene. Approximately one-third of the flask was immersed in the water of the bath, and the stirrer was occasionally raised to break the foam. Then by allowing cold water to drop on to a piece of cloth wrapped about the upper part of the flask, further foaming was prevented. The foam is produced during the mercuric oxide-thiophene-stearic acid reaction, since foaming does not occur when thiophene-free benzene is substituted.



As one would expect, stirring the benzene during the course of the treatment decreases the time required to remove the thiophene completely. We determined the time required by making the indophenine test at about 10 min. intervals on 1 cc. samples obtained by opening the cock on the tube leading from the reaction flask to the condenser long enough to permit the collection of 2 or 3 cc. of distillate. Using 100 cc. of benzene and a temperature of 80° (b.p. of benzene) 180 min. were necessary without stirring, but only 90 min. with stirring.

The effect of water was quite marked. To 100 cc. of benzene 2 cc. of water were added. The time required was 315 min. When the benzene and the reagents were carefully dried, the time requires was 105 min. When a trace of moisture was present by using benzene and reagents undried, the time required was 80 min.

In consequence, in all our experiments we used a stirrer and did not dry the benzene and the reagents. The thiophene-free benzene obtained was, as a result, sometimes slightly milky from the traces of water carried over. Shaking with a little calcium chloride removed this very readily.

To obtain a clear, water-free distillate, the mercuric oxide and stearic acid concentrations should be kept below (or at least not above) 8 mols. of mercuric oxide and 10 mols. of stearic acid to 1 mol. of thiophene. As the tabulated results and curves show, these proportions will, furthermore, remove the thiophene in almost as short a time as much larger quantities of the two reagents will require.

In the following experiments, performed to determine the influence of the relative concentrations of the reagents on the rate of the reaction, the water-bath was kept at 97—98° and the time recorded from the moment the benzene began to boil. The quantity of benzene used in every experiment was 100 cc.

All attempts to cheapen the method by substituting magnesium oxide, zinc oxide, or lead oxide for the expensive mercuric oxide gave negative results.

% thiophene in the benzene	Ratio in mols.			Minutes required to remove the thiophene
	Thiophene	Mercuric oxide	Stearic acid	
0.55	1	20	20	10
0.55	1	20	10	25
0.55	1	20	6.13	55
0.55	1	8.06	20	11
0.55	1	4.03	20	31
0.55	1	2.68	20	420*
0.27	1	20	20	12
0.27	1	20	10	27
0.27	1	20	6.13	57
0.27	1	8.06	20	18
0.27	1	4.03	20	55
0.08	1	20	20	16
0.08	1	20	30	15
0.08	1	20	13.33	26
0.08	1	20	10	40
0.08	1	20	6.13	109
0.08	1	20	4	400*
0.08	1	10.75	20	17
0.08	1	5.37	20	40
0.08	1	8.06	20	20
0.08	1	4.03	20	60

*Not completed.

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LIST OF REFERENCES

A—General articles

(1) MEYER, VICTOR—*Ber.* **15**, 2893 (1882): *J. Soc. Chem. Ind.* **2**, 37 (1883).

(2) MEYER, VICTOR—*Ber.* **16**, 1465 (1883): *J. Soc. Chem. Ind.* **2**, 325.

(3) THORPE, T. E.—*J. Chem. Soc.* **77**, 189 (1900) Victor Meyer Memorial Lecture.

(4) THORPE, SIR EDWARD—Dictionary of Applied Chemistry: 1927 Ed., Vol. VII, p. 31.

(5) BIDET, A.—*Compt. rend.* **108**, 520 (1889): *J. Soc. Chem. Ind.* **8**, 383.

B—Detection of thiophene

(6) MEYER, VICTOR—*Ber.* **16**, 1468, 1472 (1883): *J. Soc. Chem. Ind.* **2**, 325.

(7) LIEBERMANN, C.—*Ber.* **20**, 3231 (1887): *J. Soc. Chem. Ind.* **7**, 30 (1888).

(8) CLAISEN, L.—*Ber.* **20**, 2197 (1887).

(9) KREIS, H.—*Chem. Ztg.* **26**, 523 (1902): *J. Soc. Chem. Ind.* **21**, 930.

(10) BAUER, F. W.—*Ber.* **37**, 1244, 3128 (1904): *J. Soc. Chem. Ind.* **23**, 562, 1047.

(11) SCHWALBE, C.—*Ber.* **37**, 324 (1904): *J. Soc. Chem. Ind.* **23**, 273.

(12) FLEIG, C.—*Compt. rend. Soc. Biol.* **65**, 283 (1908): *C. A.* **3**, 429.

(13) WRAY, E.—*J. Soc. Chem. Ind.* **38**, 83T (1919).

C—Determination of thiophene

(14) DENIGÈS, G.—*Bull. Soc. chim. de France* [III], **13**, 538 (1895): *J. Soc. Chem. Ind.* **14**, 893.

(15) DENIGÈS, G.—*Bull. Soc. chim. de France* [III], **15**, 862, 1064 (1896): *J. Soc. Chem. Ind.* **15**, 746.

(16) DIMROTH, O.—*Ber.* **32**, 758 (1899), **35**, 2035 (1902).

(17) SCHWALBE, C.—*Chem. Ztg.* **29**, 895 (1905): *J. Soc. Chem. Ind.* **24**, 988.

(18) PAOLINI, V.—*Gazz. Chim. Ital.* **37**, I, 58 (1907): *J. Soc. Chem. Ind.* **26**, 957.

(19) PAOLINI and SILBERMAN—*Ann. Chim. Applicata* **4**, 289 (1915): *C. A.* **10**, 580: *J. Chem. Soc.* **108**, II, 806 (1915).

(20) ELLERTON, J. G.—*J. Soc. Chem. Ind.* **31**, 10 (1912).

(21) DENIGÈS, G.—*Bull. Soc. chim. de France* [IV], **17**, 353 (1915): *J. Soc. Chem. Ind.* **34**, 1142.

(22) JONES, F. B.—*J. Soc. Chem. Ind.* **37**, 324T (1918).

(23) SPIELMAN and SCHOTZ—*J. Soc. Chem. Ind.* **38**, 188T (1919).

(24) MEYER, ANDRÉ—*Compt. rend.* **169**, 1402 (1919): *C. A.* **14**, 912 (1920).

(25) LUNGE and KEANE—Technical Methods of Chemical Analysis 1911 Ed., Vol. II, Part II, p. 797.

D—Removal of thiophene from benzene

(26) MEYER, VICTOR—*Ber.* **18**, 1489 (1885):

(27) FRIEDLÄNDER, P.—Fortschritte der Teerfarbenfabrikation **9**, 96 (1908-1910): also **4**, 31 (1894-1897).

(28) LUNGE, G.—Coal Tar and Ammonia—1916 Ed., Vol. II, pp. 937, 984.

(29) HALLER and MICHEL—*Bull. Soc. chim. de France* [III], **15**, 390, 1065 (1896): *J. Soc. Chem. Ind.* **15**, 706.

(30) HEUSLER, F.—*Z. angew. Chem.* **9**, 288, 318, 750 (1896).
(31) SOC. ANON. ST. DENIS—French pat. 240,111, German pat. 79,505 (1894): *Z. angew Chem.* **8**, 114 (1895).
(32) WILLGERODT, C.—*J. prakt. Chem.* [3], **33**, 479 (1886): *J. Soc. Chem. Ind.* **5**, 480.
(33) DUTT and HAMER—English pat. 117,683 (1917): *J. Soc. Chem. Ind.* **37**, 540A (1918).
(34) HAMER, J. D.—*Gas World*, **68**, 13 (1918): *C.A.* **12**, 1828.
(35) BADISCHE ANILIN S. F.—German pat. 211,239 (1907): *J. Soc. Chem. Ind.* **28**, 790 (1909).
(36) DONATH and DITZ—*J. prakt. Chem.* [3], **60**, 575 (1899): *J. Soc. Chem. Ind.* **19**, 181 (1900).
(37) EVANS, O. B.—English pat. 112,878 (1917): *C. A.* **12**, 1118 (1918).
(38) WOOD. LOWY and FARAGHER—*Ind. Eng. Chem.* **16**, 1116 (1924).
(39) WOOD. SHEELY and TRUSTY—*Ind. Eng. Chem.* **18**, 169 (1926).
(40) DUNKEL, M.—*Brennstoff-Chem.* **5**, 145 (1924): *C.A.* **18**, 2597.
(41) I. G. FARBENINDUSTRIE A. G.—English pat. 258,576 (1925): *C. A.* **21**, 3123 (1927).
(42) DOWNS, C. R.—U. S. pat. 1,590,965 (1926): *C. A.* **20**, 3015.
(43) REESON and MOSS—English pat. 249,312 (1925): *C.A.* **21**, 972.
(44) NELLENSTEYN, F. J.—*Chem. Weekblad* **24**, 102 (1927): *C.A.* **21**, 2553.

E—Poisoning of nickel catalyst by thiophene

(45) KUBOTA and YOSHIKAWA—*Sci. Papers Inst. Phys. Chem. Research, Tokyo, Japan*, **3**, 33 (1925): *C.A.* **19**, 2771.
(46) KUBOTA and KIYOSHI—*Japan J. Chem.* **2**, 45 (1925): *C.A.* **20**, 860.